Concurrent Trajectory and Vehicle Optimization: A Case Study of Earth-Moon Supply Chain Logistics

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The objective of this paper is to demonstrate an integrated system design optimization approach for space system networks. The decisions made during the initial design phases for a complex system, will drastically affect the final product at the architectural level. Traditionally, the design process for a complex system involves the sequential design of the sub-system components, which may lead to a sub-optimal system design. In systems with a high degree of sub-system coupling, the ordering of the sub-system design decisions indirectly determines the priority of the sub-systems to the system. In addition, with the announcement of the space exploration initiative, the goal is to design a sustainable space exploration system that can accomplish multiple missions in an environment of uncertainty. As such, we can no longer consider each mission separately, and therefore must integrate the multiple missions into the initial design of the system. By viewing the set of missions as an integrated space network, we add another sub-system to the system design space. A systems level solution to this problem requires that the network, the vehicle, and the trajectory design, be considered concurrently. To illustrate a concurrent design optimization methodology, this paper considers the design of an Earth-Moon supply chain. Specifically, we consider the problem of delivering cargo units of water from low Earth orbit to lunar orbit and the lunar surface. The formulation requires that the architectural characteristics of the vehicle used to transport the packages to the destinations and the paths the vehicles travel be determined concurrently. The problem is solved using both traditional design optimization methods and a concurrent design optimization method. The vehicle optimization and concurrent optimization both achieve a minimum system mass of 58,768 kg for a single vehicle design, which is an improvement of 114,070 kg from the network optimization solution. The system objective is further reduced to 47,498 kg when multiple vehicles are designed. Initial investigations into the sensitivity of the solution to changes in demand reveal that the minimum system mass is obtained when direct routes to the demands nodes are travelled.

Nomenclature

LEO	Low Earth orbit	n_{ijk}	Number of vehicles on route (i,j,k)
EML1	First Earth-Moon Lagrange point	C_{ijk}	Capacity of vehicle on route (i,j,k)
HLO	High elliptic equatorial Lunar orbit	x_{ijk}	Number of packages on route (i,j,k)
LLO	Low circular equatorial Lunar orbit	$m_{o_{ijk}}$	Vehicle initial mass
LES	Lunar equatorial surface	$m_{w_{ijk}}$	Vehicle wet mass
LPS	Lunar polar surface	m_{pl}	Payload mass (1000 kg
ΔV	Velocity change	I_{sp}	Specific impulse (sec)

(i, j, k) Transfer starting at node i traveling to node j and terminating at node k

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I. Introduction

With the announcement of the space exploration initiative,[1] we are given a new set of challenges for designing a space transportation system. The directive given by President Bush dictates the design of a sustainable space exploration system for the Moon, Mars, and 'beyond'. This open-ended directive raises the question of how to design a space transportation system that can be maintained over an extended life-cycle, remain within the budget profile for design, development, and operation, and handle the uncertainties in the political, operational and technical realms. Given that the space exploration system has multiple mission objectives, in order to develop a sustainable exploration system it is necessary to recognize the inter-dependencies between missions. By viewing the set of missions together as an integrated network and examining how both the network of missions, and the transportation architecture are affected by each mission's objective will allow the space exploration system to fulfill the directives set forth.

Traditionally, a space transportation system is designed for a single mission objective. The first step in designing a 'point-design' mission is to select the type of vehicle used to transport the payload. Then, using these vehicle characteristics, an optimal trajectory is calculated subject to the limitations of the vehicle. There exists an extensive amount of literature detailing the computation of optimal orbit transfers, however, for the analysis considered in this work it is sufficient to use the astrodynamic relations presented in Reference 2. Using the approximations for impulsive trajectories, the velocity change (ΔV) required to transfer orbits can be computed, independent of the vehicle characteristics.

Although the astrodynamic equations decouple the vehicle and trajectory for the purpose of computing the transfer ΔV, it is important to realize that from a systems perspective, these two sub-systems possess a high degree of coupling. Therefore, when designing the optimal vehicle for a mission it is necessary to consider the trajectory and vehicle concurrently, as in Reference 3. Through the concurrent design of the trajectory and vehicle a systems level solution to the problem is obtained, however the solution is optimal only for the single mission objective.

Transportation networks are defined by two components, the vehicle and the operations, which are strongly coupled. In Reference 4, the design of an aircraft for multiple operations is examined. In this paper, the operations, namely the specified routes are defined, each with a different distance and demand. The objective is to determine the best vehicle design that satisfies the network demand. The resulting aircraft design is not optimal for a single route, but is the best compromise in range and capacity for the system as a whole. In Reference 5, the design of an air transportation network is considered. Given a set of vehicles with different ranges, capacities, and costs, the objective is to minimize the total network costs by choosing the routes through the network and allocating the appropriate vehicles to meet the given package demand. However, to develop a space transportation network, both the vehicles and the operations must be defined concurrently.

Inherent to the problem of transporting people to the Moon, Mars, and 'beyond' is sustaining the people and the operations while in transit and at the respective destinations. Especially for long-term missions, the amount of consumables required becomes a significant issue in terms of mass in LEO. In order to develop a sustainable space transportation architecture it is critical that interplanetary supply chain logistics be considered.

The goal of the supply chain logistics problem is to adequately account for and optimize the transfer of supplies from Earth to locations in space. Although the consideration of the supplies is of high importance, the commodities themselves may be of low value on Earth. As such, it is desirable to find the cheapest way to transport these supplies. The Aquarius project investigated the delivery of 1000 kg packages of water from Earth to LEO using a cheap, low-reliability launch system. Using low-priority transportation networks on Earth, such as rail and barge transportation systems, and the design of a single-stage to orbit launch vehicle, they were able to show a significant reduction in launch costs.[6]

The case study presented in this paper begins where the Aquarius project left off, in low Earth orbit. The objective is to minimize the total system mass required to deliver 1000 kg packages from LEO to lunar destinations. Using a simplified Earth-Moon network, we define the trajectory between each node and compute the transfer ΔV prior to the optimization. The design vector consists of the routes through the network, the vehicle capacities, vehicle staging locations, and the fuel used in each stage.

Before formalizing the problem, an overview of space system networks is provided in Section II. Following this, the problem formulation, including the network and vehicle models is given in Section III. Section IV outlines the traditional design approach for both network optimization and vehicle optimization, and presents the results obtained for the example problem. Section V compares the results obtained from the concurrent optimization to the results of the traditional optimization methods and discusses the sensitivity of the results to changes in demand. Section VI reviews the ideas and results presented and discusses future areas of work.

II. Space System Networks

The field of operations research has been analyzing and developing methods for solving transportation network problems on Earth.[7,8] In References 4 and 5, air transportation networks were analyzed for vehicle design and network design, respectively. However, space transportation networks differ significantly from air transportation networks, as can be seen in Figure 1. To further understand the complexity of space system networks, a comparison to air transportation networks is presented.

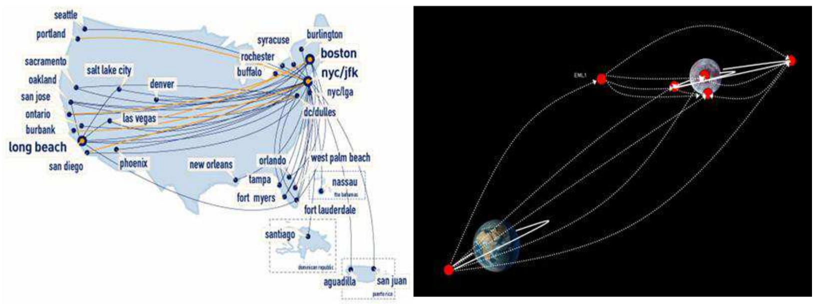

Figure 1. Representation of Air Transportation[9] and Space Transportation Networks

In transportation networks, the network definition consists of the number of nodes, the supply and demand at the nodes, the arcs between the nodes, and the costs and capacities of the arcs. In air transportation networks, the nodes represent cities, and each node may have a supply and demand. The arcs represent the routes between cities, and it is reasonable to assume a single route between two nodes. The cost of the arc can be represented as a linear function of the distance between the nodes, and therefore remains constant for a given vehicle. In space transportation networks, there are few nodes, and in general, we can assume a single supply node, Earth. In space networks, the arc cost is dependent on the ΔV between two nodes, which is time dependent. In addition, there are multiple trajectories that can be traveled between two nodes, which determines the ΔV for the transfer. Therefore, for space transportation networks, we assume that there are multiple arcs between two nodes and the arc costs are a nonlinear function of the time dependent ΔV between the nodes. The capacities on the arcs in both air and space networks are a product of the vehicle capacity and the number of vehicles.

The next important distinction is in the packages transported by the vehicles in the network. In both air and space networks, we can analyze the transfer of multiple commodities through the network. However, the distinction between the commodities in space transportation networks is more important. In air transportation networks, by virtue of the fact that we are transporting the commodity in an aircraft, rather than on a barge, we assume that the commodity is of high priority. In space transportation, all commodities, must be transferred by a spacecraft, and it is the choice of trajectory that reflects the package priority. Therefore we must explicitly denote the priority of different commodities in space transportation networks.

The final distinction between air and space networks lies in the vehicle definition and operations. In air transportation problems, the vehicle is assumed to be reusable and we can define the operations such that the system can repeat the mission continuously. In space transportation problems, the vehicle is assumed to be expendable. If a reusable space transportation system is desired, an entirely different set of modeling assumptions for the vehicle definition are required. In addition, a change in the operations for a space transportation system, once it leaves Earth, is generally not feasible, which increases the importance of both an integrated space transportation network analysis as well as the consideration of robustness to uncertainty in the model. Figure 2 summarizes the above modeling differences.

Network Definitions	Air Transportation Networks	Space Transportation Networks
Arc Costs	Arc-costs are dependant on distance between nodes	Arc-costs are dependant on ΔV between nodes
	Travel between nodes is time-invariant → Arc-costs are constant	Travel between nodes is highly time dependent → Arc-costs are a function of time
Graph Definition	Assume one arc between two nodes	Multiple arcs are available between two nodes (i.e. multiple trajectories w/ different ΔV's)
Node Balance	Assume multiple supply and demand nodes	Single source node (Earth) and multiple demand nodes
Commodities	Multiple commodities available	Multiple commodities available
	Assume commodities have same priority	Commodities may not be of the same priority.
Vehicles	Vehicles are assumed to be reusable→ vehicle can be returned to origin node	Vehicles are often expendable → vehicle does not return from destination node. Reusability requires different modeling assumptions
Operations	Network can easily be re-defined if a change in operation is desired	Difficult to restructure network for operation changes.

Figure 2. Comparison of Aircraft Transportation Networks and Space Transportation Networks

III. Problem Formulation

In this example, the Earth-Moon network consists of six nodes: low Earth orbit (LEO, node 1), the first Earth-Moon Lagrange point (EML1, node 2), a high-apoapsis elliptical equatorial lunar orbit (HLO, node 3), a low circular equatorial lunar orbit (LLO, node 4), the lunar equatorial surface (LES, node 5), and the lunar south polar surface (LPS, node 6). The nodes are topologically ordered such that only forward arcs ($i < j$) and self-arcs ($i = j$) exist in the network. In addition, lunar surface transfers are excluded. The network is depicted in Figure 3.

We assume an impulsive trajectory prior to analysis and compute the required ΔV for each allowable transfer. In the Earth-Moon system, it is reasonable to assume that the ΔV values are independent of time, however if the system were to be expanded to include other destinations, such as Mars, this assumption would no longer hold. The ΔV values are provided in Table 1.

Referring to Table 1, there are two values of ΔV for each transfer arc. The first burn allows the vehicle to enter the arc and the second burn allows the vehicle to exist the arc. For example, to transfer from LEO to LLO, a trans-lunar injection burn is performed with a ΔV of 3150 m/s and an orbit insertion burn is performed at lunar orbit with a ΔV of 850 m/s. The transfer from high lunar orbit (HLO) to low lunar orbit (LLO) consists of a single burn since the two orbits have the same periapsis.

To reach lunar orbit and the lunar surface, the vehicle can transfer directly from LEO or can transfer through another node in the network. In this example, we restrict all vehicles to begin the transfer in LEO, and allow the vehicle to visit two nodes following the initial node. For the remainder of this paper we shall note that (i, j, k) is a route that starts at node i, travels to node j and terminates at node k. If the vehicle transfers to two nodes following LEO ($j < k$) then we assume the vehicle stops at node j and can drop of payload, if desired. For a direct transfer, the intermediate node and the destination node are the same ($j = k$).

Using the network defined in Figure 3 the problem is to determine the routes through the network and

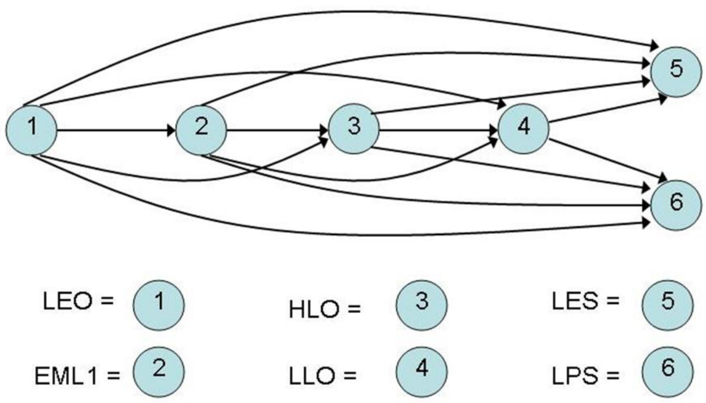

Figure 3. Earth-Moon Transportation Network

Table 1. Transfer ΔV (m/s)[10]

NODES		EML1	HLO	LLO	LES	LPS
LEO	burn 1	3100	3150	3150	3150	3150
	burn 2	750	336	850	2715	2715
EML1	burn 1		248	248	248	248
	burn 2		118	632	2497	2497
HLO	burn 1			514	514	987
	burn 2				1865	1865
LLO	burn 1				30	233
	burn 2				1865	1865

the vehicle configurations on each route that minimize the total system wet mass. Figure 4 shows the relationship of the models used to define the transportation system and a description of each model follows.

A. Network Model Formulation

The network sub-system determines the actual package flows from LEO to the destination nodes. To ensure a feasible package flow, we must define the supply, demand, and capacity constraints for the network. The supply constraints ensure that the number of packages (x_{ijk}) that leave node i are equal to the supply at node i $(s\,(i))$. If we consider LEO to be the only source in the network, we have a single supply constraint, as defined in Equation 1.

$$\sum_{j=1}^{n}\sum_{k=j}^{n} x_{1jk} = s\,(1) \tag{1}$$

Similarly, the demand constraints ensure that the number of packages that arrive at node k are equal to the demand of node k $(d\,(k))$.

$$\sum_{i=1}^{n}\sum_{j=i}^{k} x_{ijk} = d\,(k) \quad \forall k = 2, \ldots, n \tag{2}$$

In addition to the supply and demand constraints, we need to ensure that the number of packages do not exceed the capacity of the route. The capacity of a route is the product of the number of vehicles on

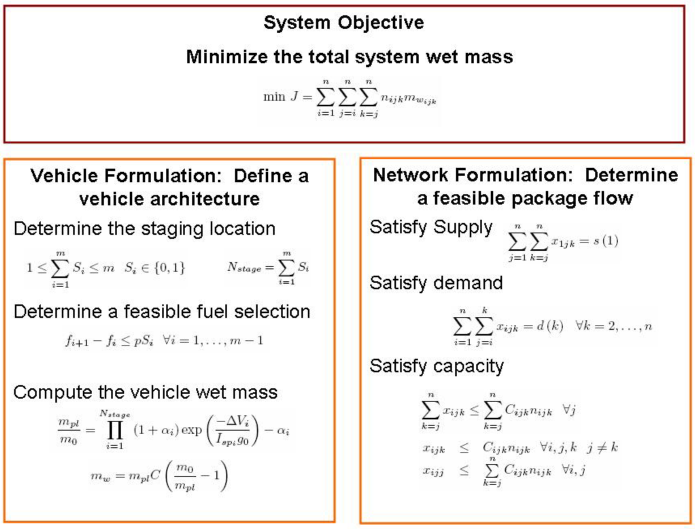

Figure 4. Concurrent Optimization Models

that route (n_{ijk}) and the capacity of the vehicle (C_{ijk}). In most cases, this is simply included as an upper bound on the number of packages on a given route (x_{ijk}). However, at the second node (node j), a vehicle can 'drop-off' packages and continue to its final destination. Equation 3 ensures that the vehicle has enough capacity to accommodate these packages.

$$\sum_{k=j}^{n} x_{ijk} \leq \sum_{k=j}^{n} C_{ijk} n_{ijk} \quad \forall j \tag{3}$$

The upper bound on the number of packages on each route enforce the remaining capacity constraints.

$$\begin{aligned} x_{ijk} &\leq C_{ijk} n_{ijk} \quad \forall i,j,k \quad j \neq k \\ x_{ijj} &\leq \sum_{k=j}^{n} C_{ijk} n_{ijk} \quad \forall i,j \end{aligned} \tag{4}$$

B. Vehicle Model Formulation

The vehicle sub-system determines the architectural characteristics of the vehicle, namely the number of burns executed by each stage and the fuel used in each stage. For the purpose of this analysis, we consider a simplified model of the spacecraft and calculate the initial mass of the vehicle. Figure 5 provides a schematic of the vehicle design. All structural components are combined into a single mass value (m_{struc}) and related to the fuel mass (m_{fuel}) through a simple proportion (α).

$$m_{struc} = \alpha m_{fuel} \tag{5}$$

We define three different types of fuels that can be chosen for each stage: liquid oxygen and kerosene ($LOX - RP1$), monomethyl hydrazine and nitrogen tetroxide ($MMH - N_2O_4$), and liquid oxygen and liquid hydrogen ($LOX - LH_2$). Table 2 lists the corresponding values of specific impulse (I_{sp}) and structural factor (α) for each fuel type. The values for each fuel property are taken from actual engine data provided in Reference 11

Having defined the vehicle components in terms of the fuel choice, it is necessary to size the vehicle based on the amount of fuel required to perform the orbit transfer burns. In the case of impulsive burns, we assume that a single stage has enough fuel to perform the entire burn. However, since each transfer requires multiple burns, it is possible for the vehicle to drop the structural mass associated with the fuel used to perform a burn after the burn is completed. Given that for every route there is a vector of ΔV burns, $[\Delta V_{b_1} \ldots \Delta V_{b_m}]$,

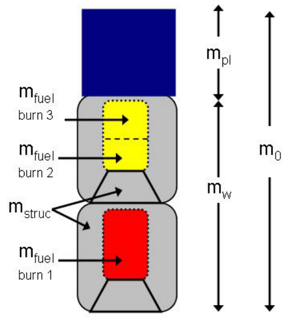

Figure 5. Schematic of Vehicle Model

Table 2. List of Chemical Fuels and Parameters

Type	Fuel	Specific Impulse	Structural Factor
1	$LOX - RP1$	290 sec	0.08
2	$MMH - N_2O_4$	330 sec	0.12
3	$LOX - LH_2$	450 sec	0.14

the problem at the vehicle level is to determine the location of the stages that minimizes the initial mass of the vehicle, given the capacity of the vehicle on that route. The following formulation applies to each route (i, j, k), however for clarity in the formulation, these subscripts will be omitted. Thus for each route, the vehicle has a payload mass equivalent to its capacity (C) and must determine the number of stages (N_{stage}), the location of the stages (S_i), and the type of fuel used for each stage (f_i).

We define a binary decision variable, S_i, which equals one if we stage after burn i and zero otherwise. We know that we can stage at most m times, where m is the total number of burns required for that route. In addition, we assume that the vehicle stages after the last burn ($S_m = 1$). Equation 6 formalizes this constraint.

$$1 \leq \sum_{i=1}^{m} S_i \leq m \quad S_i \in \{0, 1\} \tag{6}$$

In addition, the type of fuel used in each stage must be determined. Since the number of stages is unknown and the location of the stages is determined by the previous equation, we define the variable f_i to represent the type of fuel used during stage i. The variable f_i can take on integer values up to the number of different types of fuel available. In this model, we do not allow hybrid stages, so to ensure that the same type of fuel is used for consecutive burns in a single stage, we impose the following constraints.

$$f_{i+1} - f_i \leq pS_i \quad \forall i = 1, \ldots, m-1 \tag{7}$$

Here, p is the number of different types of fuel available, which in this problem is three (Table 2).

Having defined the architectural characteristics of the vehicle, the vehicle initial mass can be computed. Prior to this, we provide some initialization. First, the total number of stages is computed.

$$N_{stage} = \sum_{i=1}^{m} S_i \tag{8}$$

Using the staging locations, the amount of ΔV required for each stage (ΔV_i) can be defined. The amount of ΔV in a given stage is the sum of the ΔV for each burn up to and including the first burn for which the vehicle stages ($S_i = 1$). Finally, the initial mass (m_0) of the vehicle is calculated using the rocket equation (Equation 9).

$$\frac{m_{pl}}{m_0} = \prod_{i=1}^{N_{stage}} (1 + \alpha_i) \exp\left(\frac{-\Delta V_i}{I_{sp_i} g_0}\right) - \alpha_i \tag{9}$$

The vehicle is sized for the total capacity required on that route and neglects the mass benefits resulting from a multiple burn stage. For example, if the vehicle travels on route (1,4,5), delivers a package at node 4 and node 5, and does not stage until after the final burn, the vehicle has a capacity of two. Using Equation 9 we would compute the initial mass assuming a payload of two packages and a ΔV equal to the sum of all the burns ($\sum_{i=1}^{m} \Delta V_{b_i}$) on that route.

To vehicle wet mass is the mass of the structure and fuel without the payload mass. The vehicle wet mass is computed using Equation 10

$$m_w = m_{pl} C \left(\frac{m_0}{m_{pl}} - 1\right) \tag{10}$$

C. System Objective

The main objective of the system is to minimize the initial mass of the transportation system architecture, which is defined by Equation 11.

$$\min\ J = \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=j}^{n} n_{ijk} m_{0_{ijk}} \tag{11}$$

where n_{ijk} is the number of vehicles that start at node i travel to node j and then terminate at node k, and $m_{0_{ijk}}$ is the initial mass of a vehicle on route (i, j, k). However, the initial vehicle mass ($m_{0_{ijk}}$) is determined by the vehicle capacity for each route C_{ijk} and the actual initial mass is the wet mass ($m_{w_{ijk}}$) plus the amount of payload carried on that vehicle. Each route carries x_{ijk} packages that each weigh m_{pl}. Thus, for each route the initial mass is defined as $n_{ijk} m_{w_{ijk}} + x_{ijk} m_{pl}$ and this is summed over all routes. Upon closer inspection, we see that the summation of x_{ijk} over all routes is simply the amount of supply, which is a constant. Therefore the system objective can be re-written as Equation 12.

$$\min\ J = \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=j}^{n} n_{ijk} m_{w_{ijk}} \tag{12}$$

IV. Traditional Design Optimization

Using a traditional design optimization methodology, we have two approaches: define the vehicle and then optimize the network, and define the network and then optimize the vehicle. The first approach requires solving a transportation network problem. This approach was applied in Reference 5 to solve the aircraft allocation problem. The second approach is to design a vehicle that can satisfy the requirements of multiple missions, and mimics the approach applied in Reference 4. Each of these approaches require a decision about the system be made prior to the design optimization. To compare the different methodologies, we will consider the problem of transporting two packages to low lunar orbit (LLO, node 4), two packages to the lunar equatorial surface (LES, node 5), and two packages to the lunar polar surface (LPS, node 6).

A. Network Design Optimization

The network transportation problem requires that the arc capacities and costs be included with the supply and demand as part of the problem definition. First, we consider a single vehicle design and determine the optimal allocation of vehicles through the network. Since we do not know which routes the vehicles will travel, and therefore the required ΔV, we size the vehicles to be capable of traveling on most routes. To compute the wet mass of the vehicle, we must define the capacity, staging, and fuel. Since a single vehicle can transport packages to two nodes, we set the vehicle capacity to the upper limit, three packages. We will assume that the vehicle only stages after the last burn and uses the highest I_{sp} fuel ($LOX - LH_2$), which results in a vehicle wet mass of 53,856 kg. The vehicle is capable of traveling on any route except route (1,4,6), thus the wet mass of route this route ($m_{w_{146}}$) is set to a large number.

The objective of the network design problem is to determine the lowest total wet mass that satisfies the supply, demand, and capacity constraints. The objective is simply defined as

$$\min J = \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=j}^{n} n_{ijk} m_{w_{ijk}} \tag{13}$$

where $m_{w_{ijk}}$ is equal to 53,856 kg, except on route (1,4,6). The minimum total wet mass for the system is 161,568 kg, and the optimal configuration is shown in Figure 6.

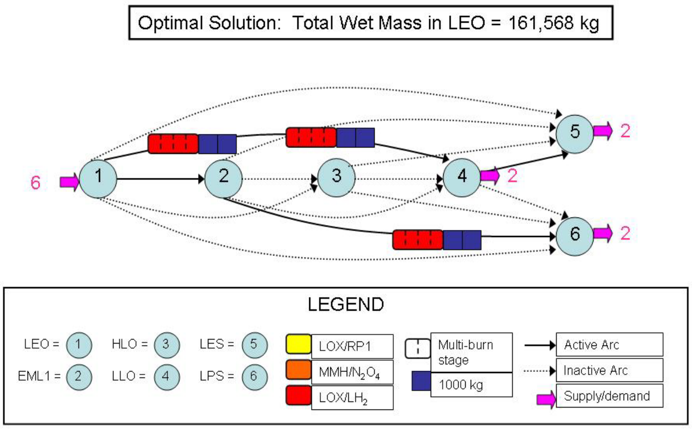

Figure 6. Optimal Allocation for a Single Vehicle Design

In Figure 6 we see that the optimal network configuration allocates one vehicle on route (1,4,4), one vehicle on route (1,4,5) and one vehicle on route (1,2,6). Although these routes are not the most efficient (in terms of ΔV), the vehicle is already sized to provide this ΔV, and therefore there is no penalty for choosing these routes. Although a route with a smaller ΔV would require less fuel, the computation of the fuel consumption for a specific route, given the structural mass, is beyond the scope of the current model.

To more accurately capture the dependencies between the vehicle and the network, we consider the problem of defining a single vehicle configuration and sizing the vehicles for each route. In this example, we define the vehicle configuration as above, but the wet mass for each route depends on the ΔV required for a vehicle to travel on that route. Using these values of wet mass, we compute the routes through the network that minimize the total system wet mass, as defined in Equation 13.

In this case, the optimal network configuration allocates one vehicle on route (1,4,4) with a wet mass of 11,479 kg, one vehicle on route (1,3,5) with a wet mass of 31,299 kg, and one vehicle on route (1,6,6) with a wet mass of 31,299 kg. The optimizer selects the routes with the least wet mass for the specified vehicle configuration that satisfy the demand. Since each vehicle is defined to have a capacity of three packages, a single vehicle can satisfy the demand of one node. The total system wet mass is 74,077 kg and the optimal configuration is shown in Figure 7

B. Vehicle Design Optimization

The vehicle design optimization problem requires the routes be selected prior to the vehicle optimization. To mimic the point design approach, the network will consist of direct routes from the source to the destination nodes (nodes 4,5,and 6). If we consider the design of a single vehicle and determine the number of vehicles

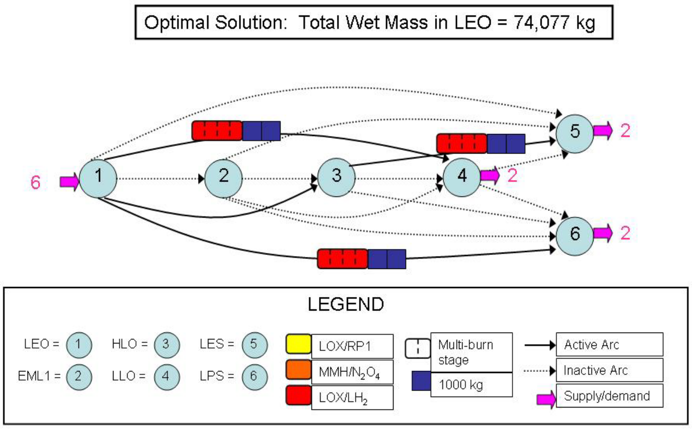

Figure 7. Optimal Allocation for a Single Vehicle Configuration

required to meet the supply and demand of the network, the objective will be to minimize the maximum vehicle wet mass.

$$\min J = (n_{144} + n_{155} + n_{166}) \max \ (m_{e_{144}}, m_{e_{155}}, m_{e_{166}}) \tag{14}$$

In this problem, we consider identical package demands, and since we are restricted to direct routes, the problem becomes a simple combinatorial optimization problem, which is solved by enumeration. The best selection is a vehicle with a capacity of two packages. The vehicle stages after both burns and uses the highest I_{sp} fuel ($LOX - LH_2$) for each stage. This configuration results in a single vehicle wet mass of 19,922 kg and a total system wet mass of 59,768 kg. This solution represents the best compromise in the vehicle staging and fuel selection given the multiple operational requirements. If we compare this result with the results obtained from the network optimization problem we see a significant reduction in system mass.

For comparison, if we optimize a vehicle for each route, the system objective reduces to 47,498 kg. The vehicle designs for routes (1,5,5) and (1,6,6) remain the same as above. However, the vehicle design for route (1,4,4) becomes a single stage vehicle employing the highest I_{sp} fuel for the transfer, and has a wet mass of 7,652 kg.

V. Integrated Transportation Network Design

For an integrated transportation network, the vehicle design and the route selection are performed concurrently. The design vector includes variables that define both the vehicle and the network. The vehicle constraints, as well as the network constraints, must all be satisfied for the current point to be evaluated by the optimizer. Since all of the variables are integer and the computation of initial vehicle mass is a non-linear analysis function, Simulated Annealing[12] is the chosen optimizer for this problem. To implement Simulated Annealing for this problem, an additional decision variable (y_{ijk}) is defined to determine the allowable routes. The use of a decision variable promotes a more effective exploration of the design space, especially as the number of variables increases. The decision variable is related to the number of vehicles by Equation 15.

$$n_{ijk} \leq M y_{ijk} \tag{15}$$

Here, M represents the maximum number of vehicles available on any route, which for this problem is four.

Again, we use the example of delivering two packages to low lunar orbit (LLO), two packages to the lunar equatorial surface (LES), and two packages to the lunar polar surface (LPS). If we consider the problem of finding a single vehicle design that minimizes the total system wet mass the objective becomes

$$\min J = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} n_{ijk} \max \left(m_{w_{ijk}}\right) \tag{16}$$

Using this objective, we optimize the vehicle design and the network configuration concurrently, and obtain the same result as with traditional vehicle design optimization for a single vehicle design.

With the concurrent formulation, however, we have the ability to design many vehicles, each optimal for the specified route, and concurrently determine the optimal routing through the network. The system objective is to minimize the total system wet mass, as defined in Equation 12. The minimum system wet mass is 47,598 kg and Figure 8 shows the optimal configuration for this problem.

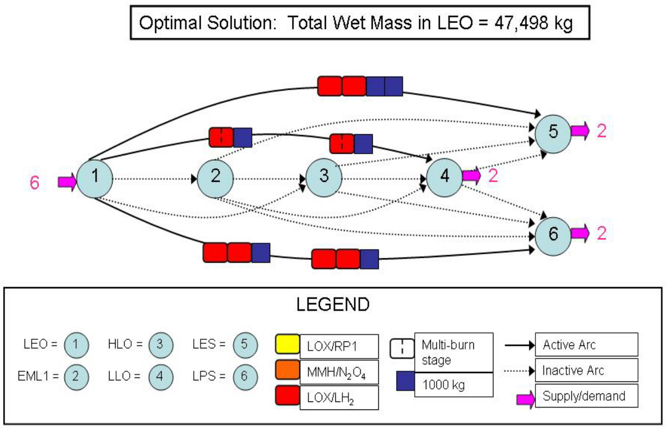

Figure 8. Concurrent Optimization of Multiple Vehicles within a Space Network

Referring to Figure 8, we see that although the objective value of the concurrent optimization is the same as with the vehicle optimization, the vehicle designs are different. In the traditional vehicle design, each route has a single vehicle with a capacity of two packages, however in the concurrent optimization design routes (1,4,4) and (1,6,6) each have two vehicles with a capacity of one package. If we closely examine Equation 9, we see that the factor of two in the capacity is multiplied through the equation. Thus, for the same configuration and route, a single vehicle with a capacity of two packages has the same wet mass as two vehicles, each with a unit of capacity.

To analyze the sensitivity of the solution to changes in demand, we perturb selected demand values at different nodes in the network. First, we consider an increase in the demand at the lunar equatorial surface (node 5) to four packages. The extra two units of demand at the LES are delivered by allocating an additional vehicle on route (1,5,5) with the same configuration as before. The minimum system wet mass is 67,421 kg.

Alternatively, we can increase the demand at the lunar polar surface (node 6) to four packages. The extra two units of demand at the LPS are delivered by increasing the capacity of the two vehicles on route (1,6,6) to two units. The minimum system mass is 67,421 kg.

Instead of perturbing the demand at existing demand nodes, we can examine the effect of introducing an additional demand node into the network. When a single package demand at the Earth-Moon Lagrangian point (node 2) is added, the optimal solution adds a single-stage vehicle with unit capacity to the network. The vehicle travels a direct route to node 2 and has a wet mass of 3561 kg. The minimum system wet mass is 51,060 kg.

Table 3 summarizes the concurrent optimization results presented in this section. The column for demand lists the nodes with demand for each case considered and the amount of demand at the nodes. The fuel-type lists one or two values, depending on the number of stages for that vehicle. As stated in Table 2, type 1 refers to $LOX - RP1$, type 2 refers to $MMH - N_2O4$ and type 3 refers to $LOX - LH_2$. The column listing vehicle wet mass refers to the wet mass of a single vehicle on the given route, and the system wet mass is the total system wet mass for that case.

Table 3. Concurrent Optimization Results

	Demand	Route	# of Vehicles	Vehicle Capacity	# of Stages	Fuel Type	Vehicle Wet Mass (kg)	System Wet Mass (kg)
Single Vehicle Design	$d(4) = 2$	$(1, 4, 4)$	1	2	2	3,3	19,922	59,768
	$d(5) = 2$	$(1, 5, 5)$	1	2	2	3,3	19,922	
	$d(6) = 2$	$(1, 6, 6)$	1	2	2	3,3	19,922	
Multiple Vehicle Designs	$d(4) = 2$	$(1, 4, 4)$	2	1	1	3	3,826	47,498
	$d(5) = 2$	$(1, 5, 5)$	1	2	2	3,3	19,992	
	$d(6) = 2$	$(1, 6, 6)$	2	1	2	3,3	9,961	
Multiple Vehicle Designs	$d(4) = 2$	$(1, 4, 4)$	2	1	1	3,3	3,826	67,421
	$d(5) = 4$	$(1, 5, 5)$	2	2	2	3,3	19,992	
	$d(6) = 2$	$(1, 6, 6)$	2	1	2	3,3	19,992	
Multiple Vehicle Designs	$d(4) = 2$	$(1, 4, 4)$	2	1	1	3	3,826	67,421
	$d(5) = 2$	$(1, 5, 5)$	1	2	2	3,3	19,992	
	$d(6) = 4$	$(1, 6, 6)$	2	2	2	3,3	19,992	
Multiple Vehicle Designs	$d(2) = 1$	$(1, 2, 2)$	1	1	1	3	3,561	51,060
	$d(4) = 2$	$(1, 4, 4)$	2	1	1	3	3,826	
	$d(5) = 2$	$(1, 5, 5)$	2	1	2	3,3	9,961	
	$d(6) = 2$	$(1, 6, 6)$	1	2	2	3,3	19,992	

As the demand at different nodes is perturbed, the system allocates additional vehicles to the network, instead of re-routing vehicles to satisfy demand at multiple nodes. Although this may be the optimal configuration, it is possible that this decision is a result of simplifications made in the vehicle model. Although the network model allows a single vehicle to deliver packages to multiple nodes, the vehicle wet mass is calculated from the vehicle capacity. Using Equation 9, the mass benefits from a multiple-burn stage, as well as the mass benefits from dropping a payload at an intermediate node are ignored. Therefore, to determine if direct transfers are optimal, it is necessary to include the package distribution directly into the formulation and modify the initial mass calculation to reflect these changes.

VI. Conclusions

In this paper, an integrated approach for designing a vehicle and network for a space transportation system was presented. By viewing multiple missions in an integrated framework, we can determine the optimal vehicle design and operations concurrently. Through the example of a simplified Earth-Moon supply chain it was shown that this method benefits the system design, as compared to a traditional network design optimization approach. The concurrent optimization of the system obtains the same results as the traditional vehicle design optimization, which confirms that, with respect to the current vehicle model, direct transfers are optimal. In addition, perturbations in demand did not affect the optimal solution structure, and simply led to the allocation of additional vehicles on existing routes.

To further understand the inter-dependencies of the trajectory, vehicle and network sub-systems for a space transportation system, it is necessary to increase the complexity of the underlying sub-system models. Reformulating the initial mass computation for impulsive transfers to benefit from multiple burns within stages and intermediate node package drops requires individual stage masses to be computed. Using these revised computations, we can then reformulate the problem to examine the individual stage masses and determine to what extent should commonality in stages be a factor in the design.

In addition, it is necessary to expand the model and incorporate different trajectories. In this paper, only impulsive trajectories were considered and a specific trajectory was defined for each route. For the transfer of cheap consumables, it is important to investigate other means of transportation, namely the use of electric propulsion for both pre-positioning and transfer. Including electric propulsion into the model will create a dynamic network where time is a factor in the transfers and result in a more diverse architectural design space.

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