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General relativistic effects in preheating

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Abstract

General relativistic effects in the form of metric perturbations are usually neglected in the preheating era that follows inflation. We argue that in realistic multi-field models these effects are in fact crucial, and the fully coupled system of metric and quantum field fluctuations needs to be considered. Metric perturbations are resonantly amplified, breaking the scale-invariance of the primordial spectrum, and in turn stimulate scalar field resonances via gravitational rescattering. This non-gravitationally dominated nonlinear growth of gravitational fluctuations may have significant effects on the Doppler peaks in the cosmic background radiation, primordial black hole formation, gravitational waves and nonthermal symmetry restoration. © 1999 Elsevier Science B.V. All rights reserved.

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As recent ideas about the end of an inflationary era have shown, post-inflation reheating was one of the most violent and explosive processes occurring in the early universe [1–3]. The nonequilibrium, nonperturbative, resonant decay of the inflaton was demonstrated in Minkowski spacetime. The first steps toward a gravitationally self-consistent treatment, which modeled inflaton decay in an expanding, dynamical background spacetime, revealed that “preheating” may proceed with qualitative, as well as quantitative, differences from the Minkowski case [2,3]. Yet even these studies neglected an essential feature of gravitational physics: the production and amplification of metric perturbations attending such a sudden transfer of energy from the oscillating inflaton to higher-momentum particles. Other papers

began the study of metric perturbations induced during preheating: in [4] a perfect-fluid analysis with Born decay was used, and crucial features of preheating were thus not incorporated; in [5] these limitations are avoided, but the effects of the amplified metric perturbations on the process of preheating itself are not considered.

In this Letter, we pursue a more self-consistent relativistic treatment of preheating, by studying both the field fluctuations (responsible for particle production) and the coupled metric perturbations (describing gravitational fluctuations in the curvature). In the process, we clarify the question of causality and the amplification of long-wavelength perturbations during preheating. We give qualitative arguments, confirmed by numerical results (see also [6]), to show

that *metric perturbations typically undergo rapid growth during multi-field preheating, and in turn act as a source and a pump for the growth of field fluctuations via gravitational rescattering*, ultimately making the preheating process even more efficient than previously realized. Neglecting this general relativistic effect can produce misleading results.

Moreover, nonlinear growth of metric perturbations is itself of potentially major significance, since it precipitates nonlinear density fluctuations, leading to strong mode-mode coupling effects, and nonlinear deviations from the conformally flat background. This could affect observable quantities such as the cosmic microwave background (CMB) spectrum, complicating the usual predictions from inflationary models. Gravitational wave power could be enhanced by gravitational bremsstrahlung, and primordial black hole production could occur without the need for special properties in the power spectrum.

We work with the gauge-invariant formalism of [7] to study the evolution of scalar perturbations of the metric. (The amplification of gravitational waves at preheating has been considered in [8].) For scalar perturbations, in the case of a scalar-field energy-momentum tensor and spatially flat background, the perturbed metric in the longitudinal gauge is

$$ds^2 = a^2(\eta) [(1 + 2\Phi) d\eta^2 - (1 - 2\Phi) dx^2],$$

where Φ is the gauge-invariant gravitational potential.

Realistic reheating models involve at least one field coupled to the inflaton $\varphi(\eta)$. First we consider models with only an inflaton, in order to give a simple illustration of some effects. The gauge-invariant field fluctuations $\delta\varphi$ and the metric perturbations Φ obey the coupled Eqs. (6.40–42) of [7]. Defining the rescaled fields $Y \equiv a\Phi$ and $X \equiv a\delta\varphi$, these become, in momentum space:

$$Y_k'' + \left[k^2 - \frac{1}{2}\kappa^2\varphi'^2 \right] Y_k = \kappa^2\varphi'' X_k, \quad (1)$$

$$X_k'' + \left[k^2 + a^2 V_{\varphi\varphi} - \frac{3}{2}\kappa^2\varphi'^2 - 2\mathcal{H}^2 \right] X_k = 2\varphi'' Y_k, \quad (2)$$

$$Y_k' = \frac{1}{2}\kappa^2\varphi' X_k, \quad (3)$$

where $V(\varphi)$ is the potential, $\mathcal{H} = d'/a$ and $\kappa^2 = 8\pi M_{\text{pl}}^{-2}$.

Eq. (3) is a constraint, showing that if there is explosive growth in the field fluctuations X_k – the

heart of preheating – then this automatically constrains the gravitational fluctuations Y_k to follow, and *vice versa*. This equation essentially says that if you “shake” the right-hand side of Einstein’s field equations $G_{\mu\nu} = \kappa^2 T_{\mu\nu}$, then unavoidably you are simultaneously “shaking” the left hand side. Clearly, neglecting the metric perturbations Y_k can be seriously misleading under many conditions, since it is tantamount to ignoring the perturbed Einstein equations. This may be reasonable in a slow-roll inflationary regime, but generally not in an oscillatory regime.

A clear illustration is provided by super-Hubble modes in the simplest model, $V = \frac{1}{2}m^2\varphi^2$. If $\delta\varphi_*$ is the field fluctuation calculated by neglecting metric perturbations, then for $k \rightarrow 0$, $X_* \equiv a\delta\varphi_*$ satisfies Eq. (2) with gravitational fluctuations eliminated, i.e.

$$X_*'' + \left[m^2 a^2 + \frac{1}{2}\kappa^2\varphi'^2 - 2\mathcal{H}^2 \right] X_* = 0.$$

This equation is then of precisely the same form as the *background* Klein-Gordon equation, so that $\delta\varphi_* \propto \varphi \approx \varphi_0 \sin(b\eta^3)/(b\eta^3)$, where φ_0 and b are constants. The approximation arises from using the time-averaged scale factor $\bar{a} \propto \eta^2$, and improves in accuracy as η increases. When metric perturbations are incorporated, the long-wavelength solution is given in general by [see [7], Eq. (6.57)] $\delta\varphi \propto \varphi' a^{-2} \int a^2 d\eta$. We find that at reheating $\delta\varphi \approx \frac{3}{5}\varphi_0 \cos(b\eta^3)$, to lowest order in η^{-1} . Gravitational rescattering produces a non-decaying term in the field fluctuations, which dominates the rapidly decaying fluctuation $\delta\varphi_*$ calculated by neglecting gravitational fluctuations. Relating in the usual way [1,3] the field fluctuations to the density of particles produced per mode, we find that this non-decaying solution indicates nonzero particle production. If gravitational perturbations are neglected, *no* particle production is found for this model. *Gravitational rescattering dramatically alters the evolution of the matter–field fluctuations even in this simplest of all models, especially on super-Hubble-radius scales.* Since preheating concerns primarily the behavior of such matter–field fluctuations after inflation, it is thus crucial to study the coupled metric–perturbation–field-fluctuation system.

Now we consider the amplification of gravitational fluctuations. The constrained system of equa-

tions Eqs. (1)–(3) has one degree of freedom, reflected in the decoupled equation

$$Y_k'' - 2(\varphi''/\varphi')Y_k' - \left(\frac{1}{2}\kappa^2\varphi'^2 - k^2\right)Y_k = 0.$$

To avoid the periodic singularities when $\varphi' = 0$, Nambu and Taruya [5] employ the rescaled Mukhanov variable

$$\tilde{Q} = a^{(2-n)/(n+1)}[X + (\varphi'/\mathcal{H})Y],$$

where n is the index in the power-law potential $V = \frac{1}{2}m^2\varphi_0^2(\varphi/\varphi_0)^{2n}$. Using the time-averaged forms of a and φ , they find that, to leading order in $\tau^{-1} \propto \eta^{3/(1-2n)} \propto a^{-3/(n+1)}$,

$$\tilde{Q}_{\tau\tau} + [1 + k^2\gamma^2\tau^{4(n-2)/3} - (4/\tau)\sin 2\tau]\tilde{Q} = 0, \quad (4)$$

where γ is a constant. This equation has Mathieu form, $y'' + [A_k - 2q \sin 2x]y = 0$, with time-dependent A_k and q , so that modes can be drawn through instability bands by the expansion of the universe, if

they were not already there initially [5,9]. Note that the resonance parameters scale as

$$q/A_k \propto a^{-3/(n+1)}(\beta^2 + k^2 a^{4(n-2)/(n+1)})^{-1},$$

where β is constant. If metric fluctuations are neglected, then $q/A_k \propto a^{-3}$. Therefore, for $n < 8$, expansion is less effective in ending resonance when metric fluctuations are incorporated.

Further discussion of single-field models is given in [10,11], which confirm the analytical conclusions of [5]: in the $n = 1$ (quadratic potential) model, there is no resonance in the long-wavelength limit, whereas large, resonant growth is found for other models, such as the massless, quartically-coupled case ($n = 2$). However, as pointed out above, the single-field case is completely inadequate as a model of reheating, and we turn now to consider the multi-field case.

The rapid growth of metric perturbations in the multi-field case (see Fig. 1) will produce a backreac-

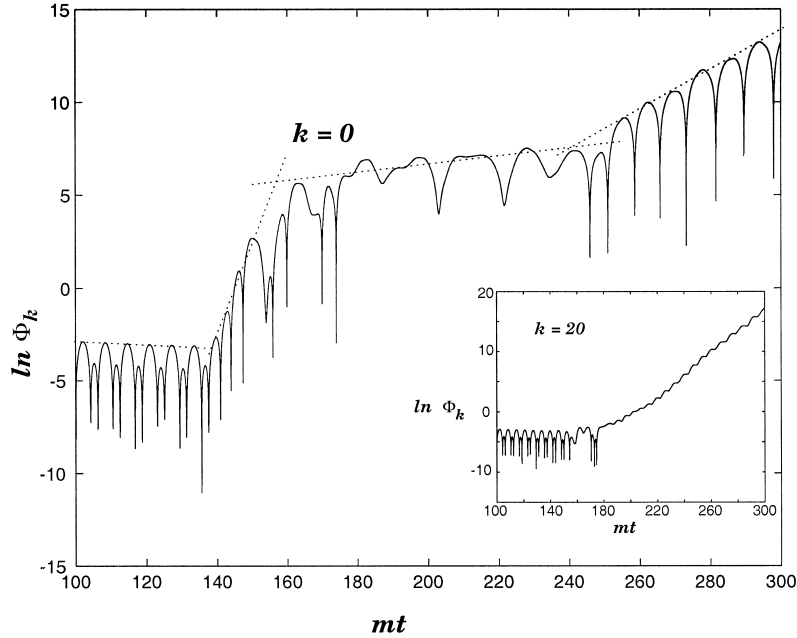


Fig. 1. Metric perturbation evolution in 2-field preheating, with $q \equiv g\varphi_1^2(t_0)/m^2 = 8 \times 10^3$. The main graph shows the $k = 0$ mode, the inset the $k = 20$ mode. Initially, $mt_0 = 100$ (well into the small amplitude phase: $\varphi_1(t_0)/m_0 = 3 \times 10^{-3}$ and $\Phi_k(t_0) = 10^{-5}$, $\dot{\Phi}_k(t_0) = 0$). The $k = 0$ mode becomes nonlinear at $mt \sim 150$ after less than 10 inflaton oscillations (i.e. well before the end of preheating) and continues growing without bound. Two strong resonance bands are evident, with different Floquet indices (given by the slopes of the dotted lines), for $140 \leq mt \leq 160$ and $250 \leq mt \leq 300$.

tion on the background quantities a and φ [12]. Similarly, the amplified field fluctuations will grow to be of the same order as the tree-level terms, such as φ^2 , and hence will damp the inflaton's oscillations, ending the parametric resonance. Let us consider an effective single-field model. We may then estimate these two time-scales, expecting the initial preheating phase to end at $\eta_{\text{end}} = \min\{\eta_m, \eta_r\}$, where $\langle \Phi^2(\eta_m) \rangle = 1$ and $\langle \delta\varphi^2(\eta_r) \rangle = \varphi^2$. In a resonance band, $\Phi_k = F_k(\eta)e^{\mu_k\eta}$, where F_k oscillates and μ_k is the Floquet exponent. From Eq. (3), and working in the saddle-point approximation, we may estimate

$$\begin{aligned} \langle \Phi^2 \rangle &= (2\pi)^{-3} \int d^3k |\Phi_k|^2 \\ &\sim \kappa^4 \varphi'^2 \langle \delta\varphi^2 \rangle / (4\mu_{k_{\text{max}}}^2), \end{aligned}$$

where $\mu_{k_{\text{max}}}$ is the maximum Floquet exponent. For single-inflaton models of chaotic inflation with polynomial potentials, the slow-roll conditions are violated, and the inflaton begins to oscillate at $\varphi_0 = \alpha M_{\text{pl}}$, with $\alpha \sim 0.3$. In models with a massive inflaton, we may further approximate $|\varphi'| \sim \alpha m M_{\text{pl}}$. The field fluctuations saturate their (linear-theory) upper limit at $\langle \delta\varphi^2 \rangle \sim \varphi^2$. Combining these yields $\langle \Phi^2(\eta_m) \rangle \sim 16\pi^2 \alpha^4 m^2 / \mu_{k_{\text{max}}}^2$. The specific spectrum of fluctuations, governed by the values of μ_k , depends on details of the potential. For modes subject to a parametric resonance, $\mu_{k_{\text{max}}} \sim \alpha m$, whereas modes subject to a negative-coupling instability may have $\mu_{k_{\text{max}}} \sim O(m)$ [1–3,13]. Thus first-order analysis reveals that gravitational backreaction will become relevant at around the same time as the backreaction of the nonlinearly-coupled field fluctuations.

Another important issue is to demonstrate how super-Hubble modes may be amplified causally. In inflation, modes which had been deep within the Hubble radius *during* inflation become amplified and stretched to super-Hubble scales [7,14]. Reheating was believed not to be able to affect these super-Hubble scales. In preheating, however, the coherence of the inflaton condensate immediately after inflation *does* allow for super-Hubble dynamics. At first, such behavior might appear to violate causality. This is not the case; consider the following: (1) the field equations are relativistic, hence causality is automatically built into the solutions; (2) causality concerns space-like related events, and does not translate into direct constraints on individual modes

in Fourier space [15]; (3) explicit calculation of the unequal-time two-point correlation function reveals that no mass or energy is being transported superluminally by these super-Hubble resonances. See [6] for this calculation and further discussion. Similar conclusions regarding the possibility for the causal amplification of super-Hubble modes at preheating have been reached in [10]. The main point we wish to emphasize is that causality restricts the *shape of the spectrum* of amplified modes, but not directly the wavelengths that can be amplified.¹

Preheating in single-field models is typically restricted to the narrow-resonance regime, and we may expect much larger effects in models with multiple scalar fields coupled to the oscillating inflaton, because the resonance parameter, q , may be much greater than unity [3,6]. The dynamics of such coupled-oscillator systems are in general chaotic [16] and lead to enhanced particle production [17]. The multi-field generalization of Eqs. (1)–(3) can be given as [18,6]

$$\begin{aligned} 3H\dot{\Phi} + [(k/a)^2 + 3H^2]\Phi \\ = -\frac{1}{2}\kappa^2\Sigma[\dot{\varphi}_i(\delta\varphi_i) - \Phi\dot{\varphi}_i^2 + V_i\delta\varphi_i], \end{aligned} \quad (5)$$

$$\begin{aligned} (\delta\varphi_i)'' + 3H(\delta\varphi_i)' + (k^2/a^2)\delta\varphi_i \\ = 4\dot{\Phi}\dot{\varphi}_i - 2V_i\Phi - \Sigma V_{ij}\delta\varphi_j, \end{aligned} \quad (6)$$

$$\dot{\Phi} + H\Phi = \frac{1}{2}\kappa^2\Sigma\dot{\varphi}_i\delta\varphi_i, \quad (7)$$

dropping the mode label k , using proper time t , and writing $V_i = \partial V / \partial\varphi_i$. Eq. (7) shows that resonant amplification of field fluctuations is accompanied by similar behavior of gravitational fluctuations. In the single-field case, any resonant growth occurs with the same characteristics for metric and field fluctuations, and stability bands are thus also the same. In the multi-field case, this simple relation is broken, and the stability band structure is much more complicated. The non-inflaton fields φ_i ($i > 1$) grow rapidly under resonance, while the inflaton φ_1 is strongly damped. Thus *metric fluctuations Φ grow more quickly than any of the field fluctuations $\delta\varphi_i$*

¹ The convenient approximation sometimes used in preheating, that the distribution of amplified modes falls as a spike, $\delta(k - k_{\text{resonance}})$, violates causality, since this requires that the field fluctuation contain correlations on all length scales, even for modes with $k/a \gg H$.

due to the quadratic products $\varphi_i \delta\varphi_i$. In models which include a substantial broad resonance regime for the coupled field fluctuations, resonance parameters are much larger than in the single-field case, typically in the range $q \sim 10^2 - 10^6$, rather than $q \leq O(1)$ [2], so the associated Floquet indices are much larger [2,3].

These qualitative remarks are illustrated in Fig. 1 and amplified in the extensive simulations in [6], arising from integrating Eqs. (5)–(7) for the potential $V = \frac{1}{2}m^2\varphi_1^2 + \frac{1}{2}g\varphi_1^2\varphi_2^2$, which describes decay of the massive inflaton φ_1 into the boson field φ_2 . The graph shows the resonant amplification and nonlinear growth of gravitational fluctuations on both super- and sub-hubble scales. Also clear is the validity of the Floquet index as a characterizer of growth at strong resonance in an expanding universe (the slopes in the resonance bands are very nearly constant). The expansion has the effect of pulling modes through the resonance bands leading to phases of explosive growth and quiescence. Note that even with the expansion of the universe included, the Floquet index is a useful concept at broad resonance – the growth of Φ_k is almost exactly exponential in the resonance bands, as is evident in Fig. 1. A further crucial point, with wide-ranging implications, is that the non-gravitationally dominated evolution of perturbations during preheating breaks the scale-invariance of the primordial spectrum, as is evident from comparing the evolution of the $k = 0, 20$ modes in Fig. 1 [6].

The numerical simulations in [6] reveal that inclusion of the coupled metric perturbations *enhances* the strength of the resonances in $\delta\varphi_i$, compared with when Φ is neglected. This can be understood by the fact that the gravitational field has negative specific heat. For this reason, Φ does not act as a ‘parasite’ or ‘competitor’ with the $\delta\varphi_i$ for the energy of the oscillating inflaton, but rather can serve as a source or pump for field fluctuations growth.

Preheating in the narrow resonance and broad resonance regimes shows another important qualitative difference: broad-resonance preheating in an expanding universe proceeds *stochastically* [3]. The phases of the amplified field modes are virtually uncorrelated between each moment when the oscillating inflaton passes through zero. Both the field modes and the metric perturbations will have

stochastic driving terms. It has been shown that such stochastic terms in general remove all stability bands, so that *all* modes $k \geq 0$ grow subject to a parametric resonance, with, in general, larger characteristic exponents than in the simple periodic case [19,17].

Note that once the fluctuations have become strongly nonlinear, the linearized perturbation equations are no longer valid, and mode-mode coupling, arising due to convolutions which are ignored in the linearized Eqs. (5)–(7), must be included [3].

Nonlinear gravitational fluctuations could produce various observable signatures, in particular on the CMB. Firstly, if the preheating phase is followed by a second round of inflationary expansion, then sub-hubble amplified modes would get stretched into observationally-interesting scales. Such double-inflation is typical for most realistic scenarios based on supergravity or supersymmetry [20]. Secondly, acoustic Doppler peaks at $\ell \geq 100$ could also be affected. These are a key prediction of most inflationary models. In contrast, defect models often predict no secondary peaks and a shift in the position of the first peak, because the metric perturbations are produced randomly, receiving out of phase ‘kicks’ due to the mode-mode coupling inherent in the non-linearity of defect models [21].

In the multi-field, broad-resonance case, stochastic amplification leads to nonlinear metric fluctuations, producing mode-mode coupling which drives the unequal-time correlation function $\Delta \rightarrow 0$ for $|\tilde{\eta} - \eta| > \eta_c$, the coherence time of the system. As $\eta_c \rightarrow 0$ we get δ -correlated (white) noise [17], memory of initial conditions (and hence coherence) is lost, and the field evolution mimics the *active*, incoherent evolution of defects. The crucial question is whether the nonlinear mode-mode coupling survives local interactions and persists up to nucleosynthesis and photon decoupling. If so, there could be significant limits placed on inflationary reheating by observed element abundances and small-scale CMB anisotropies. By smearing out the Doppler peaks, surviving nonlinear mode-mode coupling would greatly reduce the effectiveness of small-angle CMB observations for differentiating inflationary scenarios from defects models. The implied nonlinearity would lead to hybrid CMB anisotropies – passive and typically inflationary on large angular scales, active and defect-like on smaller scales.

Another possible observational consequence arises from gravitational bremsstrahlung in scattering of the large scalar perturbations [8]. Combined with our analysis of causality above, the extra gravitational wave power due to the previously neglected metric fluctuations implies that the probability of detection in instruments such as LIGO may be higher than previously estimated, at least for preheating following chaotic inflation. Nonlinear gravitational fluctuations could also create the large density contrast necessary to produce primordial black holes (PBHs) [22], without the need for a blue spectrum or a large peak in the power spectrum at some k . PBH limits may then be able to constrain the nature of preheating and the associated amplification of metric perturbations.

Finally, amplified metric perturbations affect non-thermal symmetry restoration at preheating [23], by altering the inflaton's effective potential via addition of the terms (see Eq. (10.68) of [7]):

$$\Delta V = a^2 V_{\varphi\varphi} \langle \delta\varphi^2 \rangle + 2a^2 V_{\varphi} \langle \delta\varphi \Phi \rangle.$$

The first term was originally considered in [23], while the second is due to the direct (gravitational) coupling between $\delta\varphi$ and Φ . It is of the same perturbative order as the first term, and is missed if gravitational fluctuations are neglected in preheating.

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References

- [1] L. Kofman, A. Linde, A. Starobinsky, Phys. Rev. Lett. 73 (1994) 3195; Y. Shtanov, J. Traschen, R. Brandenberger, Phys. Rev. D 51 (1995) 5438; D. Boyanovsky et al., Phys. Rev. D 51 (1995) 4419; 52 (1995) 6805; M. Yoshimura, Prog. Theo. Phys. 94 (1995) 8873; H. Fujisaki et al., Phys. Rev. D 53 (1996) 6805; earlier perturbative studies of resonance included J. Traschen, R.H. Brandenberger, Phys. Rev. D 42 (1990) 2491; A. Dolgov, D. Kirilova, Sov. J. Nuc. Phys. 51 (1990) 172.
- [2] D. Kaiser, Phys. Rev. D 53 (1996) 1776; S.Y. Khlebnikov, I.I. Tkachev, Phys. Rev. Lett. 77 (1996) 219; 79 (1997) 1607; Phys. Lett. B 390 (1997) 80; T. Prokopec, T.G. Roos, Phys. Rev. D 55 (1997) 3768; D. Boyanovsky et al., Phys. Rev. D 56 (1997) 1939; I. Zlatev, G. Huey, P.J. Steinhardt, Phys. Rev. D 57 (1998) 2152; B.A. Bassett, S. Liberati, Phys. Rev. D 58 (1998) 021302.
- [3] L. Kofman, A. Linde, A. Starobinsky, Phys. Rev. D 56 (1997) 3258.
- [4] H. Kodama, T. Hamazaki, Prog. Theo. Phys. 96 (1996) 949, 1123.
- [5] Y. Nambu, A. Taruya, Prog. Theo. Phys. 97 (1997) 83; Phys. Lett. B 428 (1998) 37.
- [6] B.A. Bassett, F. Tamburini, D.I. Kaiser, R. Maartens, hep-ph/9901319; <http://www-thphys.physics.ox.ac.uk/WWW/Users/BruceBassett/reheating/>
- [7] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, Phys. Rep. 215 (1992) 203.
- [8] S.Y. Khlebnikov, I.I. Tkachev, Phys. Rev. D 56 (1997) 653; B.A. Bassett, Phys. Rev. D 56 (1997) 3429.
- [9] H.E. Kandrup, astro-ph/9903434.
- [10] F. Finelli, R.H. Brandenberger, Phys. Rev. Lett. 82 (1999) 1362.
- [11] M. Parry, R. Easther, Phys. Rev. D 59 (1999) 061301.
- [12] V. Mukhanov, L. Abramo, R. Brandenberger, Phys. Rev. Lett. 78 (1997) 1624; Phys. Rev. D 56 (1997) 3248.
- [13] P.B. Greene, T. Prokopec, T.G. Roos, Phys. Rev. D 58 (1997) 6484.
- [14] A.D. Linde, Particle Physics and Inflationary Cosmology Harwood, Chur, 1990; E.W. Kolb, M.S. Turner, The Early Universe, Addison-Wesley, Redwood City, CA, 1990; A. Liddle, D. Lyth, Phys. Rep. 231 (1993) 1.
- [15] J. Robinson, B.D. Wandelt, Phys. Rev. D 53 (1996) 618; N. Turok, Phys. Rev. D 54 (1996) 3686; Phys. Rev. Lett. 77 (1996) 4138.
- [16] N.J. Cornish, J.J. Levin, Phys. Rev. D 53 (1996) 3022; R. Easther, K. Maeda, Class. Quantum Grav. 16, in press, 1999 (gr-qc/9711035).
- [17] B.A. Bassett, F. Tamburini, Phys. Rev. Lett. 81 (1998) 2630.
- [18] L. Kofman, D. Pogosyan, Phys. Lett. B 214 (1988) 508.
- [19] V. Zanchin et al., Phys. Rev. D 57 (1998) 4651; B.A. Bassett, Phys. Rev. D 58 (1998) 021303.
- [20] J.A. Adams, G.G. Ross, S. Sarkar, Nucl. Phys. B 503 (1997) 405; R. Jeannerot, Phys. Rev. D 53 (1996) 5426; D.H. Lyth, E.D. Stewart, Phys. Rev. Lett. 75 (1995) 201.
- [21] A. Albrecht et al., Phys. Rev. Lett. 76 (1996) 1413; J. Magueijo et al., Phys. Rev. Lett. 76 (1996) 2617; Phys. Rev. D 54 (1996) 3727.
- [22] J. Bullock, J. Primack, Phys. Rev. D 55 (1996) 7423.
- [23] L. Kofman, A. Linde, A. Starobinsky, Phys. Rev. Lett. 76 (1996) 1011; I.I. Tkachev, Phys. Lett. B 376 (1996) 35.