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A ψ is just a ψ? Pedagogy, Practice, and the Reconstitution of General Relativity, 1942–1975

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1. Introduction: Problems and the Training of 'Renaissance Relativists'

> No one expects a guitarist to learn to play by going to concerts in Central Park or by spending hours reading transcriptions of Jimi Hendrix solos. Guitarists practice. Guitarists play the guitar until their fingertips are calloused. Similarly, physicists solve problems. And hopefully, physicists practice solving problems until doing so seems easy. (Then they find harder problems.)[1]

When Albert Einstein penned a foreword for his friend and Princeton-neighbour Peter Bergmann's 1942 textbook, *Introduction to the Theory of Relativity*, he exhorted its readers: 'I believe that more time and effort might well be devoted to the systematic teaching of the theory of relativity than is usual at present at most universities.'[2] Bergmann himself similarly remarked that in the 1940s: 'You only had to know what your six best friends were doing, and you would know what was happening in general relativity.'[3] Einstein's elegant theory of gravitation, completed in 1915–1916, had by the 1940s nearly disappeared from the training of American graduate students in physics.

A quick survey of the course offerings at several universities confirms the dearth of attention to general relativity noted by Einstein, Bergmann and others. At Princeton, home of these gravitational physicists, general relativity was taught in the mathematics department until 1954, when Math 570 became

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[1] This is the colourful opening declaration to the recently published compilation of physics problems, *Princeton Problems in Physics, with Solutions* (Newbury *et al.*, 1991, p. xv).

[2] Albert Einstein, 'Foreword', in Bergmann (1942, p. v).

[3] Bergmann is quoted in Will (1993 [1986], p. 11). Similar remarks were made by Robert Dicke in his 1964 Les Houches lectures (Dicke, 1965, pp. 1–3), and in his interview in Lightman and Brawer (1990, p. 204).

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Physics 570, taught by the physicist John Wheeler. At Harvard, the mathematician Birkhoff taught his idiosyncratic approach to general relativity in the mathematics department through the 1940s, but no physicist taught the subject until Sidney Coleman began to teach 'The Theory of Relativity' (Physics 210) in 1967. Sounds of 'gee-mu-nu' were hardly echoing through the halls of MIT's 'infinite corridor': not a single physics or mathematics course at MIT focussed on general relativity throughout the 1950s.[4] Similarly, in 1961, neither Columbia University nor the University of California at Berkeley required or recommended coursework in general relativity for their physics graduate students.[5] Even when coursework was offered to physics graduate students, as at Princeton in the late 1950s, no questions on the theory of general relativity were posed on the graduate student general examinations.[6]

And yet, like so many changes during the tumultuous 1960s, general relativity experienced a 'revolution'. The physicist Clifford Will has termed the changes during this decade 'the renaissance of general relativity'. On Will's reading, a number of discoveries (quasars, pulsars, cosmic microwave background radiation) throughout the 1960s thrust the study of general relativity and gravitational fields into a new spotlight.[7] By the late 1960s, 'gravity groups' of physicists and graduate students could be found at places like Princeton and Caltech, working actively in experimental and theoretical problems in general relativity. After 'begging' for names and addresses of physicists specialising in general relativity in 1961, the Swiss-based International Society for General Relativity and Gravitation could count over 220 members in its ranks by 1974; and over 800 participants crowded into the conference halls for the Ninth Texas Symposium on Relativistic Astrophysics, held in December 1978.[8] From very

[4] The information on course offerings is taken from each of the university's course catalogues, deposited in the various university archives. General relativity was listed as one topic among several to be covered in some applied mathematics courses during this period, but was not the subject of its own course in the physics department. This matches the general pattern noted by Eisenstaedt (1989).

[5] See Appendix 15 in the unpublished report by W. C. Kelly, 'Survey of Education in Physics in Universities of the United States', dated 1 December 1962. Copies of his report may be found in the American Institute of Physics, 'Education and Manpower Division', *Records, 1951–1973, Box* 9; deposited in the American Institute of Physics Archives, Niels Bohr Library, College Park, Maryland.

[6] Interview of Professor Robert H. Dicke with the author, 10 March 1995. The 1961 qualifying examinations and general examinations from Columbia, Berkeley, and MIT, included in Appendix 19 of the Kelly Report, contain no questions on general relativity or gravitation. Harvard physics graduate students had to pass an oral, rather than a written, qualifying examination, a list of 'suitable topics' for which, drafted in 1962, similarly revealed the absence of gravational physics. This memorandum may be found in the AIP's unpublished "Institutional Histories" Harvard University, catalogue number IH95, Niels Bohr Library.

[7] See Will (1993 [1986], especially Chapter 1, pp. 3–18). Cf. 'Introduction to Modern Cosmology' in Lightman and Brawer (1990, pp. 1–49).

[8] See the form letter from Professor André Mercier addressed 'To Scientists throughout the World active in the field of Theories of Relativity and Gravitation', dated January 1961. A decade later, in July 1971, Mercier and others had established the International Society for General Relativity and Gravitation. A list of its membership from 1974 includes 221 names and addresses, organised by country. Seventy-one of these members were working in the United States. These papers are

humble beginnings, a community of physicists dedicated to the study of general relativity had been formed.

Throughout this period of explosive growth in the numbers of practitioners, however, the theory of 'general relativity' did not remain a constant, static physical theory. Instead, it was reworked, and its methods of calculation reconsidered. Readers of the colossal 1973 textbook by Charles Misner, Kip Thorne, and John Wheeler, entitled simply *Gravitation* experienced a theory which *looked different* from what had passed under the same title in Bergmann's 1942 text. In the process, differences in the practice of gravitational physics emerged at every level: conceptually, ontologically, calculationally, pedagogically, and sociologically.

By studying how various textbooks presented the subject of general relativity, and what kinds of problems these texts asked students to calculate, we can follow a transition from a predominantly geometrical conception of gravitation to a more dynamical one. The geometrical conception, as typified by Bergmann's text, focussed on the mathematical properties of the non-Euclidean nature of spacetime, as described by the Riemann curvature tensor and its contractions. Here the tools and approach came from mathematicians' differential geometry and tensor calculus. The latter form of general relativity shifted attention to a more field-theoretic approach, emphasising Lagrangian techniques for formulating, approaching, and solving problems in gravitation. On this view, gravitation arose as other forces did, from the exchange of a dynamical field propagating through the flat spacetime of special relativity.[9] During the 1960s, strains of both of these approaches to general relativity could be found, although with increasing influence felt from the Lagrangian-toting field theorists. In this brief paper, I will limit attention to the contrast between Einstein's and Bergmann's pedagogical route to general relativity with that of the particle theorists Richard Feynman and Sidney Coleman.[10]

[8] (*continued*)
deposited in the 'International Society for General Relativity and Gravitation', *Records, 1961–1982*, held in the American Institute of Physics Archives, Niels Bohr Library. On the Ninth Texas Symposium, see Will (1993 [1986], p. 15).

[9] This is not to suggest that there was a clear, uniform transition from a geometrical to a dynamical view. The novelty of the 1973 Misner, Thorne, and Wheeler textbook, *Gravitation*, for example, lay in its new-found geometrical emphases, now pursued with more modern mathematical representations of tensors. Their new approach famously made gravitation synonymous with 'geometrodynamics'. See Misner *et al.* (1973).

[10] This paper is part of a larger project on changes in the practice of gravitational physics during the middle decades of this century. The more complete study aims to relate these pedagogical changes (in textbooks, lecture notes, syllabi, problem sets, and general examinations) to changes in the published research articles within gravitational physics. Of course there are limits in studying published textbooks apart from other elements of the 'pedagogical apparatus'—for one, such a focus tends to downplay the *active* nature of teaching and reading within a classroom setting, in favour of assuming some transparent, single meaning carried by the text itself. It is not my intention to restrict attention to this level, though in this brief paper the texts themselves may be read and compared profitably to begin to tease out specific changes in the kinds of calculational tools and practices emphasised within various pedagogical traditions.

This study of changes in the practice of general relativity thus proceeds with the understanding that the history of physical theories cannot be formulated as a problem of 'conception' and 'reception'. Instead, as emphasised by Andrew Warwick's study of British responses to Einstein's 1905 paper on special relativity, I will treat physical theories as collections of *practices*—calculational approaches and techniques.[11] As we will see, Feynman denied any role to Einstein's beloved geometry, while proclaiming all the while that he was teaching students to *do* gravitational physics. Thus, rather than telling this story as one of a passive reception of a more or less 'correct' understanding of Einstein's equivalence principle, I see the episode instead as an active encounter and deployment, on many of the field theorists' part, of specifically field-theoretic practices. Every step in the process of teaching and studying gravitational physics, from the initial foundation and derivation of the governing equations, to the later calculation of specific problems, proceeded along distinct directions, calling on the manipulation of distinct mathematical quantities, in analogy to other, quite distinct mathematical and physical theories.

This theme of different approaches to the practice of theory encourages a focus on pedagogy; after all, theory as 'practice' is something which must be *practised.* The burgeoning membership in general relativity associations and conferences during the 1970s required intense efforts to train students in the calculational specificities of gravitational physics. A careful attention to lectures, textbooks, and their rewriting during this period, and to the problems assigned to graduate students as an integral part of their training, provides a means with which to trace the reconstitution of general relativity during the middle decades of this century.

2. Einstein, Bergmann and the Geometrical Foundations of Gravitation

Einstein's own route to finding the field equations governing his theory of general relativity between 1911–1915 was notoriously convoluted.[12] Though flirting briefly in the fall of 1914 with a derivation based on an action principle and variational techniques, his final derivation in November 1915 eschewed this approach.[13] Later, Einstein consistently chose to rely on a simplified version of

[11] See Warwick (1992, 1993). Much of the motivation for this kind of approach stems from Thomas Kuhn's brief discussion of exemplars and the community-building function of problem-solving in his 1969 postscript to *The Structure of Scientific Revolutions* (Kuhn, 1970 [1962], pp. 187–204). A similar move has been made by philosophers Nancy Cartwright and Ronald Giere; see Cartwright (1983), especially Chapters 6–7; and Giere (1988), Chapter 3.

[12] See Norton (1984) and Stachel (1979). See also Eisenstaedt and Kox (1992); Pais (1982, pp. 177–265); and Cao (1997, pp. 45–103). Einstein himself provided some recollections on this process in 'Notes on the Origin of the General Theory of Relativity', originally published in *Mein Weltbild* (1934), and translated and reprinted in Einstein (1982 [1934], pp. 285–290).

[13] Einstein's post-Grossmann attempt to formulate his theory by means of an action principle may be found in his paper, Einstein (1914). Cf. Norton (1984, pp. 293–298).

his own more geometrical derivation when presenting his theory in pedagogical settings. This strong emphasis on geometrical foundations carried over to nearly all of the prominent textbooks and popular accounts of Einstein's famous theory through the 1950s.[14] And it was this means of derivation which his friend and former student, Peter Bergmann, chose to highlight in his 1942 textbook.

A clear example of Einstein's pedagogical means of deriving his field equations comes from his May, 1921 Stafford Little Lectures at Princeton University. In brief, Einstein 'derived' his famous theory of gravitation by beginning with the principle of equivalence, which united inertial and gravitational mass. Einstein explained that this foundational principle necessitated a shift from the Euclidean flat space of Newton's gravitational theory, *and* a shift from the Minkowski flat spacetime of special relativity, to *curved* spacetime. To describe this spacetime mathematically, Einstein introduced the notion of a metric tensor for the four-dimensional spacetime, $g_{\mu\nu}$, and the Riemann–Christoffel curvature tensor, $R^{\rho}_{\mu\sigma\tau}$. This curvature tensor, Einstein continued, contained combinations of $g_{\mu\nu}$ and its first two derivatives. If the components of the metric tensor were interpreted as gravitational potentials, then the curvature tensor might play the analogue of $\Delta\phi$, the Laplacian of the Newtonian potential. From here, working in explicit analogy to the Poisson equation governing Newtonian gravity, Einstein worked out the single possible candidate for his field equations: they had to be linear in the second derivatives of the $g_{\mu\nu}$ and have a vanishing divergence. Putting all this together, and fixing the constant of proportionality between the left-hand (geometrical curvature) side and the right-hand (energy–momentum) side by means of the Poisson equation, Einstein presented for his eager listeners his new theory of gravitation:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}. \tag{1}$$

Gravity, in Einstein's hands, was merely the local curvature of spacetime in response to a source of energy density. To study gravity, 'all' one needed to master was the geometrical description of non-Euclidean spaces, exemplified by the Riemann–Christoffel curvature tensor.

Following Einstein's lead, Bergmann sought to equip students of physics and mathematics with just this 'specialized mathematical apparatus', including 'tensor calculus and Ricci calculus' (Bergmann, 1942, p. vii).[15] Part II of his

[14] Prominent textbooks from this period include Weyl (1952 [1918]); Pauli (1958 [1921]); Eddington (1930 [1923]); Tolman (1934); Landau and Lifshitz (1951 [1941]); Møller (1952); and McVittie (1956). Popularisations of general relativity, many of which were read by students who would later pursue the topic in graduate school, similarly stressed its geometrical core: see Einstein (1961 [1916]); Eddington (1920); and Gamow (1965 [1947]). In this sense, Bergmann's text was a well-known example of a well-established tradition.

[15] The presence of such close similarities between Bergmann's and Einstein's pedagogical presentations should not be surprising: Einstein noted in his foreword to Bergmann's textbook that '[m]uch effort has gone into making this book logically and pedagogically satisfactory, and Dr. Bergmann has spent many hours with me which were devoted to this end' (p. v).

textbook, treating the general theory of relativity, proceeded along the same series of steps as had Einstein's Stafford Lectures. Chapter X, 'The Principle of Equivalence', provided nine pages of text (with almost no equations) to motivate to students that the equivalence of inertial and gravitational mass led necessarily to the notion of curved spacetime. In order to be able, eventually, to study the physical consequences of the equivalence principle, students of Bergmann's text next had to master Chapter XI, 'The Riemann–Christoffel Curvature Tensor'. Here, in fifteen pages, Bergmann expanded upon Einstein's brief introduction to the mathematics of Riemannian spaces. The metric tensor g_{ik} was introduced, and the Christoffel symbols $\{ik, l\}$ were defined as convenient combinations of g_{ik} and its first derivatives. Next the Riemann–Christoffel curvature tensor, now written as R^{n}_{ikl}, was defined and shown to be equivalent to a 'commutation law for covariant differentiation'. Six more pages spelled out, line by line, the detailed symmetry properties of the curvature tensor and its contractions (*ibid.*, pp. 161–174; quotation on p. 166).

In the next chapter, Bergmann swiftly proceeded to use precisely the same series of analogies to Poisson's equation as had Einstein in order to arrive at the gravitational field equations. It was a matter of pure geometry to find an appropriate divergenceless tensor consisting of no higher than the second derivatives of the $g_{\mu\nu}$, which might be proportional to $T_{\mu\nu}$. With this, he wrote down the gravitational field equations as Einstein had. Much like his mentor, Bergmann conducted his students directly from the mathematical geometry of curved spacetime to the governing equations of gravitation. Equipped now with Einstein's field equations, Bergmann proceeded posthaste to study the equations in their 'linearized' form, calculating the non-vanishing Christoffel symbols in this perturbative approximation, and arriving (ten pages later) at the geodesic equation for point-particle motion.

It was only at this point, after the reader had studied fifteen pages on the curvature tensor and another sixteen pages on the geometrical derivation of the field equations, that Bergmann offered a contrasting means of deriving Einstein's famous equations. In fewer than four pages, Bergmann outlined what had essentially been David Hilbert's 1915 derivation of the field equations, based on the variation of an action integral with respect to a variation of the metric, $\delta g_{\mu\nu}$.

Tellingly, Bergmann noted in this section merely that 'the relativistic field equations [(1)] *can be represented* as the Euler–Lagrange equations of a Hamiltonian principle' (*ibid.*, p. 100; emphasis added). Unlike his original derivation, in which each step was at least motivated by physical considerations (and more often than not calculated at length explicitly), here he simply posited the particular action integral to be manipulated:

$$I = \int R\sqrt{-g}\, d\xi^1 d\xi^2 d\xi^3 d\xi^4, \qquad \delta I = 0. \tag{2}$$

Bergmann quickly noted that 'straightforward' (though tacit) computation reveals that the variation of the Ricci tensor, $\delta R_{\mu\nu}$, was in fact the covariant divergence of a vector. One term, then, became the integral of an ordinary divergence, which vanished due to Gauss' theorem. In a similarly terse way, Bergmann continued (*ibid.*, p. 193) : '$\delta(\sqrt{-g}\,g^{\mu\nu})$ is simply

$$\delta(\sqrt{-g}\,g^{\mu\nu}) = \sqrt{-g}\left(\delta g^{\mu\nu} - \frac{1}{2}g_{\rho\sigma}g^{\mu\nu}\delta g^{\rho\sigma}\right).' \tag{3}$$

After multiplying through by $R_{\mu\nu}$, the variation of I became

$$\delta\int R\sqrt{-g}\,d\xi = \int G_{\mu\nu}\delta g^{\mu\nu}\sqrt{-g}\,d\xi. \tag{4}$$

Without fanfare, Bergmann concluded this section with the single sentence: 'The equations [(1)] are the Euler–Lagrange equations of the Hamiltonian principle [(2)]' (*ibid*).[16]

This quick demonstration was the only point throughout Bergmann's entire presentation of general relativity in which any attention was drawn to variational techniques. The following chapter, 'Rigorous Solutions of the Field Equations of the General Theory of Relativity', presented the Schwarzschild metric and its variants. These solutions were found by postulating a general list of terms for the line element ds^2, and then removing or evaluating their coefficients based on geometrical constraints (such as spherical symmetry) and the field equations. The next two chapters, surveying the famous experimental tests of Einstein's general relativity and the equations of motion for test particles in the theory, proceeded in a similar way, often exploiting features like spherical symmetry within a linearised expansion of the field equations (*ibid.*, pp. 198–242).

Readers of Bergmann's 1942 textbook, *Introduction to the Theory of Relativity*, much like auditors of Einstein's 1922 Stafford Lectures, thus were presented with a very particular view of what it mean to *do* gravitational physics. Building on a careful study of the mathematical properties of the curvature tensor, and proceeding in close analogy with Poisson's equation for Newtonian gravity, students could grasp Einstein's hard-won field equations for general relativity. With these governing equations in hand, specific problems could be formulated, such as the notorious perihelion shift in the orbit of Mercury around the Sun. In order to calculate this shift as predicted by general relativity, students were led to expand carefully the full geometrical metric tensor in a perturbative expansion, calculate first and second derivatives, and arrange these terms within Christoffel symbols and Riemann–Christoffel curvature tensors. Contracting these and applying the field equations produced a series of non-linear differential equations. And so it went.

[16] Equations (2) to (4) lead to the field equations in the absence of sources, $G_{\mu\nu} = 0$. Bergmann next spent two and a half pages (pp. 195–198) amending this Hamiltonian derivation to include the case of a non-vanishing $T_{\mu\nu}$, although he never explicitly completed this derivation by arriving at the full field equations (1).

How are we to interpret Bergmann's fleeting side-thoughts on the alternative derivation of Einstein's field equations, by means of variational techniques? Clearly Professor Bergmann himself was adept at calculating in this manner. His own skills might account for his swift progression from the variation $\delta R_{\mu\nu}$ to the covariant derivative of a vector, or from the quantity $\delta(\sqrt{-g}\,g^{\mu\nu})$ to equation (3). Yet what remained 'simply' an equivalence for Bergmann received fourteen lines of explicit derivation and explanation in the 1975 *Problem Book in Relativity and Gravitation* (Lightman *et al.*, 1975, p. 575). Reading *Introduction to the Theory of Relativity* as a pedagogical text, then, we are steered toward an alternate interpretation: Bergmann's brief, clipped presentation of the field equations as the Euler–Lagrange equations of an action integral indicates that these were not the kinds of mathematical skills which students of general relativity were to cultivate. When the four pages of this section are compared with Bergmann's methodical, protracted and explicit derivation of the Bianchi identities and the contracted Bianchi identities within his chapter on the curvature tensor, it becomes all the more clear what kinds of skills students of Bergmann's text were to develop (cf. Bergmann, 1942, pp. 169–172).[17]

In choosing this pedagogical path, Bergmann followed more than only his friend Einstein. His treatment retraced a pattern set by Arthur Eddington, Richard Tolman and several others, who similarly included very brief sections on the derivation of the field equations from the variation of an action only after the full machinery of tensor calculus and non-Euclidean geometry had been brought to bear on questions of gravitation. Eddington further nullified the Lagrangian approach in his famous 1923 textbook, immediately following its fleeting introduction, remarking in 'retrospect' that the use of Least Action 'does not appear at present to admit of any very general application. In any case it seems better adapted to give neat mathematical formulae than to give physical insight' (Eddington, 1930 [1923], p. 147).[18] This last phrase nearly constituted a kiss of death for Lagrangian methods in the study of gravitation, ushered as it was from the pen of so influential a relativist. For Bergmann, as for Einstein and these others, general relativity rested upon geometrical foundations. To master it, students had to work hard to acquire speaking knowledge of a very particular kind of mathematical language and practice.

[17] For this reason, it does not seem that Bergmann's terseness with respect to the variational techniques belies any confidence on his part that students would not need a fuller explanation.

[18] The only textbooks from this period to present the primary derivation of the gravitational field equations by means of varying an action integral were those by Weyl and Pauli; yet even with these texts, the overwhelming bulk focussed on the mathematical tools of tensor calculus and Riemannian geometry. Unlike later field theorists, neither denied or downplayed the fundamental role of non-Euclidean geometry in the study of gravitation. Furthermore, Pauli's text, first published in a German encyclopaedia, was only first translated into English in 1958.

3. Particle Theorists and the World of Gravitation: Feynman and Coleman

In his very first lecture on gravitation for his 1962–1963 course at Caltech, Richard Feynman made explicit his intended break from earlier pedagogical approaches to general relativity: 'The usual course in gravitation […] starts by stating the laws just as Einstein did. This procedure is, however, unnecessary, and for pedagogical reasons we shall here take a different approach to the subject'. Feynman explained his reason for change: 'Today, physics students know about quantum theory and mesons and the fundamental particles, which were unknown in Einstein's day (Feynman, [1962–1963] p. 1)'.[19] Taking into account the skills and interests of his students, Feynman hoped to craft an approach to general relativity which would be the most natural for those students who had been trained in the tradition of interacting, dynamical quantum fields.

A few moments later, Feynman mused for his listeners about a possible second route to general relativity. Suppose, he began, that scientists on Venus had developed the same knowledge of nucleons and mesons as had Feynman and his colleagues, but had not learned about gravity. 'And suddenly, an amazing new experiment is performed [by the Venutians], which shows that two large neutral masses attract each other with a very, very tiny force. Now, what would the Venutians do with such an amazing extra experimental fact to be explained? They would probably try to interpret it in terms of the field theories which are familiar to them' (*ibid.*, p. 2). Much like Feynman's imaginary Venutian physicists, most students in Feynman's class were likely to have been versed in the skills and practices of particle physics and quantum field theory, as Feynman himself made clear:

> Our pedagogical approach [to gravitation] is more suited to meson theorists who have gotten used to the idea of fields, so that it is not hard for them to conceive that the universe is made up of twenty-nine or thirty-one other fields all in one grand equation; the phenomena of gravitation add another such field to the pot, it is a new field which was left out of previous considerations, and it is only one of the thirty or so; explaining gravitation therefore amounts to explaining three percent of the total number of known fields (*ibid.*).

With this background and training, Feynman and his students would therefore study gravitation by deploying the full range in their arsenal of field theory techniques.

In his second lecture, Feynman demonstrated that many phenomenological aspects of gravity could be reproduced in a particle model in which the force of

[19] From the first lecture of Richard P. Feynman's 'Lectures on Gravitation', delivered at Caltech in 1962–1963, p. 1. These typed notes were prepared for distribution to the class members by Fernando B. Morinigo and William G. Wagner. They have recently been published as Feynman (1995). Few changes have been made between the unpublished typescript and the new published version; all references below are to the original typescript.

gravity arose from the exchange of one or two massless neutrinos. The third lecture focussed more attention on such a particle-exchange model of gravitation, noting that (unlike his toy neutrino models) the spin of the 'graviton' would have to be 2 (*ibid.*, pp. 26–28). Again Feynman paused to note the difference between the course he was charting and the canonical one: 'Our program is now to construct a spin 2 theory in analogy to the other field theories that we have. We could at this point switch to Einstein's viewpoint on gravitation, since he obtained the correct theory, but it will be instructive and *possibly easier for us to learn* if we maintain the fiction of the Venutian scientists in order to guess at the properties of the correct theory' (*ibid.*, pp. 32–34, quotation on p. 34; emphasis added).

In the remainder of Lecture 3, Feynman developed a scattering theory for a spin-2 symmetric tensor potential, $h_{\mu\nu}$. Proceeding 'by simple analogy to [quantum] electrodynamics', he first developed amplitudes for the exchange of a graviton, deriving along the way an expression for the graviton's propagator. Soon he turned to a Lagrangian formulation of the emerging theory. Thus, as had long been the standard procedure in quantum field theory, Feynman's theory of gravity relied on the specification of a Lagrangian and the derivation of Euler–Lagrange equations of motion for the dynamical field h. Feynman raised this particle-theorist approach to what he called 'a rule of thumb about theories of physics: Theories not coming from some kind of variational principle, such as Least-Action, may be expected to eventually lead to trouble and inconsistencies'. He and his students would thus '*insist* that our equations be deducible from a variational principle such as Least Action' (*ibid.*, pp. 74; emphasis added).

Feynman next studied how combinations of the dynamical field $h_{\mu\nu}$ and its derivatives behaved under infinitesimal transformations (again in direct analogy to quantum field theorists' practice of studying local gauge transformations). From these algebraic transformations, Feynman proceeded in several steps to derive the only scalar-invariant action containing at least two derivatives of the field $h_{\mu\nu}$, arriving at:

$$F = -\frac{1}{2\lambda^2}\int g^{\mu\nu}R^{\tau}_{\mu\nu\tau}\sqrt{-\operatorname{Det} g_{\mu\nu}}\,d\tau. \tag{5}$$

With this, Feynman signalled the completion of his task: 'The function F which we have just deduced results in a Venutian theory of gravitation which is identical to that developed by Einstein. [...] We may say therefore that our Venutian viewpoint has succeeded in its aim to construct a self-consistent theory of gravitation by means of successive logical steps guessed at by analogy' to other field theories (*ibid.*, pp. 80–85, quotation p. 85).

As Feynman confirmed later in his lectures, this approach was entirely ungeometrical. The theory was based on the assumption that 'space is describable as the space of Special Relativity' (*ibid.*, p. 108). The tensor $g_{\mu\nu}$, rather than serving as a metric tensor to define a curved Riemannian manifold, was simply a convenient combination of the Kronecker delta, $\delta_{\mu\nu}$, and the fundamental gravitational field, $h_{\mu\nu}$. Similarly, the Christoffel symbols and the Riemann curvature tensor (neither being explicitly named such during the derivations) were merely convenient algebraic combinations of the h and their derivatives. Feynman had thus made a complete break with the geometrical conception of gravitation: just like the exchange of photons in quantum electrodynamics, gravitation was due to the exchange of spin-2 gravitons. There was no need to interpret the various symbols ($\{\mu\nu, \sigma\}$ and R) as anything more than the necessary algebraic apparatus of a non-linear field theory.[20] Feynman concluded rather succinctly: 'The geometric interpretation is not really necessary or essential to physics' (*ibid.*, p. 110).[21]

Feynman's particle-theorist approach was not alone. Beginning in 1967, Sidney Coleman began teaching his course on relativity at Harvard, and many of the problem sets and examination questions he assigned to his students were similarly based on treating gravitation as a Minkowski-space theory of non-linearly coupled dynamical fields. Foremost in nearly every assigned problem were Lagrangians and specific variational-principle techniques.

Consider, for example, a problem on the final examination for Coleman's course from 1970. For this problem, students were asked to 'investigate a Minkowski-space theory of gravity, as a possible alternative to general relativity'. Like Feynman, Coleman led his students through the steps of constructing a field theory of gravitation which eschewed the geometrical interpretations of older treatments. Again following Feynman, the problem began by specifying an action: [22] 'In this theory, the gravitational field is a scalar field φ, and the action integral is of the form

$$I = \frac{1}{2}\int d^4x \partial_\mu\varphi\partial_\nu\varphi\delta^{\mu\nu} + I_m\text{'}. \tag{6}$$

The action integral for matter, I_m, was to be obtained by defining a new tensor in terms of the Minkowski spacetime metric, $\delta_{\mu\nu}$, and the gravitational

[20] In Lecture 8, Feynman discussed gravity as a (non-linear) Yang–Mills field, corresponding to a gauge invariance with respect to spacetime-dependent displacement transformations (pp. 110–111). After deriving his gravitational action, equation (5), and the Euler–Lagrange equations arising from it, Feynman did spend portions of the next three lectures describing a more 'Einsteinian', geometrical interpretation of such terms as the metric and curvature tensors, concluding, however, that such a parallel, geometrical description was 'not really necessary or essential to physics'. And in Lecture 10, Feynman returned to a strictly field-theoretic approach, constructing possible actions for scalar and spinor fields in his gravitational theory, including cases of direct coupling between the two fields, φ and h (via a φ^2R non-minimal coupling in the Lagrangian; see pp. 137–143). He used explicit Feynman diagrams to aid in the calculation of the 'quantum mechanical amplitudes' of the multiple φ–h vertices.

[21] Note that the delta function $\delta_{\mu\nu}$ was not exactly the standard Kronecker delta function; instead, it was what others would have called the Minkowski metric: $\delta_{00} = -1$, $\delta_{ii} = +1$, all other terms vanishing.

[22] Problem 3 of the Final Examination for Physics 210, dated January 1970.

scalar field, φ:

$$g_{\mu\nu} \rightarrow \delta_{\mu\nu}(1 + c_1\sqrt{G}\varphi + c_2 G\varphi^2 + \ldots). \tag{7}$$

In this problem, Coleman had his students develop a scalar field theory of gravitation much as Feynman had constructed a rank-two tensor theory of gravitation, each time defining the tensor $g_{\mu\nu}$ as a convenient combination of the Minkowski metric and the dynamical field. With this action and definition of $g_{\mu\nu}$, the assignment began: derive the field equations resulting from this action, and show that they agree with Newtonian gravity when particular choices for the constants c_1 and c_2 are made.

When treating general relativity, Coleman's students were required to practise the tools of calculation in ways which readers of Bergmann's text had not been. On homework problem 13, students had to retrace, on their own, the many complicated steps involved in manipulating the variation of the Lagrangian density $R\sqrt{-g}$: 'Show, by integrating by parts and discarding surface terms, that the quantity which occurs in the action integral for the gravitational field, $\int d^4xR\sqrt{-g}$, is equivalent to an integral of a quadratic function of the Christoffel symbols (times $\sqrt{-g}$). Find this function explicitly'.[23] Here the students had to produce on their own the steps which had been hidden under Bergmann's phrase 'straightforward computation'.

Meanwhile, not one problem from the course, either assigned as homework or on an examination, encouraged the students to practise manipulating the geometrical curvature tensor, establish its symmetry and antisymmetry properties, or construct the Bianchi identities, all of which Bergmann had belaboured in his text.

Perhaps the starkest evidence that Physics 210 trained graduate students to approach gravitation with a 'bag of tricks' derived from field theory comes from other assignments within this same course. On a final examination question from 1969–1970, students were asked to consider a problem which contained no gravitational physics, sandwiched between two problems on general relativity. Here the students were to study an interacting field theory which contained two scalar fields and two vector fields. Given the Lagrangian density, students were asked to demonstrate that the Lagrangian remained invariant under particular transformations of the fields $\phi^{(i)}$ and $V_\mu^{(i)}$. From the invariance of the Lagrangian under these symmetries, students next had to derive the conserved currents, and to calculate the energy–momentum tensor for the system. Similarly, homework problem 8 contained no gravitational physics, but instead provided a lesson in relativistic quantum field theory. Given a Lagrangian density for a free scalar field, students found general solutions to the equation of motion in terms of as-yet unspecified 'coefficients' a_k and a_k^*. Next they calculated the four components of the energy–momentum four-vector as integrals over the unknown

[23] This problem set is dated Thursday, April 20 [1970].

coefficients. As Coleman prompted his students: 'Once you have the answer, you may understand why, in quantum field theory, $a_k^* a_k$ is called the number density of particles in momentum space'. Remember, these problems appeared in a course entitled 'The Theory of Relativity'!

During the late 1960s, Coleman's Physics 210 course was the only physics course at Harvard teaching general relativity. And in this course, much as in Feynman's 1962–1963 course at Caltech, graduate students learned to formulate and discuss gravitation in the same Lagrangian-based terms as they treated problems in quantum field theory. Students in these courses practised a mathematical language and developed a set of calculational techniques which could not have been further removed from Bergmann's 1942 treatment.[24]

A useful gauge to measure how these explicitly quantum field theoretic approaches to gravitation affected the practice of general relativity comes from the 1975 compendium of problems and solutions, *Problem Book in Relativity and Gravitation* (Lightman *et al.*, 1975). The book's four authors had recently emerged from Kip Thorne's late-1960s Caltech 'gravity group'. As laid out in their compendium, the tools required in the study of gravitation, to be developed and practised by students in working out the five hundred problems included in the *Problem Book*, often now included an explicit admixture of calculations from particle physics and general relativity.

In Feynman-like style, the first section of the *Problem Book* to focus on 'gravitation generally' asked students to consider gravity as a theory of spin-2 particle exchange (Lightman *et al.*, p. 74). In the section on the gravitational field equations, students were asked to consider possible quantum field theoretic origins of a cosmological constant (*ibid.*, p. 77). Similarly, much as Feynman had done in his Lecture 10, students had to study non-minimally coupled scalar fields, with φR terms in the action, and to determine whether or not the resulting field equations violated the strong equivalence principle (*ibid.*, pp. 84–85).

To ensure that graduate students were well-versed in the Lagrangian-based methods of field theory, the authors of the *Problem Book* included a final section focussed exclusively on variational techniques. The first problem asked students to derive the identity quoted, without proof, in Bergmann's text (see equation (2) above). The next two problems practised the derivation of Euler–Lagrange equations from an action, when no gravitational effects were included. Problem 21.4 then applied these techniques to the usual gravitational action, asking

[24] Steven Weinberg was another particle theorist teaching gravitation in this period; his textbook, *Gravitation and Cosmology* (Weinberg, 1972), included much explicit discussion of the use of Lagrangians in gravitational physics; and like Feynman and Coleman, he similarly downplayed the role of geometry in the practice of general relativity. His 'dissatisfaction' with 'the usual approach to the subject' stemmed from the fact that 'in most textbooks geometric ideas were given a starring role', and that 'the geometrical approach has driven a wedge between general relativity and the theory of elementary particles' (pp. vi–viii).

students to practise deriving the field equations in two distinct ways:

> *Problem* 21.4. Consider the action
>
> $S = (16\pi)^{-1}\int(-g)^{1/2}Rd^4x + \int L_m(-g)^{1/2}d^4x$. [...] (a) Treat the g's and the Γ's as independent field variables ("Palatini method"), and show that $\delta S = 0$ leads to the Einstein field equations and the usual formula for the Γ's in terms of the g's. [...] (b) Now assume Γ's are Christoffel symbols used to define covariant derivatives in the usual way. Show that $\delta S = 0$ (where now $\delta\Gamma^{\alpha}_{\beta\nu}$ is not independent of $\delta g^{\alpha\beta}$) leads to the Einstein field equations.

After having completed each of these derivations of Einstein's field equations from the action S, students were now prepared to tackle some alternative theories of gravitation, such as the Brans–Dicke theory. Problem 21.7 presented Brans and Dicke's Lagrangian, instructing the students to '[d]erive the field equations from $\delta\int L(-g)^{1/2}d^4x = 0$ by varying $g_{\alpha\beta}$ and φ' (*ibid.*, pp. 125–127).[25]

Thus, although Feynman's and Coleman's overtly field-theoretic approaches to gravitation did not edge out the more traditional pedagogical routes, distinct signs of their particular emphases had indeed entered the emerging mainstream studies of general relativity and gravitation by the mid-1970s. Between the endpoints of Bergmann's 1942 *Introduction to the Theory of Relativity* and the 1975 *Problem Book in Relativity and Gravitation*, the language and practice of general relativity had changed dramatically.

4. Conclusions

> If you want to find out anything from the theoretical physicists about the methods they use, I advise you to stick closely to one principle: don't listen to their words, fix your attention on their deeds (Einstein, 1982 [1933], p. 270).

The practice of general relativity changed from what had been geometrical foundations and approaches in the 1940s, to an incorporation of quantum field theory techniques by the 1970s.[26] This transition is illuminated most clearly by watching how physicists chose, pedagogically, to derive Einstein's field equations for gravitation—either from a non-Euclidean generalisation of Poisson's

[25] The solutions for these problems were quite detailed, occupying pp. 575–584. The Brans–Dicke theory had first been published in Brans and Dicke (1961).

[26] This should not be understood as equating Lagrangian techniques exclusively with elementary particle physics or quantum field theory; as we saw above, Einstein himself published papers in which he derived his own field equations from an action principle. However, in the period 1942–1975, those practitioners who pressed their students to learn and to practise the Lagrangian techniques did so as field theorists, usually with a strong particle physics background. Lagrangians, though in principle a general tool in theoretical physics, became emblematic of a certain kind of physics practice.

equation, or from the variation of an action integral.[27] For Einstein and Bergmann, there simply was *no* gravitational theory separate from the characterisation of geometrically-curved spacetimes; for Feynman, there simply was *no* gravitational theory separate from Lagrangians and equations of motion for dynamical fields (hence his 'rule of thumb').

These differing visions of general relativity emerged on many levels at once. Ontologically, where Einstein and Bergmann described gravitation as the bending and twisting of an elastic, four-dimensional curved manifold, Feynman and Coleman lectured instead of the exchange of non-linearly coupled quanta on a flat spacetime. Calculationally, where Einstein and Bergmann tackled gravitation as a problem of non-Euclidean geometry requiring the 'specialized mathematical apparatus' of tensor calculus, Feynman and Coleman taught their students that gravitation was really just one of thirty other fields racing around the universe, to be studied as any other Lagrangian field theory would be. Pedagogically, where Einstein and Bergmann led their readers to expand the geometrical quantities like the metric tensor and the curvature tensor perturbatively, Feynman and Coleman drilled their students to derive Euler–Lagrange equations from foundational Lagrangians and to construct conserved quantities under gauge-like transformations. Sociologically, where Einstein and Bergmann struggled to expand the circle of dedicated relativists beyond their 'six best friends', Feynman and Coleman instructed roomfuls of meson theorists to approach general relativity in their own terms.

In the eyes of at least some more traditionally-trained relativists, the changing background of physicists engaged in gravitational questions, and their new means of approaching these questions, signalled an unwelcome intrusion. Roger Penrose, for one, has recently described the influx of particle physicists into gravitational studies in terms of a descent of 'locusts', who proceeded in ways 'very foreign to my way of thinking—very ungeometrical'.[28] Penrose's comments serve to remind us that the transition in the practice of general relativity did not always entail a smooth cooperation between physicists of different training. From the examples of changing textbooks and problem sets, then, we can learn as much about the reconstitution of the *community* practicing general relativity as we can about the reconstitution of the theory itself—indeed, we can even question how distinct 'general relativity' ever has been from any given community pursuing its study.

[27] As noted in the introduction, the various examples discussed here should not be read as a unilateral 'march of the Lagrangians' into the domain of general relativity. Some of the most influential texts, such as Misner, Thorne and Wheeler's 1973 *Gravitation*, resist such depictions, emphasising instead a more thorough-going geometrisation than even the Einstein and Bergmann examples. Cf. Robert Wald, *General Relativity* (Chicago: University of Chicago Press, 1984). The quantum field theoretic turns within the practice of general relativity represented only one of the most obvious changes in gravitational physics.

[28] Interview of Roger Penrose by Alan Lightman, in Lightman and Brawer (1990, pp. 415–434); quotation on p. 429.

This study can thus serve to broaden the historiographical sense of the term 'community'. Unlike Andrew Warwick's study of the exceedingly local Cambridge reactions to Einstein's special relativity, the case we have outlined above reveals the possibility of continuities in language use and approach to physical theories beyond narrowly geographical groups of physicists. If we rightly hesitate to label such textbooks as Bergmann's *Introduction to the Theory of Relativity* as Latourian 'immutable mobiles', the books and their methods were at least stable enough to be carried under one's arm when moving from one institution to another.[29] Thus, students of John Wheeler's at Princeton, such as Kip Thorne, could establish their own 'centres of calculation' at Caltech, in turn training such physicists as Bill Press, who would make their way back to the East Coast, there to train still more students.[30] Or, from the more particle-oriented crew, Sidney Coleman could venture from Feynman's Caltech to Harvard's Lyman Laboratory, and there continue to teach graduate students in their shared field-theory way.

The story of general relativity in the middle decades of this century, then, highlights several issues for the historian's study of physical theories. 'General relativity' became a playing field upon which many different physicists, speaking different kinds of mathematical languages, could renegotiate what it meant to do gravitational physics. These negotiations, at times friendly and at times agonistic, were carried out in the lecture hall and by means of the problem set. The crafting of a field-theoretic general relativity, and the training of graduate students to learn and to practise this new set of calculations and techniques, bespoke the reconstitution of general relativity between 1942 and 1975.

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[29] On Bruno Latour's notion of 'immutbale mobiles', see his book *Science in Action* (Latour, 1987), especially pp. 227–237; see also (Latour, 1990) on pp. 26–35.

[30] See the unpublished letters to John Wheeler from his former students, collected in 'John Archibald Wheeler: A family gathering. Students & collaborators of John Archibald Wheeler gather some recollections of their work with him, and of his influence on them and through them on their own students.' The collection was presented to Wheeler on 11 August 1977. Copies of these two volumes may be found in the Niels Bohr Library of the American Institute of Physics.

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