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Preface

In this research monograph we will discuss the solution of large and challenging multistage decision problems using methods of reinforcement learning, also referred to by other names such as *approximate dynamic programming*, and *neuro-dynamic programming*. We will focus on a subset of methods which are based on the idea of policy improvement and policy iteration, i.e., starting from some policy and generating one or more improved policies. If only one improved policy is generated, this is called *rollout*. If multiple successively improved policies are generated, we obtain forms of approximate policy iteration, which we will also refer to as *perpetual rollout*. This is one of the most prominent types of reinforcement learning methods, and is central in the implementation of recent high-profile successes, such as the AlphaGo and AlphaZero programs. They can be implemented using data generated by the system itself, a process known as *self-training*, and value and policy approximation architectures, including neural networks.

Fundamentally, our methods draw their validity from the algorithmic theory of dynamic programming, but they also rely on more modern approximation methods that originated in large part in learning ideas from artificial intelligence, such as the simulation-based training of compact approximation architectures, and the use of neural networks. Consequently we selectively summarize background or related material, some of which is covered in greater depth in the author's RL book [Ber19a] (see also the slides and videolectures [Ber19d]).

On the other hand, we will aim to develop rollout and approximate policy iteration methods beyond the book [Ber19a]. In particular, we will present new research, relating to systems involving multiple agents, partitioned architectures, and distributed asynchronous computation. We will also apply our methods to the solution of challenging combinatorial/discrete optimization problems and partially observed Markov Decision Problems (POMDP).

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