Constraints in the context of induced-gravity inflation

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Constraints on the required flatness of the scalar potential $V(\phi)$ for a cousin model to extended inflation are studied. It is shown that, unlike earlier results, induced-gravity inflation can lead to successful inflation with a very simple Lagrangian and $\lambda \sim 10^{-6}$, rather than 10^{-15} as previously reported. A second order phase transition further enables this model to escape the "big bubble" problem of extended inflation, while retaining the latter's motivations based on the low-energy effective Lagrangians of supergravity, superstring, and Kaluza-Klein theories.

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I. INTRODUCTION

Since Guth's first paper on inflation a decade ago [1], there has been an explosion of effort dedicated to developing a natural theory of inflationary cosmology. These efforts have established inflation as a ubiquitous program of research among cosmologists and particle physicists, but have not yet produced any particular model which satisfies all of the practitioners. As Kolb and Turner recently remarked, inflation is by now "a paradigm in search of a model" [2]. Much of the trouble concerns the so-called "fine-tuning" required to force the various models into agreement with measurements of the small anisotropy of the cosmic microwave background radiation.

Adams, Freese, and Guth [3] developed a quantitative means of measuring the amount of "fine-tuning" necessary to make inflationary models agree with observations. They define a dimensionless parameter Λ as the ratio of the change in the scalar field's potential to the change in the scalar field:

$$\Lambda \equiv \frac{\Delta V(\phi)}{(\Delta\phi)^4} \ . \tag{1}$$

In this way, Λ functions as a measure of the flatness of a given potential. (Reference [3] denotes this ratio by λ, rather than Λ; the upper-case letter is used here to avoid confusion with the closely related quartic self-coupling parameter.) In [3], they evaluate this ratio for generic inflation scenarios (without a curvature-coupled scalar field) and for the original version of extended inflaton [4]. They show that extended inflation requires a fine-tuning *eight* orders of magnitude more stringent than the general inflation schemes: whereas they find $\Lambda \leq 10^{-6}$–10^{-8} for the inflation potential of new inflation, they calculate that a potential for the Brans-Dicke-like scalar field in extended inflation would require $\Lambda \leq 10^{-15}$. Although the authors of [3] are quick to point out that their method of calculating Λ is highly model-dependent, others have taken this result to indicate that adding a potential $V(\phi)$ for the curvature-coupled scalar in *any* type of extended inflation model necessarily entails this sort of extreme fine-tuning (see, e.g., [5]).

This paper examines the constraints on $V(\phi)$ for a cousin model to the original extended inflation. Building on Zee's early ideas [6] about uniting spontaneous symmetry breaking with the Brans-Dicke reformulation of general relativity, Accetta, Zoller, and Turner [7] developed a model of "induced-gravity inflation." In this model, a single scalar boson does all the work of inflation: it couples to the scalar curvature R *and* drives inflation with $V(\phi)$ (unlike ordinary extended inflation, which requires one field to couple to R while a separate and unrelated field drives the expansion). This kind of one-boson model can agree with observations with about the same degree of "fine-tuning" as the generic models of inflation examined in [3]: as will be shown below, "Induced-gravity Inflation" requires $\Lambda \lesssim 10^{-6}$, a far cry from the 10^{-15} of Ref. [3]! Furthermore, by employing a *second*-order phase transition to exit the inflationary epoch, rather than the first-order transition of [4], the "big bubble" or "ω problem" which plagued original extended inflation [8,9] may be avoided.

In Sec. II, we calculate Λ for induced-gravity inflation, and consider why earlier attempts to determine the required flatness of the potential have led to much more constrained results. Section III examines the accuracy of the slow-rollover solutions upon which the calculation of Λ is based. And in Sec. IV, we briefly consider benefits and difficulties of placing the inflationary epoch at such a high energy scale.

II. CALCULATING Λ

We begin with the Lagrangian density[1]

[1]The sign conventions follow those of Misner, Thorne, and Wheeler [11], which are based upon the 1962 edition of Landau and Lifshitz [11]. Thus, $g_{tt} < 0$, the full Riemann tensor is $R^{\lambda}_{\mu\nu\kappa} = \partial_\nu \Gamma^{\lambda}_{\mu\kappa} - \partial_\kappa \Gamma^{\lambda}_{\mu\nu} + \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\kappa} - \Gamma^{\lambda}_{\sigma\kappa}\Gamma^{\sigma}_{\mu\nu}$, and the Ricci tensor is $R_{\mu\kappa} \equiv R^{\lambda}_{\mu\lambda\kappa}$. The original Brans-Dicke papers followed these sign conventions. Note that these definitions for the Riemann and Ricci tensors are opposite in sign from those in Weinberg's text [11] (and thus opposite to some of the recent literature on extended inflation).

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$$\mathcal{L}=f(\phi)R-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)+L_M \ , \tag{2}$$

where the L_M term here only includes contributions from "ordinary" matter—there is no separate Higgs sector as in [4]. If we choose $f(\phi)=\phi^2/(8\omega)$, we find the coupled field equations:

$$H^2+\frac{k}{a^2}=\frac{4\omega}{3\phi^2}[\rho+V(\phi)]+\frac{2\omega}{3}\left[\frac{\dot\phi}{\phi}\right]^2-2H\left[\frac{\dot\phi}{\phi}\right] ,$$
$$\ddot\phi+3H\dot\phi+\frac{\dot\phi^2}{\phi}=\frac{2\omega}{3+2\omega}\phi^{-1}[(\rho-3p)+4V(\phi)-\phi V'(\phi)] \ . \tag{3}$$

In Eq. (3), $a(t)$ is the cosmic scale factor of the Robertson-Walker metric, and is related to the Hubble parameter by $H\equiv\dot a/a$. From Eq. (2), one can see that $f(\phi)\to(16\pi G_{\text{eff}})^{-1}$, which leads to $4\omega\phi^{-2}=8\pi G_{\text{eff}}$. Note that we have parametrized the Lagrangian slightly differently from the 1985 "Induced-gravity" paper: our Brans-Dicke parameter ω is inversely proportional to their coupling strength $\varepsilon(\omega=(4\varepsilon)^{-1})$. That paper also makes the minor approximation that $(2\omega)/(3+2\omega)\to1$, whereas we have kept this term explicit. During the inflationary epoch, the k term becomes negligible; similarly, since ρ and p now only include contributions from ordinary matter, they too may be neglected. If we now impose the appropriate slow-rollover approximations [7,10]:

$$\left|\frac{\dot\phi}{\phi}\right|\ll H \ ,$$
$$|\ddot\phi|\ll 3H\dot\phi \ ,$$
$$|\dot\phi^2|\ll V(\phi) \ , \tag{4}$$

the field equations enter a more tractable form:

$$H^2\simeq\frac{4\omega}{3}\frac{V(\phi)}{\phi^2} \ ,$$
$$3H\dot\phi\simeq\frac{2\omega}{3+2\omega}\frac{[4V(\phi)-\phi V'(\phi)]}{\phi} \ . \tag{5}$$

In order to calculate the required flatness (Λ) of the potential, we must now assume a particular form for $V(\phi)$. To achieve a second-order phase transition with a minimum of fine-tuning, we may choose the simplest form for the potential at the tree level:

$$V(\phi)=\frac{\lambda}{4}[\phi^2(t)-v^2]^2 \ , \tag{6}$$

where λ is the quartic self-coupling, and v is the vacuum expectation value for ϕ. In addition to being a qualitatively simple form for the desired potential, this is also the optimal form found in [3] for the case of generic new inflation. It also matches the potential of [7], with the minor difference that it is parametrized with $\lambda/4$ rather than $\lambda/8$.

Using this expression for $V(\phi)$, combined with the field equations (5), we may solve for $\phi(t)$ and $a(t)$:

$$\phi(t)=\phi_0+\left[\frac{\lambda\omega}{3\gamma^2}\right]^{1/2}v^2t \ ,$$
$$\frac{a(t)}{a_B}=\left[\frac{\phi(t)}{\phi_0}\right]^\gamma\exp\left[\frac{\gamma}{2v^2}[\phi_0^2-\phi^2(t)]\right] \ . \tag{7}$$

The factor $\gamma\equiv(3+2\omega)/2$ is slightly different from the exponent found in [7] [because of their earlier approximation that $(2\omega)/(3+2\omega)\to1$]. The quantities ϕ_0 and a_B are values at the beginning of the inflationary epoch. Note that at early times, when $\phi(t)\sim\phi_0$, these equations yield the familiar power-law solution, with $a(t)\propto t^\gamma$. As t increases, the rate of expansion slows due to the $\exp(\phi_0^2-\phi^2)$ term. With these analytic expressions for $\phi(t)$ and $a(t)$, we may now calculate Λ.

Following [3], we begin with the two basic constraints on the scalar potential: it must provide for sufficient inflation to solve the flatness, horizon, and monopole density problems, and it must allow for the proper amplitude of density perturbations to act as seeds for the evolution of large scale structure. The first of these requirements takes the form

$$\frac{1}{H_Na_N}\le\frac{1}{H_Ba_B} \ , \tag{8}$$

where the subscript N refers to present values ("now"), and B refers to values at the beginning of the inflationary epoch. Using the standard cosmological model's assumption of adiabatic expansion following the end of inflation, we may write

$$\frac{a_{\text{end}}}{a_N}\simeq\frac{T_N}{T_{\text{RH}}} \ , \tag{9}$$

where a_{end} is the value of the scale factor at the end of inflation, and T_{RH} is the reheat temperature following thermalization of the foregoing false vacuum energy density. (Reference [3] assumes $T_{\text{RH}}\approx M_F$, where M_F^4 is the false vacuum energy density of the second boson which drives inflation.) The Hubble parameter may then be parametrized as

$$H_B^2=\frac{8\pi}{3}\frac{V_f}{m_P^2} \ ,$$
$$H_N^2=\frac{8\pi}{3}\frac{\beta^2T_N^4}{M_P^2} \ , \tag{10}$$

where V_f is the initial value of the false vacuum energy density; m_P is the effective value of the Planck mass at the beginning of inflation [related to the initial value of the field ϕ: $\phi_0=(\omega/2\pi)^{1/2}m_P$]; and M_P is the present value of the Planck mass, $M_P\approx1.22\times10^{19}$ GeV. The quantity β is the ratio of the energy density in matter to the energy density in radiation today, which [3] takes to be around 81. In the original extended inflation framework, $V=V_f=M_F^4$ was constant, due to the energy density of the metastable state of the second boson before it completed its first-order phase transition. In the present model, $V(\phi)$ changes in time as $\phi(t)$ changes; yet even in this new context, V_f is still a constant, and is simply equal to $V(\phi_0)$; its use here is thus independent of any slow-roll approximation.

Using Eqs. (8)–(10), the condition for sufficient inflation takes the form

$$\frac{a_{\text{end}}}{a_B}\ge\frac{V_f^{1/2}}{\beta T_NT_{\text{RH}}}\frac{M_P}{m_P} \ . \tag{11}$$

Combining with Eq. (7), we get

$$\frac{a_{\rm end}}{a_B}=\left[\frac{\phi_{\rm end}}{\phi_0}\right]^{\gamma}\exp\left[\frac{\gamma}{2v^2}(\phi_0^2-\phi_{\rm end}^2)\right]\geq\frac{V_f^{1/2}}{\beta T_N T_{\rm RH}}\frac{M_P}{m_P}\ , \tag{12}$$

where we have followed [7] in utilizing the explicit analytic solution for $a(t)$ based on the slow-roll approximation right up to $a_{\rm end}$; as we shall see below, this approximation is a good one: the slow-rollover solutions employed above do not break down until $\phi(t)\simeq 0.98v$.

Equation (12) may be used to place a bound on the change in the scalar field, $\Delta\phi=(\phi_{\rm end}-\phi_0)$. Writing ϕ_0 and $\phi_{\rm end}$ in the exponent in terms of their associated masses, we find

$$\left[\frac{\phi_{\rm end}}{\phi_0}\right]\geq\left[\frac{V_f^{1/2}}{\beta T_N T_{\rm RH}}\frac{M_P}{m_P}\right]^{1/\gamma}\exp\left[\frac{-\omega}{4\pi v^2}(m_P^2-M_E^2)\right]\ , \tag{13}$$

where we have followed [3] in writing M_E for the value of the Planck mass at the end of inflation. In the simplest case, ϕ would not evolve any more after the end of inflation, so M_E would equal M_P; yet for the time being the more general value M_E will be used. Since $\phi_0=(\omega/2\pi)^{1/2}m_P$ and $v\simeq\phi_{\rm end}=(\omega/2\pi)^{1/2}M_E$, Eq. (13) may be rewritten:

$$\Delta\phi\geq\sqrt{\omega/2\pi}\,m_P\left\{\left[\frac{V_f^{1/2}}{\beta T_N T_{\rm RH}}\frac{M_P}{m_P}\right]^{1/\gamma}\times\exp\left[\frac{1}{2}\left[1-\left[\frac{m_P}{M_E}\right]^2\right]\right]-1\right\}\ . \tag{14}$$

Taking $\Delta V=V_f$ and defining $\mu\equiv(m_p/M_E)$, the ratio Λ thus becomes

$$\Lambda\leq\left[\frac{2\pi}{\omega}\right]^2\frac{V_f}{m_P^4}\left\{\left[\frac{V_f^{1/2}}{\beta T_N T_{\rm RH}}\frac{M_P}{m_P}\right]^{1/\gamma}\times\exp\left[\frac{1}{2}(1-\mu^2)\right]-1\right\}^{-4}\ . \tag{15}$$

Incorporating the constraint that V_f must exceed the kinetic energy of the ϕ field's de Sitter space fluctuations without exceeding the effective Planck scale at the beginning of inflation [3], we may bound V_f by

$$V_f\leq\left[\frac{3}{8\pi}\right]^2 m_P^4\ , \tag{16}$$

which may then be used to eliminate the coefficient of V_f/m_P^4:

$$\Lambda\leq\left[\frac{3}{4\omega}\right]^2\left\{\left[\frac{V_f^{1/2}}{\beta T_N T_{\rm RH}}\frac{M_P}{m_P}\right]^{1/\gamma}\times\exp\left[\frac{1}{2}(1-\mu^2)\right]-1\right\}^{-4}\ . \tag{17}$$

Thus far we have only relied upon the requirement of sufficient inflation. The second requirement, based on the amplitude of density perturbations, may now be used to place a bound on the ratio $T_{\rm RH}^2/V_f^{1/2}$. This bound can then be used to remove nearly all mass-scale dependence from Λ. The post-COBE (Cosmic Background Explorer) parametrization of the density perturbation constraint may be written as [12]

$$\left[\frac{H^2}{\dot\phi}\right]_{\rm |hor}\leq 5\pi\delta_H\ , \tag{18}$$

where the quantity $H^2/\dot\phi$ is to be evaluated at the time of last horizon-crossing of the perturbations (during the inflationary epoch), and $\delta_H\simeq 1.7\times 10^{-5}$. (Reference [3] used 5×10^{-4}, rather than $5\pi\delta_H$.) As noted in [13], the quantities pertaining to the time of last horizon crossing in Eq. (18) should be evaluated in the Einstein frame; when compared with the "naive" calculations in the Jordan frame, a correction factor $F(\omega)$ must be introduced. Yet this factor $F(\omega)$ decreases monotonically with ω, and converges to unity rather quickly: $F(\omega=25)=1.052$, and $F(\omega=500)=1.002$. For the particular values of ω with which we shall be concerned below, $F(\omega)$ would thus be negligible, and so we may continue to calculate $H^2/\dot\phi$ in the Jordan frame.[2] (This is also the approach adopted in [5].) Since we want to relate Eq. (18) to a bound on V_f, we should first rewrite $\dot\phi(t_*)$ in terms of V_f, where t_* is the time of horizon crossing. From Eq. (7) we have

$$\dot\phi(t)=\left[\frac{\lambda\omega}{3\gamma^2}\right]^{1/2}\frac{\omega}{2\pi}M_E^2\ , \tag{19}$$

and from Eq. (6) we have:

$$V_f=V(\phi_0)=\frac{\lambda\omega^2}{16\pi^2}M_E^4(1-\mu^2)^2\ . \tag{20}$$

Using these expressions, we may then rewrite (the time-independent) $\dot\phi$ in terms of the constant V_f:

$$\dot\phi=\left[\frac{4\omega}{3\gamma^2}\right]^{1/2}V_f^{1/2}|(1-\mu^2)^{-1}|\ . \tag{21}$$

The absolute value for the $(1-\mu^2)$ term comes from taking the positive square root of M_E^4. Since $m_P<M_E$, the pole at $\mu^2=1$ is excluded.

Now we need to calculate the Hubble parameter at the time t_*. Following [3], we may write

$$\frac{1}{H(t_*)a(t_*)}=\frac{1}{H_N a_N}\ , \tag{22}$$

which, after employing Eqs. (9), (10), and (21), leads to

[2]Note that Eq. (18) follows the assumption, expresssed in [13], that the magnitude of the density perturbations should be calculated as $H^2/\dot\phi$ even for models with a Brans-Dicke, nonminimal ϕR coupling [and hence a complicated $H(\phi)$ evolution]. The limits to the accuracy of this assumption are currently under study by the author.

$$\frac{H^2(t_*)}{\dot{\phi}(t_*)} \simeq \frac{8\pi}{3} \frac{\beta^2 T_N^2 T_{\rm RH}^2}{M_P^2 V_f^{1/2}} \sqrt{3\gamma^2/4\omega} \left[\frac{a_{\rm end}}{a(t_*)}\right]^2 (1-\mu^2) \le 5\pi\delta_H \; . \tag{23}$$

Rewriting this as a bound on $(T_{\rm RH}^2/V_f^{1/2})$, we get

$$\left[\frac{T_{\rm RH}^2}{V_f^{1/2}}\right] \le \frac{5\delta_H M_P^2}{4\beta^2 T_N^2} \sqrt{3\omega/\gamma^2} \left[\frac{a_{\rm end}}{a(t_*)}\right]^{-2} |(1-\mu^2)^{-1}| \; . \tag{24}$$

This may be substituted into Eq. (17) for Λ:

$$\Lambda \le \left[\frac{3}{4\omega}\right]^2 \left[\left[\frac{4\beta T_N}{5\delta_H M_P} \frac{T_{\rm RH}}{m_P} \sqrt{\gamma^2/3\omega} \left[\frac{a_{\rm end}}{a(t_*)}\right]^2 (1-\mu^2)\right]^{1/\gamma} \exp\left[\frac{1}{2}(1-\mu^2)\right] - 1\right]^{-4} . \tag{25}$$

We have taken the equality in Eq. (24) as a worst case for Λ; if $(T_{\rm RH}^2/V_f^{1/2})$ were much less than the right-hand side of Eq. (24), the bound on Λ would increase (and the resultant need for "fine-tuning" would therefore decrease). The remaining factor of $T_{\rm RH}/m_P$ may now be removed by combining Eqs. (16) and (24). Following these substitutions, the expression for Λ becomes

$$\Lambda \le \left[\frac{3}{4\omega}\right]^2 \left[\left[\sqrt{1/10\pi\delta_H} \left[\frac{3\gamma^2}{\omega}\right]^{1/4} \left[\frac{a_{\rm end}}{a(t_*)}\right] (1-\mu^2)^{1/2}\right]^{1/\gamma} \exp\left[\frac{1}{2}(1-\mu^2)\right] - 1\right]^{-4} . \tag{26}$$

The time of last horizon crossing can be calculated from Eq. (22), and from t_* one could then find the ratio $a_{\rm end}/a(t_*)$. Yet the dependence of Λ on $a_{\rm end}/a(t_*)$ is very weak (being suppressed by the exponent γ^{-1}), so some simplifying approximations may be made. In most models of inflation, t_* is around 60 e-folds before the end of inflation (although some recent models have $t_* \sim 50$ e-folds before the end of inflation, e.g. [14]); this means that the ratio $a_{\rm end}/a(t_*)$ is simply e^{60} (or, perhaps, e^{50}). For the calculation of Λ, we will assume $a_{\rm end}/a(t_*) \sim e^{60}$ in this model. As Fig. 1 shows, Λ increases monotonically with increasing μ for a given value of ω; a *lowest* bound on Λ thus comes from taking the limit $\mu \to 0$ (i.e., $m_P \ll M_E$). When this is done, Λ becomes a function of ω alone. Figure 2 shows a plot of Λ versus ω in the limit $\mu \to 0$: the maximum value of Λ (corresponding to the least amount of "fine-tuning" required) is 5.4×10^{-6}, for the value $\omega_{\rm cr}=240$. We can check the dependence of Λ on $a_{\rm end}/a(t_*)$ by defining a parameter α as $a_{\rm end}/a(t_*)=e^{\alpha}$. Figure 3 shows a plot of Λ versus α in the limit $\mu \to 0$ for a particular value of $\omega(\omega=500)$. The dependence on α is indeed weak: Λ evaluated at $(\alpha=50, \omega=500)$ gives 4.5×10^{-6}, whereas Λ evaluated at $(\alpha=60, \omega=500)$ gives 3.8×10^{-6}. Similarly, for $\alpha=50$ rather than 60, $\Lambda(\omega_{\rm cr})=7.3\times10^{-6}$, instead of 5.4×10^{-6}.

In the original model of extended inflation, ω was constrained to be less than 25 in order to avoid observable inhomogeneities coming from the large range in bubble sizes, even though present tests of Brans-Dicke gravitation versus general relativity limit ω to the range $\omega \ge 500$ (hence the "big bubble" or "ω problem" of old extended inflation). Yet in the present model, the second-order phase transition lifts this constraint on ω; ω can now be as large as necessary to meet the experimental limits. As one can see in Fig. 2, $\Lambda(\omega)$ falls off slowly from its maximum with increasing ω. It is interesting to note that $\Lambda_{\rm max}$ occurs within a factor of 2 of the value $\omega=500$. Because of its agreement with present day observations, and its proximity to $\omega_{\rm cr}$, $\omega=500$ appears to be a good candidate for the Brans-Dicke parameter.

Equation (1) may be used to relate Λ to the quartic self-coupling constant λ. For the form of $V(\phi)$ considered here, we find:

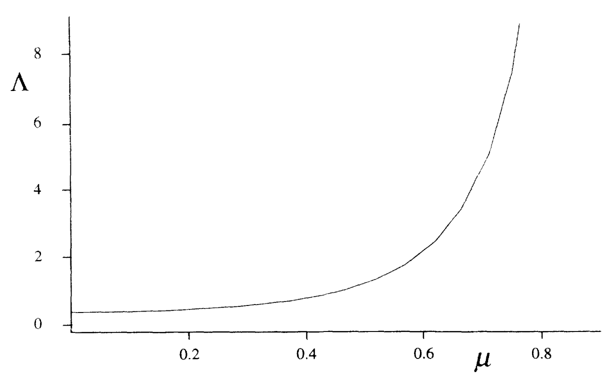

FIG. 1. Plot of Λ as a function of $\mu \equiv (m_P/M_E)$, based on Eq. (26) with $\omega=500$. The vertical scale is in units of 10^{-5}, and the assumption that $a_{\rm end}/a(t_*) \simeq e^{60}$ has been used. The intercept at $\mu=0$ is $\Lambda=3.8\times10^{-6}$.

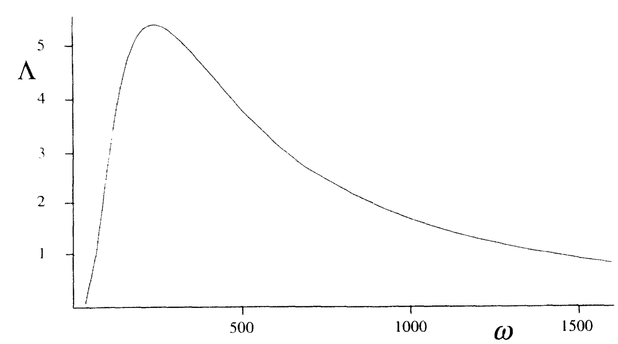

FIG. 2. Plot of $\Lambda(\omega)$ vs ω, based on Eq. (26) with $\mu \to 0$. The vertical scale is in units of 10^{-6}, and the assumption that $a_{\rm end}/a(t_*) \simeq e^{60}$ has been used. Note that Λ reaches its maximum value of 5.4×10^{-6} at $\omega_{\rm cr}=240$; the value at $\omega=500$ is $\Lambda(500)=3.8\times10^{-6}$.

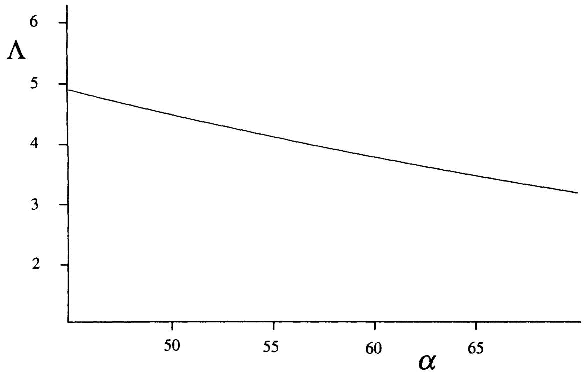

FIG. 3. Plot of Λ vs α, where $a_{\rm end}/a(t_*)=e^{\alpha}$, based on Eq. (26) with $\mu\to 0$ and $\omega=500$. The vertical scale is in units of 10^{-6}.

$$\Lambda=\frac{\Delta V}{(\Delta\phi)^4}=\frac{\lambda}{4}\frac{(\phi_0^2-v^2)^2}{(\phi_0-v)^4}\ , \tag{27}$$

or, if we keep terms only up to $O(\mu)$,

$$\Lambda\simeq\frac{\lambda}{4}\frac{[1+O(\mu^2)]}{[1-4\mu+O(\mu^2)]}\ . \tag{28}$$

Thus, λ is of the same order of magnitude as Λ, which, for the model under study, means $\lambda\sim 10^{-6}$. This value of λ is much larger than the results in [3] for the original model of extended inflation, indicating far less of a need for "fine-tuning."

We should pause here to consider why this relatively large value for Λ has not been noted before. The most important reason is because the calculation of Λ is highly model-dependent. Both papers of [3], for example, assumed that a separate Higgs sector would drive inflation; this meant that their $V_f^{1/4}=M_F$ was constrained to lie at the GUT scale, with such ratios as $(M_F/v)\sim 10^{-5}$. Furthermore, by insisting upon a first-order phase transition in the Higgs sector, ω was constrained to $\omega\le 25$. It is interesting to note that an attempt in 1989 to unite the original induced-gravity inflation model with extended inflation [15] similarly relied upon a separate Higgs sector to drive inflation until it underwent a first order phase transition.

The authors of the 1985 paper introducing induced-gravity inflation [7] studied constraints on the quartic self-coupling λ, based also on the twin requirements of sufficient inflation and a proper amplitude for density perturbations. Yet their result indicated that $\lambda\le 10^{-14}$ for $\omega\sim 500$. Several factors help to explain this low result. First, their pre-COBE parametrization of the amplitude of density perturbations leads to an increase of an order of magnitude for λ when compared with present, post-COBE values. Most important, however, is their approach to bounding λ: they solved for λ in terms of the ratio $(v/\phi(t_*))$, where $\phi(t_*)$ is the value of the field at the time of last horizon crossing. Their result for λ, which in their analysis is proportional to $\sinh^{-4}[\ln(v/\phi(t_*))]$, is thus *very* sensitive to the value of $\phi(t_*)$. Because this value cannot be solved exactly (even in the slow-roll approximation), two extreme limiting regimes were studied: $\phi(t_*)\approx v$ versus $\phi(t_*)\ll v$. Yet small differences in the approximation of $\phi(t_*)$ lead to order-of-magnitude differences in their estimation of λ: a difference of 0.01 in the assumed value of $\phi(t_*)$ leads to a difference in λ of two orders of magnitude. The method of calculating bounds on Λ employed in this paper avoids expanding in terms of the unknown ratio $(v/\phi(t_*))$; the only mass ratios involved here are of order $(\phi_0/v)^2$, and their inclusion *raises* the bound on Λ. The information regarding $\phi(t_*)$ is now contained in the ratio $(a_{\rm end}/a(t_*))$, and we saw above that changing this ratio from e^{60} to e^{50} leads to a change in Λ by a factor of only ~ 1.18 (see Fig. 3). This appears to be the major reason for the large split in values of λ between this paper and the 1985 analysis.

III. ACCURACY OF SLOW-ROLLOVER APPROXIMATE SOLUTIONS

We may now check the accuracy of our slow-rollover approximate solutions by following Steinhardt and Turner's "prescription" for successful slow-rollover [16]. The analysis is easiest by rewriting Eq. (2) in terms of a Brans-Dicke field Φ, where $\Phi\equiv f(\phi)=\phi^2/(8\omega)$. The "prescription" of [16] concerns finding conditions for when the $\ddot\Phi$ term may be neglected. In the present model, when $\ddot\Phi$ is negligible, the Φ equation becomes

$$\dot\Phi=\frac{1}{3H}\frac{1}{(3+2\omega)}[2V(\Phi)-\Phi V'(\Phi)]\ , \tag{29}$$

where the prime now indicates differentiation with respect to Φ. Using this expression for $\dot\Phi$, we may calculate $\ddot\Phi$, and then write the ratio $\ddot\Phi/(3H\dot\Phi)$, which becomes

$$\frac{\ddot\Phi}{3H\dot\Phi}=\frac{1}{9H^2}\frac{1}{(3+2\omega)}[V'-\Phi V'']-\frac{1}{9H^3}\frac{1}{(3+2\omega)}\left[\frac{\partial H}{\partial\Phi}\right][2V-\Phi V']\ . \tag{30}$$

From Eq. (30), it is consistent to neglect the $\ddot\Phi$ term when

$$|V'-\Phi V''|\ll(3+2\omega)(9H^2)\ ,$$
$$\left|\left[\frac{\partial H}{\partial\Phi}\right](2V-\Phi V')\right|\ll(3+2\omega)(9H^3)\ . \tag{31}$$

These conditions may be used to solve for when the slow-rollover approximation breaks down; that is, solved for values of Φ for which the left-hand side of each inequality roughly *equals* the right-hand side (rather than being much less than it). Since $\Phi=\phi^2/(8\omega)$, the potential for our particular model may be written $V(\Phi)=\lambda/4(8\omega\Phi-v^2)^2$, which leads to

$$|V'-\Phi V''|\to 4\lambda\omega v^2\ . \tag{32}$$

After reexpressing H in terms of Φ [see Eq. (5)], we find the value of the field for which the consistency of the slow-roll approximation breaks down ($\Phi_{\rm bd}$) to be

$$\Phi_{\rm bd}=\frac{v^2}{8\omega}\left[1+\frac{1-\sqrt{1+6\gamma}}{3\gamma}\right]\rightarrow$$

$$\phi_{\rm bd}=v\left[1+\frac{1-\sqrt{1+6\gamma}}{3\gamma}\right]^{1/2} . \tag{33}$$

If $\omega=500$, $\phi_{\rm bd}=0.98v$. It is interesting to compare this with the result for $\phi_{\rm bd}$ based on the second condition for slow-rollover. This condition leads to the assignment:

$$\phi_{\rm bd}=v\left[1+\frac{\sqrt{72}}{288}\frac{1}{\gamma}\right]^{1/2} . \tag{34}$$

In other words, the second condition does not break down until $\phi>v$! Thus, the result based on the first condition will be used.

We may calculate a maximum reheating temperature for the model by finding the value of $V(\phi)$ at the point where the field begins its damped oscillations around the true minimum of the potential. Taking this point to be $\phi_{\rm bd}$ leads to

$$V(\phi_{\rm bd})=\frac{\lambda v^4}{18\gamma^2}(3\gamma+1-\sqrt{1+6\gamma}) . \tag{35}$$

If $\omega=500$, $V(\phi_{\rm bd})=(3.2\times10^{-4})\lambda v^4$, so $T_{\rm RH,max}=V^{1/4}(\phi_{\rm bd})=(0.13)\lambda^{1/4}v$. Furthermore, if $\lambda\sim10^{-6}$, then $T_{\rm RH,max}=(4.1\times10^{-3})v$. Assuming the simplest case, that ϕ does not evolve after the end of inflation, then $v=\sqrt{\omega/2\pi}M_P$, which (for $\omega=500$) leads to $T_{\rm RH,max}\simeq4.5\times10^{17}$ GeV. We will consider possible interpretations of inflation at this energy scale below.

IV. CONCLUSIONS

Induced-gravity inflation, which combines properties from the "new inflation" schemes of 1982 [17] (such as a slowly rolling field leading to a second-order phase transition) with characteristics from the original version of the extended inflation [4] (including a nonminimal ϕR coupling), can lead to successful inflation with potentially acceptable limits on "fine-tuning." The Lagrangian of Eq. (2) requires only a qualitatively simple scalar potential associated with a single curvature-coupled scalar field; there is no need for adding special phenomenologically-inspired "extra" terms by hand to $\mathcal{L}$, as in [5,10]. Induced-gravity inflation can also get all of the "work" of inflation done with only one boson, thereby helping to slow the proliferation of "specialty" bosons, each of which is invented to complete specific and unrelated tasks in the early universe. In addition to this simplicity, the model retains many of the motivations for extended inflation, based on the appearance of Brans-Dicke-like couplings in the low-energy effective theories for various Kaluza-Klein, superstring, and supergravity theories (see [18]).

The calculations of Λ in this paper depends only on the amplitude of density perturbations, δ_H. Yet the character of the spectrum of perturbations may also help to rule out various inflationary schemes [10,14,19]. Ordinary extended inflation, for example, predicts a rather steep tilt away from a scale-invariant (Harrison-Zel'dovich) spectrum of density perturbations, which appears to contradict COBE data. Determination of the tensor mode contributions versus scalar modes in the density perturbation spectrum of induced-gravity inflation is made more difficult because of the deviation of Eq. (7) from a simple power-law solution, and is the subject of further study. At early times at least, when the evolution of $a(t)$ is roughly proportional to t^γ, the present model would yield a tiny tilt away from scale-invariance: when $a(t)\propto t^\gamma$, the spectral index goes as $n=1-2/(\gamma-1)$ [9], which in this case (with $\omega\simeq500$) would give $n=0.996$. For more on the possibility of "observing" the inflation potential based on the contributions from tensor mode perturbations, see [14,19].

One point of concern for induced-gravity inflation is the scale at which it operates: unlike most other inflationary schemes, which study phase transitions associated with the breaking of a GUT symmetry (at an energy of around 10^{14} to 10^{16} GeV), induced-gravity inflation is associated with the Planck scale. This could lead to conflict with the value of $V(\phi)$ at the time of last horizon crossing. References [12,14] show that present COBE data appear to limit $V^{1/4}(t_*)\sim(3\text{–}4)\times10^{16}$ GeV, which, for the present model (with $\omega\simeq500$ and $\lambda\simeq10^{-6}$), would require $\phi(t_*)$ to be *very* close to v. Yet, as pointed out in [14], uncertainties in the data lead to an entire order of magnitude range in the value for $V^{1/4}(t_*)$, so the present model cannot be ruled out by these COBE results. For more on constraints on the energy scale of inflation, see [20].

A theoretical difficulty for induced-gravity inflation stemming from its high energy scale is how to combine it with a "realistic" particle physics sector. (Recent work with extended technicolor as a means of achieving Planck-scale unification of gauge couplings [21] might offer a means of connecting an induced-gravity inflation model with realizable particle physics models.) Yet what it might lose on the particle side, it gains on the gravitational side: it should be much easier to relate the present model to a specific higher-energy gravitational theory. Or the model might be useful as part of a "double-inflation" scheme, in which the induced-gravity phase transition (which, as we have seen above, could solve the flatness and horizon problems rather easily, and lead to an acceptable amplitude of density perturbations) is followed by a related GUT transition at a lower energy (which would then only need to solve the monopole density problem, so the requirements for this second epoch of inflation would be greatly relaxed). (For earlier attempts to use the original model of extended inflation in a double-inflation scenario, see [22].) Although these details have yet to be worked out, the prospect of a well-motivated inflationary scenario which requires $\Lambda\sim10^{-6}$ rather than $\sim10^{-15}$ remains an encouraging result.

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