TEN SIMPLE RULES
FOR
MATHEMATICAL WRITING

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ON WRITING

• “Easy reading is damn hard writing” (Hawthorne)

• “Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought” (Knuth)

• “I think I can tell someone how to write but I can’t think who would want to listen” (Halmos)
WHAT IS MATH WRITING?

• Writing where mathematics is used as a primary means for expression, deduction, or problem solving.

• Examples that are:
  – Math papers and textbooks
  – Analysis of mathematical models in engineering, physics, economics, finance, etc

• Examples that are not:
  – Novels, essays, letters, etc
  – Experimental/nonmathematical scientific papers and reports
WHAT IS DIFFERENT ABOUT MATH WRITING?

• Math writing blends two languages (natural and math)
  – Natural language is rich and allows for ambiguity
  – Math language is concise and must be unambiguous

• Math writing requires slow reading
  – Often expresses complex ideas
  – Often must be read and pondered several times
  – Often is used as reference
  – Usually must be read selectively and in pieces
WHY THIS TALK?

• Experience is something you get only after you need it …

• One current model: The conversational style
  – “Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods” (Halmos)
  – “Talk to your readers as you write” (Strang)
  – Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe …” Halmos)
  – Controversial (where do proofs start and end? … I am not sure what the assumptions are … I can’t find what I need … etc)

• Instead we will advocate a structured style
  – Offers specific verifiable rules that students can follow and thesis advisors can check
  – Allows room to develop and improve over time
SOURCES

- General style books
  - Strunk and White, “The Elements of Style” (www)
  - Fowler and Aaron, “The Little Brown Handbook”
  - Venolia, “Write Right!”

- Halmos, “How to Write Mathematics”
- Knuth, et al, “Mathematical Writing” (www)
- Kleiman, “Writing a Math Phase Two Paper,” MIT (www)
- Krantz, “A Primer of Mathematical Writing”
- Higham, “Handbook of Writing for the Mathematical Sciences”
- Alley, “The Craft of Scientific Writing”
RULES OF THE GAME

• Small rules:
  – Apply to a single sentence (e.g., sentence structure rules, mathspeak rules, comma rules, etc)

• Broad rules:
  – Apply to the entire document
  – General style and writing strategy rules
  – Are non-verifiable (e.g., organize, be clear and concise, etc)

• Composition rules (our focus in this talk):
  – Relate to how parts of the document connect
  – Apply to multiple sentences
  – Are verifiable
SOME EXAMPLES OF SMALL RULES I

• Break up long blocks of text into simpler ones:
  – Few lines and verbs per sentence; few sentences per paragraph.
  – 2-3-4 rule: Consider splitting every sentence of more than 2 lines, every sentence with more than 3 verbs, and every paragraph with more than 4 "long" sentences.

• Mathspeak should be “readable”
  – BAD: Let $k>0$ be an integer.
  – GOOD: Let $k$ be a positive integer or Consider an integer $k>0$.
  – BAD: Let $x \in \mathbb{R}^n$ be a vector.
  – GOOD: Let $x$ be a vector in $\mathbb{R}^n$ or Consider a vector $x \in \mathbb{R}^n$.

• Don’t start a sentence with mathspeak
  – BAD: Proposition: $f$ is continuous.
  – GOOD: Proposition: The function $f$ is continuous.
SOME EXAMPLES OF SMALL RULES II

- Use active voice ("we" is better than "one")
- Minimize "strange" symbols within text
- Make proper use of "very," "trivial," "easy," "nice," "fundamental," etc
- Use abbreviations correctly (e.g., cf., i.e., etc.)
- Comma rules
- "Which" and "that" rules
- ... ETC
BROAD RULES
SOME EXAMPLES

• Language rules/goals to strive for: precision, clarity, familiarity, forthrightness, conciseness, fluidity, rhythm
• Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)
• “Down with the irrelevant and the trivial” (Halmos)
• “Honesty is the best policy” (Halmos)
• “Defend your style” (against pushy copyeditors -Halmos)
• … ETC
MATH WRITING WITHOUT MATH PROOFS

• Is it OK to skip proofs?
  – Rigorous proofs are the essence of mathematical writing
  – A mathematician relies on proofs to gain intuition
  – ... but many readers prefer no detailed proofs

• Intuitive math writing: An alternative to a proof based development
  – Explain mostly in words (some) math results, and give refs
  – State precisely a few (if any) theorems ... place (some) proofs in appendixes
  – Use suggestive natural language to describe the intuition behind theorems/algorithms
  – A challenge: Intuitive math writing is trickier/more demanding than rigorous proof-based writing

• Example: Bertsekas/Tsitsiklis probability book
PRINCIPLES OF INTUITIVE MATH WRITING

• Do not cut corners
  – Better to skip a proof than to give a sloppy proof

• Maintain math rigor in natural language
  – Without math, precise language becomes more important

• Use natural language and logic to replicate math arguments (without excessive formality)

• Explain the range and limitations of your development, and use well-chosen (simple and intuitive) examples to demonstrate

• Aim to provide enough explanation/intuition so mathematicians can believe your argument and even construct a rigorous proof
PRACTICAL TIPS FOR INTUITIVE MATH WRITING

- Loose language is the easiest way to confuse the reader

- Define terms rigorously, use them consistently
  - Avoid ambiguous or undefined terms
  - Don’t use multiple terms with essentially the same meaning, e.g., don’t use “likelihood” or “chance” (in place of “probability”), “random values” (in place of “random variables”), etc

- Use well-designed and suggestive figures

- Overexplain when addressing a cross-disciplinary audience (but don’t overdo it)

- Consider taking a rigorous math course as preparation for intuitive math writing!
THE TEN COMPOSITION RULES

• Structure rules (break it into digestible pieces)
  – Organize in segments
  – Write segments linearly
  – Consider a hierarchical development

• Consistency rules (be boring creatively)
  – Use consistent notation and nomenclature
  – State results consistently
  – Don’t underexplain - don’t overexplain

• Readability rules (make it easy for the reader)
  – Tell them what you’ll tell them
  – Use suggestive references
  – Consider examples and counterexamples
  – Use visualization when possible
1. ORGANIZE IN SEGMENTS

• “Composition is the strongest way of seeing” (Weston)

• Extended forms of composition have a fundamental unit:
  - Novel
  - Film
  - Slide presentation
  - Evening news program

• Key Question: What is the fundamental unit of composition in math documents?

• Answer: A segment, i.e., an entity intended to be read comfortably from beginning to end

• Must be not too long to be tiring, not too short to lack content and unity
SEGMENTATION PROCESS

• Examples of segments:
  – A mathematical result and its proof
  – An example
  – Several related results/examples with discussion
  – An appendix
  – A long abstract
  – A conclusions section

• A segment should “stand alone” (identifiable start and end, transition material)

• Length: 1/2 page to 2-3 pages
### EXAMPLE OF SEGMENTATION: A SECTION ON PROB. MODELS

- Sample space - Events: (1 page)
- Choosing a sample space: (0.5 page)
- Sequential models: (0.75 page)
- Probability laws - Axioms: (1.25 page)
- Discrete models: (2 pages)
- Continuous models: (1 page)
- Properties of probability laws: (2 pages)
- Models and reality: (1.25 page)
- History of probability: (1 page)

See Sec. 1.2 of Bertsekas and Tsitsiklis probability book
2. WRITE SEGMENTS LINEARLY

• Question: What is a good way to order the flow of deduction and dependency?

• General rule: Arguments should be placed close to where they are used (minimize thinking strain)

• Similarly, definitions, lemmas, etc, should be placed close to where they are used

• View ordering as an optimization problem

• A linear/optimal order is one that positions arguments (definitions, lemmas) so as to minimize the total number of “crossings” over other arguments (definitions, lemmas), subject to the dependency constraints. Depth-first order is usually better.
EXAMPLES OF ORDERING

Dependency Graph of Arguments

Level 1 Arguments

Level 2 Arguments

Nonlinear

Linear

1

2

3

4

T

1

2

3

4

T

1

3

4

T

Ten Simple Rules, D. P. Bertsekas
3. CONSIDER A HIERARCHICAL DEVELOPMENT

- Arguments/results used repeatedly may be placed in special segments for efficiency

- Possibly create special segments for special material (e.g., math background, notation, etc)

- Analogy to subroutines in computer programs
4. USE CONSISTENT NOTATION

- Choose a notational style and stick with it
- Examples:
  - Use capitals for random variables, lower case for values
  - Use subscripts for sequences, superscripts for components
- Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc
- Use simple notation. Example: Try to avoid parenthesized indexes: \( x(m,n) \) vs \( x_{mn} \)
- Avoid unnecessary notation:
  - BAD: Let \( X \) be a compact subset of a space \( Y \). If \( f \) is a continuous real-valued function over \( X \), it attains a minimum over \( X \).
  - GOOD: A continuous real-valued function attains a minimum over a compact set.
5. STATE RESULTS CONSISTENTLY

- Keep your language/format simple and consistent (even boring)
- Keep distractions to a minimum; make the interesting content stand out
- Use similar format in similar situations
- Bad example:
  - Proposition 1: If A and B hold, then C and D hold.
  - Proposition 2: C’ and D’ hold, assuming that A’ and B’ are true.
- Good example:
  - Proposition 1: If A and B hold, then C and D hold.
  - Proposition 2: If A’ and B’ hold, then C’ and D’ hold.
6. DON’T OVEREXPLAIN - DON’T UNDEREXPLAIN

• Choose a target audience level of expertise/background (e.g., undergraduate, 1st year graduate, research specialist, etc)
• Aim your math to that level; don’t go much over or under
• Explain potentially unfamiliar material in separate segment(s)
• Consider the use of appendixes for background or difficult/specialized material
7. TELL THEM WHAT YOU’LL TELL THEM

- Keep the reader informed about where you are and where you are going
- Start each segment with a short introduction and perhaps a road map
- Don’t string together seemingly aimless statements and surprise the reader with “we have thus proved so and so”
- Announce your intentions/results, e.g., “It turns out that so-and-so is true. To see this, note …”
- Tell them what you told them
8. USE SUGGESTIVE REFERENCES

- Frequent numbered equation/proposition referencing is a **cardinal sin**
- It causes page flipping, wastes the reader’s time, and breaks concentration
- Refer to equations/results/assumptions by content/name (in addition to number), e.g., Bellman’s equation, weak duality theorem, etc
- Repeat simple math expressions
- Remind the reader of unusual notation, and earlier analysis
- Dare to be repetitive (but don’t overdo it)
9. CONSIDER EXAMPLES AND COUNTEREXAMPLES

• “Even a simple example will get three-quarters of an idea across” (Ullman)

• Examples should have some spark, i.e., aim at something the reader may have missed

• Illustrate definitions/results with examples that clarify the boundaries of applicability

• Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions
10. USE VISUALIZATION WHEN POSSIBLE

• “A picture is worth a thousand words”
• Keep figures simple and uncluttered
• Use substantial captions
• Captions should reinforce and augment the text, not repeat it
• Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality
• Prefer graphs over tables
THE END

“Bad thinking never produces good writing”
(Lamport)

Good writing promotes good thinking …