Restoring the sting to metric preheating

Bruce A. Bassett
Department of Theoretical Physics, Oxford University, Oxford OX1 3NP, England
and Relativity and Cosmology Group, Division of Mathematics and Statistics, Portsmouth University, Portsmouth PO1 2EG, England

Christopher Gordon and Roy Maartens
Relativity and Cosmology Group, Division of Mathematics and Statistics, Portsmouth University, Portsmouth PO1 2EG, England

David I. Kaiser
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

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The relative growth of field and metric perturbations during preheating is sensitive to initial conditions set in the preceding inflationary phase. Recent work suggests this may protect super-Hubble metric perturbations from resonant amplification during preheating. We show that this possibility is fragile and sensitive to the specific form of the interactions between the inflaton and other fields. The suppression is naturally absent in two classes of preheating in which either: (1) the vacua of the noninflaton fields during inflation are deformed away from the origin; or (2) the effective masses of noninflaton fields during inflation are small but during preheating are large. Unlike the simple toy model of a $g^2\phi^2\chi^2$ coupling, most realistic particle physics models contain these other features. Moreover, they generically lead to both adiabatic and isocurvature modes and non-Gaussian scars on super-Hubble scales. Large-scale coherent magnetic fields may also appear naturally.

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I. INTRODUCTION

Standard inflationary models must end with a phase of reheating during which the inflaton $\phi$ transfers its energy to other fields. Reheating itself may begin with a violently nonequilibrium “preheating” era, when coherent inflaton oscillations lead to resonant particle production (see [1] and Refs. therein). Until recently, preheating studies implicitly assumed that preheating proceeds without affecting the space-time metric. In particular, causality was thought to be a “silver bullet,” ensuring that on cosmologically relevant scales the nonadiabatic effects of preheating could be ignored.

In fact, exciting, super-Hubble effects are possible during preheating, and metric perturbations may be resonantly amplified on all length scales [2–4]. Causality is not violated precisely because of the huge coherence scale of the inflaton immediately after inflation [2] (see also [5]). Strong preheating (with resonance parameter $q \gg 1$; see [1,2] for overviews and notation) typically leads to resonant amplification of scalar metric perturbation modes $\Phi_k$, including those on super-Hubble scales (i.e., $k/aH \ll 1$, where $a$ is the scale factor and $H$ the Hubble rate). One of our aims is to answer the question “how typical is typical?”

The answer is crucial since preheating can lead to distortions in the anisotropies in the cosmic microwave background (CMB). Observational limits rule out those models that produce unbridled nonlinear growth, but models which pass the metric preheating test on Cosmic Background Explorer (COBE) scales may nevertheless leave a nonadiabatic signature of preheating in the CMB. Hence one can no longer universally avoid consideration of reheating when analyzing inflationary predictions for cosmology, even if the final effect of reheating in some particular models is small.

In this vein, it has been argued recently [6,7] that metric perturbations on super-Hubble scales are in fact immune to metric preheating in the archetypal two-field potential typically used in earlier studies [1,2]. The claim arises because the initial value of the fluctuations in the created bosonic field $\chi$ at the start of preheating is much smaller than that used in [2]. The basic argument is as follows. For the coupling $\frac{1}{2}g^2\phi^2\chi^2$, strong preheating typically requires $q = g^2\phi^2/m^2 \gg 1$ (exceptions exist in which $q$ is small but metric preheating is strong [4]). This increases the effective $\chi$ mass relative to the Hubble rate during inflation, $m_{\chi,\text{eff}} \sim g\phi\sim H \sim m$, where $m$ is the inflaton mass. This leads to an exponential suppression $\sim a^{-3/2}$ of both $\chi$ and $\delta\Phi_k$ during inflation; hence these fields would have values at the start of preheating around $10^{-36}$ smaller than those used in all previous simulations. This would stifle any growth in the small-$k$ modes of $\Phi$ until late times. Initial conditions for large-$k$ modes, in contrast, are claimed to be unaffected, so that they would grow nonlinear first. Their resulting backreaction would then end the resonance before any interesting effects occur on cosmologically significant scales [6,7]. Irrespective of super-Hubble behavior, we note that nonperturbative metric-preheating effects are vital on smaller scales [6,8], and this in itself is a major departure from the old theory that neglects metric perturbations in preheating. Metric preheating leads to interesting possibilities, such as significant primordial black hole formation [2] (see also [8,6]).

Returning to super-Hubble scales, $k/aH \ll 1$, we will show that the above suppression mechanism is highly sensitive to the particular form of interaction Lagrangian, while metric preheating is not. Indeed, the suppression of $\chi$ and $\delta\chi_k$ at the start of preheating argued for in [6,7] is absent for models in either of the following two classes:

Class I. Models in which the vacuum expectation value (VEV) of $\chi$ is nonzero during inflation.
Class II. Models in which the $\chi$ effective mass is small during inflation but undergoes a transition and becomes large during preheating.

Since these possibilities arise naturally in a variety of realistic particle physics models, we conclude that the suppression mechanism proposed recently [6,7] is fragile, i.e., unstable to small changes in the potential. On the other hand, resonant growth of super-Hubble metric perturbations in preheating is robust, since it persists under small changes of the potential.

The fields split into a homogeneous part and fluctuations: $\phi_f(t, x) = \phi_f(t) + \delta \phi_f(t, x)$. The background equations are

\[
H^2 = \frac{1}{2} \kappa^2 \left[V + \frac{1}{2} \sum_{i} \dot{\phi}_i^2\right], \quad \ddot{\phi}_i + 3H \dot{\phi}_i + V_i = 0, \tag{1}
\]

where $\kappa^2 = 8\pi M_{\text{pl}}^2$ and $V_i = \partial V / \partial \phi_i$. The linearized equations of motion for the Fourier modes of field ($\delta \phi_{kl}$) and scalar metric fluctuations ($\Phi_k$) are

\[
(\delta \phi_{ik})' + 3H(\delta \phi_{ik})' + (k^2 / \alpha^2) \delta \phi_{ik} = -\sum_{jl} V_{ijl} \delta \phi_{jk} + 4\phi_r \delta \Phi_k - 2V_i \Phi_k, \tag{2}
\]

\[
\Phi_k + H \Phi_k = \frac{1}{2} \kappa^2 \sum_{lj} \dot{\phi}_l \delta \phi_{ik}. \tag{3}
\]

This system is subject to the constraint

\[
\left[ \frac{k^2}{\alpha^2} - \frac{1}{2} \kappa^2 \sum_{lj} \dot{\phi}_l^2 \right] \Phi_k = -\frac{1}{2} \kappa^2 \sum_{lj} \dot{\phi}_l^2 (\delta \phi_{lk} / \dot{\phi}_l)', \tag{4}
\]

which we use to check the accuracy of our numerical integrations of Eqs. (2) and (3) and to set $\Phi_k$ initial conditions.

We envisage a model with the inflaton $\phi_i = \phi = \phi(t) + \delta \phi(t, x)$ coupled to a massless scalar field $\phi_f = \chi = X(t) + \delta \chi(t, x)$ (assumed to be in its vacuum state near the end of inflation). This schematically represents the particle content of the inflationary and preheating eras. More realistic models should consider the gauge group, nonminimal coupling and fermionic effects, and of course an accurate phenomenology of metric preheating must begin to study these issues [9]. However, since we are interested only in essential conceptual points, this simple picture will suffice for now.

II. SUPER-HUBBLE METRIC PREHEATING

The often-used interaction term $\frac{1}{2} g^2 \phi^2 \chi^2$ is not the only coupling appropriate to preheating, but is rather one simple example for which resonance occurs. As we show below, additional couplings linear in $\chi$, as well as quadratic couplings in which $g^2 < 0$, provide a mechanism for escaping the inflationary suppression claimed in [6,7]. Essentially, these alternatives produce a nonzero attractor $X \neq 0$, to which inflation drives the $\chi$ field, so that the initial values of $X$ and $\delta \chi(t \to 0)$ at preheating are not suppressed. These possibilities are incorporated in the effective potential

\[
V = \frac{1}{2} m^2 \phi^2 + \lambda_1 \phi^4 + \lambda_2 (\chi^2 - \sigma^2)^2 + \frac{1}{2} \epsilon g^2 \phi^2 \chi^2 + \frac{g^2}{2} \kappa^{n-3} \phi^n, \tag{5}
\]

for the constants $\epsilon = 0 \pm 1$ and $n = 2,3$. The $\lambda_1, \lambda_2$ terms ensure that $V$ is bounded from below. The various terms in this potential are phenomenologically well motivated:

In theories where supersymmetry (SUSY) is softly broken, the potential will only acquire logarithmic radiative corrections and the suppression may apply. However, in realistic models with gravity, SUSY is replaced by supergravity (SUGRA), and SUGRA models often contain couplings of the form $\phi^n \chi, n = 2,3$ [10].

Even if $\tilde{g} = 0$ initially, if the $\chi$ field exhibits symmetry-breaking ($\sigma \neq 0$), shifting the field $\chi \to -\chi - \sigma$ generates the linear term $\sigma g^2 \phi^2 \chi$ via the quadratic coupling. The possible importance of symmetry breaking of this sort has long been noted [11] for its role in generating single-body inflaton decays and hence complete inflaton decay. If we choose $\sigma$ to correspond to the grand unified theory (GUT) scale, then $\sigma / M_{\text{pl}} \sim g^2 / g_s \sim 10^{-3}$.

Negative coupling instability (NCI) models ($\epsilon = -1$) are dominated by the coupling $-g^2 \phi^2 \chi^2$, and the $\chi$ field is driven to a nonzero VEV during inflation [12].

A fermionic coupling $h \bar{\psi} \chi \psi$ would lead to a driving term $h(\bar{\psi} \psi)$ in the $\chi$ equation of motion. This would have a similar effect of giving a nonzero VEV for $\chi$.

A. Class I: unsuppressed initial conditions

We now present analytical arguments (assuming for simplicity that $\sigma = 0$) to show that the new couplings avoid the claimed suppression of super-Hubble $\chi$ fluctuations. By Eq. (1), the background $X$ field obeys

\[
\dot{X} + 3HX + \epsilon g^2 \phi^2 X + \lambda_1 X^3 = -\tilde{g}^2 \kappa^{n-3} \phi^n, \tag{6}
\]

and by Eq. (2), its large-scale fluctuations satisfy

\[
(\delta \chi_k)' + 3H(\delta \chi_k)' + [\epsilon g^2 \phi^2 + 3\lambda_1 X^2] \delta \chi_k = 4X \Phi_k + [(2 - n) \tilde{g}^2 \kappa^{n-3} \phi^{n-1} + 2\lambda_1 X^3 / \phi] \delta \phi_k, \tag{7}
\]

where we used the slow-roll relation $\Phi \sim -\delta \phi / \phi$. We now consider the two separate cases with similar results:

Case 1: $\epsilon = 1, \tilde{g} > 0, \lambda_1$ negligible

Using the fact that $\phi, H \approx \text{constant}$ during inflation, we see that while the solution of the homogeneous part of Eq. (6) decays rapidly towards zero as $a^{-3/2}$, the particular solution arising from the inhomogeneous term is approximately constant. It follows that $\chi$ emerges at the end of inflation ($t = t_0$) with background part

\[
X(t_0) \approx -\tilde{g}^2 / g_s \kappa^{n-3} \left[\phi(t_0)\right]^{n-2}, \tag{8}
\]

where $\phi(t_0) \approx 0.3 M_{\text{pl}}$. Similarly, the fluctuations also have a nontransient solution. For $n = 2$, we need to include the small
term $\dot{X}P_i$, which is not straightforward to evaluate, but for $n = 3$ we can neglect this term, and Eq. (7) implies

$$\delta \chi(t_0) = -\left(\frac{g}{g}ight)^2 \dot{\delta} \phi(t_0).$$

Thus the super-Hubble $\chi$ fluctuations emerge from inflation unsuppressed, though smaller than the inflaton fluctuations by a factor $(\dot{g}/g)^2$.

**Case 2: $\epsilon=-1, \tilde{g}=0$**

NCI coupling gives rise to a nonzero VEV, since Eq. (6) has an attractor solution ($X=0$), and then using this solution, the fluctuations governed by Eq. (7) are also seen to have an approximate attractor solution:

$$X(t_0) = (g/\sqrt{\Lambda}) \varphi(t_0), \quad \delta \chi(t_0) = (g/\sqrt{\Lambda}) \delta \phi(t_0).$$

In both cases, for consistency, inflation should be dominated by the $\frac{1}{2} m^2 \dot{\phi}^2$ term in the potential, and the super-Hubble fluctuations should be dominated by adiabatic inflaton fluctuations. The equations for $\varphi$ and $\dot{\phi}$ show that this will be secured if $\lambda$ is negligible and $|e^{-2X} + n g^2 \kappa^{-2}\varphi^{2-n}X| \ll m^2$, given Eqs. (8)–(10). In summary, our analytical arguments show that by the end of inflation, the $\chi$ field and its super-Hubble fluctuations are not negligibly small; the linear couplings ($\tilde{g}>0, \epsilon=1$) and the negative quadratic coupling ($\epsilon=-1, \tilde{g}=0$) each provide a mechanism to evade the super-Hubble suppression of $\chi$ fluctuations.

**B. Class I: numerical simulations**

In order to confirm and extend the analytical arguments above, we performed numerical simulations in one Class I model, with $\tilde{g}^2 \dot{\phi}^2 \chi$ coupling ($\epsilon=1, \lambda$, and $\lambda$ negligible). To avoid subtleties associated with matching inflation to preheating, we numerically integrated Eqs. (1)–(3) starting deep inside the inflationary era. Our primary interest is in cosmologically relevant scales, so we follow the evolution of a scale that crosses the Hubble radius at $t=t_{\text{H}}$, about 55 $e$ folds before the start of preheating at $t=t_0$.

The slow-roll approximation gives $N = \kappa^2 (\varphi_m^2 - \varphi_0^2)/4$ for the number of $e$ folds before the end of inflation, so we choose $\varphi_m = 3M_{\text{pl}}$ to get $N \approx 55$. For the background $\chi$ field we use the approximate dominant solution in Eq. (8) and take $X_m = -(\dot{g}/g)^2 \varphi_m$. We follow [6] and take the field fluctuations at Hubble-crossing ($k = a H$) as

$$k^3 |\delta \phi_k|^2 = H^2 (2 \omega_{\text{ik}}) \left| \omega_{\text{ik}} | \delta \phi_k \right|^2,$$

where $\omega_{\text{ik}}^2 = (k/a)^2 + m_k^2$, with $m_k = g \varphi$. We also take $X_m = \omega_k X_{m}$. The initial metric perturbation ($\Phi_{\text{ik}}$) in is then fixed by Eq. (4). The comoving wavenumber is $k = m a(t_0) e^{-N} \kappa \varphi_m^{0.5}/6$. We also take $\dot{g}/g \ll 10^{-2}$, with $g = \frac{4}{5} \pi 3 \times 10^{-3}$ and $m = 10^{-6}M_{\text{pl}}$. This yields a resonance parameter $q = 3.8 \times 10^5$ which is used for all our simulations here.

As well as tracking a scale that crosses the Hubble radius at $t_{\text{H}}$, we consider scales that are within the Hubble radius at $t_0$, i.e., at the start of preheating, with $k/a_0 H_0 > 1$. Although fluctuations on these small scales are cosmologically insignificant, we need to compare their evolution with those on very large scales, since this has a bearing on the question of backreaction. The initial amplitude is given by $\delta \phi_k(t_0) = 1/(2 \omega_{\text{ik}})$, and for $k/a_0 \gg g \varphi_0 \equiv m$ we find that $|\delta \phi_k(t_0)| = 1/(a_0 \sqrt{k})$.

The numerical results summarized in Fig. 1 confirm the analytical discussion above. The field and metric fluctuations on cosmological scales are resonantly amplified as expected. The curvature perturbation $\zeta = \Phi - H(\dot{\Phi} + H \Phi)/H$, which would remain constant in standard reheating on super-Hubble scales in adiabatic models, instead shows violently nonadiabatic growth. This resonant amplification will be terminated by backreaction effects, which are governed by the growth of the variance $\langle \chi^2 \rangle \times \int dk k^2 |\delta x_k|^2$ (suitably renormalized and regularized [13]). Resonant growth on small scales will reinforce the backreaction, since the $k^2$ factor will weight the sub-Hubble contribution more strongly. Our simulations indicate that for the chosen value of $q$, resonance occurs for sub-Hubble scales with $1 \approx k/a_0 H_0 < 100$ at the start of preheating, which occurs at $t_{\text{H}} \approx 20$. In Fig. 2 we plot the fluctuations for a mode with $k/a_0 H_0 = 10$. In addition to the resonance, this shows that nonlinear growth in the sub-Hubble mode occurs before that of the super-Hubble mode of Fig. 1. Nonlinear growth of the super-Hubble modes may therefore be prevented, but since these modes begin to grow resonantly soon after the sub-Hubble modes ($\Delta t \approx 20$), we can expect some preheating growth in the power spectrum. For other values of $q$ we expect that super-Hubble modes may grow the quickest, as explicitly occurs in some models that can be studied analytically [4]. The study of backreaction (including both gravitational and matter-field contributions), and of the preheating imprint on the power spectrum, is currently in progress.

An indication of how the strength of the super-Hubble resonance in $\Phi_k$ is affected by changes in the coupling...
that we expect independently noted in recent work. This occurs naturally in various models: small during inflation but then becomes large at preheating.

nl increases in response to the suppression of initial conditions that occurs as $g$ is decreased. Note that synchronization occurs: all fluctuations share roughly the same $t_{nl}$ values so that we expect $M^2_{nl} \langle \Phi^2 \rangle \sim \langle \Phi^2 \rangle \langle \chi^2 \rangle$. The importance of metric perturbations in determining backreaction has been independently noted in recent work.

**C. Class II models**

In Class II models, the $\chi$ effective mass is simply very small during inflation but then becomes large at preheating. This occurs naturally in various models:

Globally SUSY hybrid models based on the superpotential $W = \alpha S \bar{\varphi} - \mu^2 S$, where the singlet $S$ plays the role of the inflaton. The corresponding unbroken potential is $V = \alpha^2 |S|^2 (|\varphi|^2 + |\bar{\varphi}|^2) + |\alpha \varphi \bar{\varphi} - \mu^2|^2$, together with $D$ terms which vanish along the flat direction $|\varphi| = |\bar{\varphi}|$. For $S \gg \mu/\sqrt{\alpha}$, inflation occurs with the minimum of the potential at $\langle \varphi \rangle = \langle \bar{\varphi} \rangle = 0$. However, for $S \ll \mu/\sqrt{\alpha}$, $V$ has a new minimum at $\langle S \rangle = 0$, $\langle \varphi \rangle = \mu/\sqrt{\alpha}$, and preheating occurs via oscillations around this minimum [14]. Now let us couple $\chi not to the inflaton $S$, but to the field $\varphi$ through the term $g^2 \chi^2 |\varphi|^2$. Then the $\chi$ effective mass $g |\varphi|$ vanishes during inflation (up to logarithmic corrections) —and hence so does the suppression mechanism of [6,7]. The effective mass only departs strongly from zero once inflation ends and reheating begins, leading to a huge increase in the value of the resonance parameter $q$.

In models with strong running of coupling constants, where the beta function is negative, such as occurs in QCD, the theory is asymptotically free and the coupling increases at lower energies. Perhaps the strongest examples of this are based on $S$-type dualities, where the coupling $g^2$ is very small during inflation but is very large during reheating, which occurs in the strongly coupled phase with dual coupling $\sim 1/g^2 \gg 1$. An example is provided by “dual inflation” [15], where $m_{\chi, \text{eff}} \sim g < H$, and $\chi$ fluctuations are similar to those in the inflaton, and not strongly suppressed. In fact, it is arguable that models of this sort are needed if preheating is to be viable in non-SUSY theories, since large $g$ leads to radiative corrections to the potential which may violate the slow-roll conditions for inflation.

**III. NEW COSMOLOGICAL EFFECTS**

Our eventual goal must be to calculate physical quantities such as the power spectrum of $\Phi_k$. Since $P_\Phi = k^3 \langle |\Phi_k|^2 \rangle / 2\pi^2$, one might be concerned that these strong preheating effects at $k \rightarrow 0$ would be made irrelevant by the $k^3$ phase space factor. Perhaps the easiest way to see that this is not so is to look at the evolution of $\xi_k$. Since $\xi_k$ is not conserved for small $k$ (see Fig. 1), the standard normalization of the CMB spectrum is increased. This can only take place if the power spectrum of metric fluctuations is strongly affected as $k \rightarrow 0$. This is understandable since preheating acts only as a nontrivial transfer function $T(k)$.

Beyond the effects discussed in [2], metric preheating can lead to a host of interesting new effects.

The growth of $\xi_k$ implies amplification of isocurvature modes in unison with adiabatic scalar modes on super-Hubble scales. Preheating thus yields the possibility of inducing a post-inflationary universe with both isocurvature and adiabatic modes on large scales. If these are uncorrelated and of roughly equal strength, the corresponding Doppler peaks will tend to cancel [16]. (This mechanism is independent of the one discussed in [2], which requires nonlinearity to persist until decoupling.) However, if the adiabatic and isocurvature modes are strongly correlated, this would create the possibility of a “smoking gun” fingerprint of preheating. The challenge remains to distinguish such correlations from
those induced in hybrid inflation.

Because the metric perturbations can go non-linear, whether on sub- or super-Hubble scales, the corresponding $\chi$ density perturbations $\delta \chi$ typically have non-Gaussian statistics. This is simply a reflection of the fact that $-1 \leq \delta \chi < \infty$, so that the distribution of necessity becomes skewed and non-Gaussian. Further, in Class II models, where $\langle \chi \rangle = 0$ during inflation, $\chi$ perturbations in the energy density will necessarily be non-Gaussian (chi-squared distributed), even if $\delta X_4$ is Gaussian distributed, since stress-energy components are quadratic in the fluctuations (see, e.g., [17]). Non-Gaussian effects are therefore an intrinsic part of many metric preheating models (particularly those in Class II), and open up a potential signal for detection in future experiments.

Another new feature we would like to identify is the breaking of conformal invariance. Once metric perturbations become large on some scale, the metric on that scale cannot be thought of as taking the simple Friedmann-Lemaître-Robertson-Walker (FLRW) form, and conformal invariance is lost. This is particularly important for the production of primordial magnetic fields, which are usually strongly suppressed due to the conformal invariance of the Maxwell equations in a FLRW background. The coherent oscillations of the inflaton during preheating further provide a natural cradle for producing a primordial seed for the observed large-scale magnetic fields. A charged inflaton field, with large-scale magnetic fields. A charged inflaton field, with electric charge $Q$ and mass $m$, will couple to electromagnetism through the gauge covariant derivative $D_\mu = \nabla_\mu - ieA_\mu$. This will naturally lead to parametric resonant amplification of the existing magnetic field, which could produce large-scale coherent seed fields on the required super-Hubble scales without fine-tuning [18]. (Note that a tiny seed field must exist during inflation due to the conformal trace anomaly and one-loop QED corrections in curved spacetime [19].)

In conclusion, the suppression discussed in [6, 7] is highly sensitive to the form of the particle interactions considered; when couplings are considered which are found in most realistic particle physics models, the effects of [6, 7] recede. Instead, in models from either of the two general classes highlighted here, preheating can produce a strong amplification of metric perturbations on cosmologically significant scales. Metric preheating thus allows us to rule out models in which backreaction effects fail to prevent super-Hubble non-linear growth, and shows that in the surviving models, there will typically be some signature of preheating imprinted on the power spectrum. The robustness of the amplification further demonstrates the need to move towards more realistic models of preheating in order to develop a realistic understanding of the predictions of inflation for observational cosmology.

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