16.06 Lab #2 Quanser Controller Design

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Abstract

The purpose of this lab is to design a pitch controller for the Quanser system in Lab #1.

We decided on a PI + phase-lead controller which contains two zeros and two poles. One of the zeros and both poles are known. The missing zero was found using angle conditions to be $z_1 = 0.83$ and the controller gain was found using SISO in Matlab to be $K_{RL} = 14.22$. These values were entered into the block diagram in Simulink and the Quanser's response was observed. Minor adjustments were made. Finally, these values were found to be optimal to obtain a fast response that has somewhat small overshoot:

$$z_1 = 0.5$$
$$K = 13$$

Thus the controller transfer function is:

$$G_c(s) = \frac{13\,(s+0.5)}{s(s+20)}$$

such that the overall open-loop transfer function is

$$G_c(s)G(s) = \frac{3882\,(s+0.5)(s+0.4)}{s(s+20)(s+10)\left(s^2+0.4987s+1.7064\right)}$$

The controlled response has an overshoot of 5 degrees, and a settling time of about 5 seconds.

1. Introduction

The purpose of this lab is to design a pitch controller for the Quanser system in Lab #1.

1.1 Plant

The plant model has been improved to consider the simple lag pole that is due to the motor. The open-loop system is given in the form

$$\frac{\Theta(s)}{V(s)} = \frac{K\omega_n^2 / T_m}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)\left(s + \frac{1}{T_m}\right)}$$

For Quanser 1, the values for gain, natural frequency, damping ratio, and motor delay time are as follows:

$K = 17.5$

$\omega_n = 1.3063$

$\zeta = 0.1909$

$T_m = 0.1$

The values for gain, natural frequency, and damping ratio were obtained from a revised version of Lab #1. Please see Appendix A for details of the revised second-order system model in obtaining K, ω_n, and ζ.

Using the above values, the open-loop plant transfer function becomes:

$$\frac{\Theta(s)}{V(s)} = \frac{298.6}{\left(s^2 + 0.4987s + 1.7064\right)\left(s + 10\right)}$$

1.2 Controller

The controller technique suggested to us is a PI + phase-lead controller of the following form to design a compensator that places the complex conjugate poles with a damping ratio of 0.6 and undamped natural frequency of 2.5 rad/s:

$$G_c(s) = K_c \frac{(s + z_1)(s + z_2)}{s(s + p_1)}$$

Where for Quanser #1,

$p_1 = -20$

$z_2 = -0.4$

Thus I need to use root locus to find z_1 that would place complex poles such that the damping ratio is 0.6 and undamped natural frequency is 2.5 rad/s.

1.3 Design process

The goal is to obtain a system response that reaches steady state as fast as possible without overshooting too much. These two goals are unfortunately tradeoffs, so the parameters need to be adjusted such that both of these goals are met to an acceptable degree.

The design process shall be as follows:

- The first zero z_1 shall be calculated as stated in 1.2.
- The SISO tool in Matlab will be used to determine the gain.
- The controller transfer function will be fully defined, and can be added to the block diagram in Simulink.
- I will run the Quanser and observe the response
- Controller gain and/or zeroes will be adjusted as necessary until satisfactory response is reached (i.e. A system response that reaches steady state as fast as possible without overshooting too much).

2. Results and Discussion

2.1 Setup

The Quanser to be controlled is #Q1.

2.2 *Determining* z_1

The open-loop plant transfer function is

$$G(s) = \frac{298.6}{\left(s^2 + 0.4987s + 1.7064\right)\left(s + 10\right)}$$

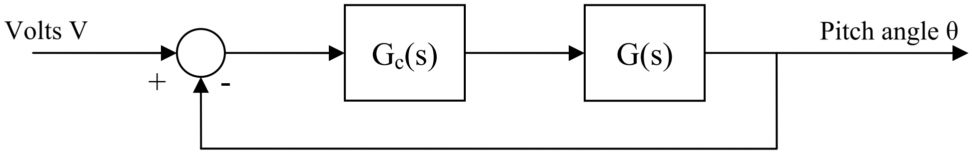
And the controller transfer function is

$$G_c(s) = K_c \frac{(s + z_1)(s + 0.4)}{s(s + 20)}$$

So the overall open-loop transfer function is

$$G_c(s)G(s) = K_c \frac{(s + z_1)(s + 0.4)}{s(s + 20)} \frac{298.6}{\left(s^2 + 0.4987s + 1.7064\right)\left(s + 10\right)}$$

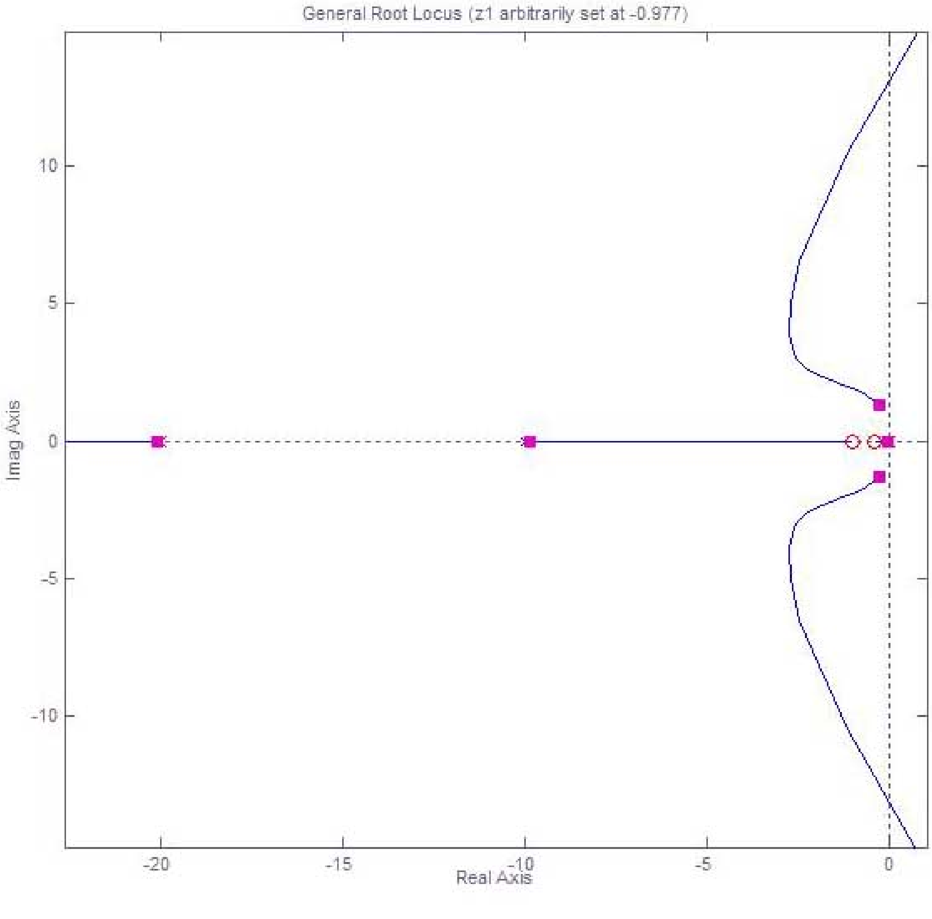
For the feedback closed-loop:



The transfer function is

$$\frac{\Theta(s)}{V(s)}=\frac{298.6\,K_c(s+z_1)(s+0.4)}{s(s+10)(s+20)\left(s^2+0.4987s+1.7064\right)+298.6\,K_c(s+z_1)(s+0.4)}$$

The pole-zero plot looks like



We want

$\zeta = 0.6$

$\omega_n = 2.5$

which corresponds to complex poles at $-1.5 \pm 2j$

Now use angle condition to solve for z_1:

$$\sum \angle zeros - \sum \angle poles = -180^\circ$$

$$\tan^{-1}\tfrac{2}{z_1-1.5} + \tan^{-1}\tfrac{2}{0.4-1.5} - \tan^{-1}\tfrac{2}{0-1.5} - \tan^{-1}\tfrac{2}{10-1.5} - \tan^{-1}\tfrac{2}{20-1.5} - \tan^{-1}\tfrac{2-\omega_d}{\zeta\omega_n-1.5} - \tan^{-1}\tfrac{2+\omega_d}{\zeta\omega_n-1.5} = -180$$

$$\tan^{-1}\tfrac{2}{z_1-1.5} + (-61.19) - (-53.13) - (13.24) - (6.17) - (-29.85) - (-69.14) = -180$$

$$\tan^{-1}\tfrac{2}{z_1-1.5} = -251.52$$

$$\frac{2}{z_1-1.5} = -2.992$$

$$\boxed{z_1 = 0.83}$$

2.3 *Use SISO to find gain*

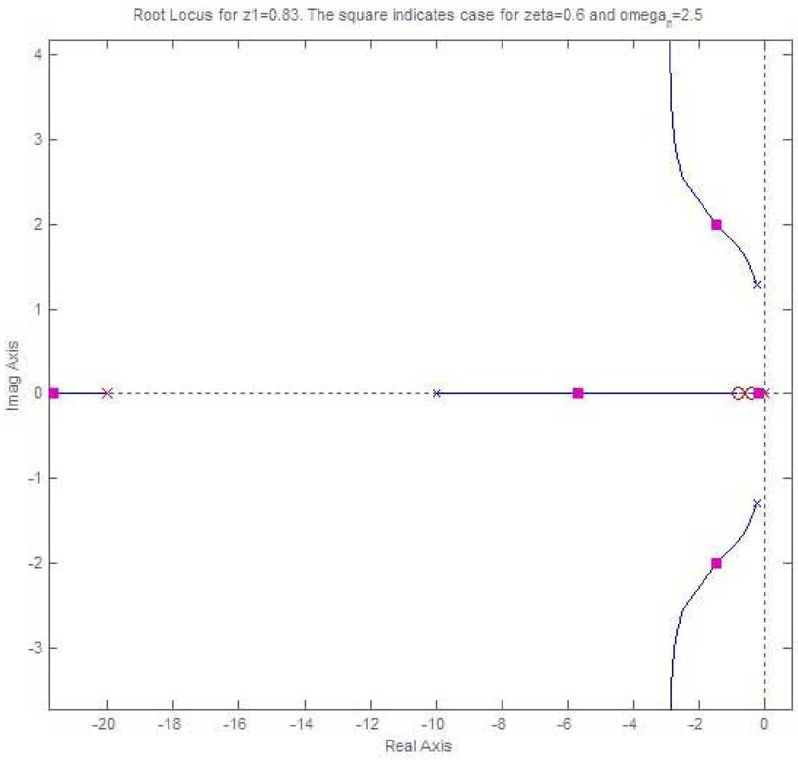
Now the open-loop plant transfer function can be entered into the SISO tool in Matlab as

$$G(s) = \frac{298.6}{\left(s^2+0.4987s+1.7064\right)\left(s+10\right)}$$

And the controller transfer function as

$$G_c(s) = \frac{(s+0.83)(s+0.4)}{s(s+20)}$$

With these poles and zeros, the pole-zero plot looks like:



We can read off the standard gain from the SISO program:

$K_{std} = 0.237$

So the root locus gain is

$K_{RL} = K_{std} \times 1.2 \times 2.5 \div 0.05$

$\boxed{K_{RL} = 14.22}$

2.4 *Run the Quanser*

The overall open-loop system transfer function with

$z_1 = 0.83$
$K = 14.22$ is

$$G_c(s)G(s) = \frac{4246\,(s+0.83)(s+0.4)}{s(s+20)(s+10)\left(s^2+0.4987s+1.7064\right)}$$

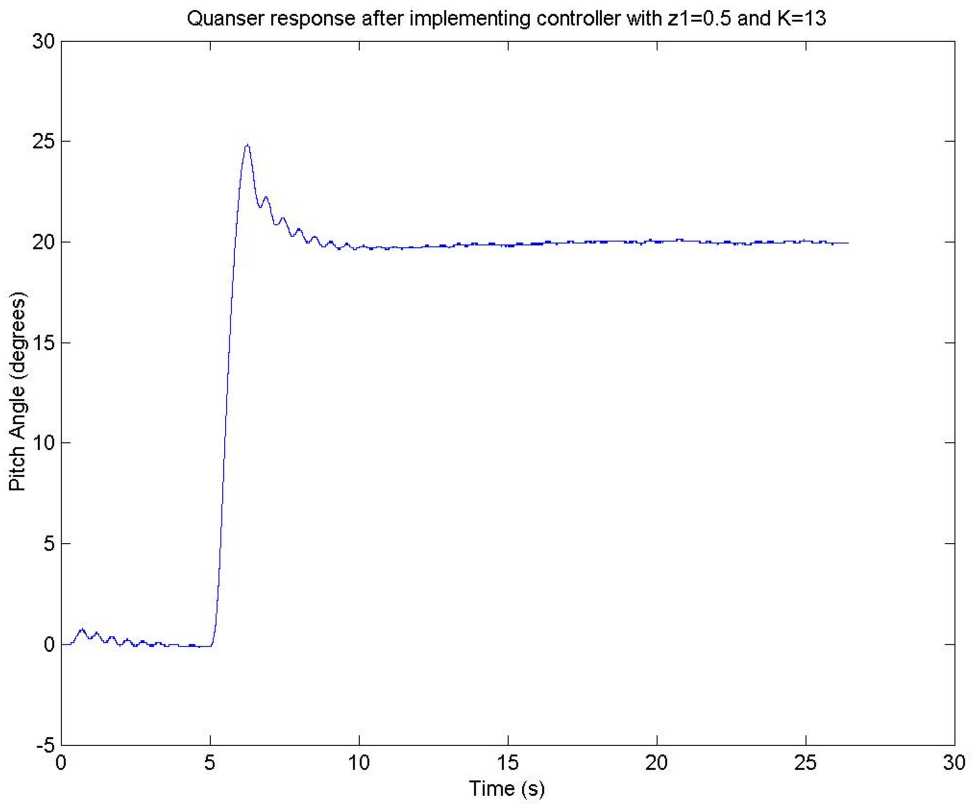
After entering z_1 and K into the system transfer function (block diagram) in Simulink and running the Quanser, we see that the response takes a while to reach steady-state, and that over-shoot is quite large. (Unfortunately I had not remembered to save what this response looks like exactly). So in order to get a quicker response, I moved the zero (z_1) closer to the $j\omega$ axis to try to cancel out the slow pole. I adjusted gain accordingly to obtain the correct steady-state response. After a few adjustments, I found that these values are optimal to obtain a fast response that has somewhat small overshoot:

$z_1 = 0.5$
$K = 13$

This gives an open-loop transfer function of

$$G_c(s)G(s) = \frac{3882\,(s+0.5)(s+0.4)}{s(s+20)(s+10)\left(s^2+0.4987s+1.7064\right)}$$

And the Quanser response looks like the following:



2.5 *Is the controller acceptable?*

From the response plot, we see that overshoot is less than 5 degrees, giving a P.O. of 25%. This is much better than the uncontrolled response (from Lab #1) which had a P.O. of 42%. Also, it takes about 5 seconds for the response to reach steady-state, much better than in the uncontrolled case where settling time is much greater than 15 seconds (our plot only went as far as 15 seconds, and the response had still not settled). This controller accomplishes the goal to obtain a system response that reaches steady state as fast as possible without overshooting too much.

3. Conclusion

Quanser #1 can be controlled by a PI + phase-lead controller with the following transfer function:

$$G_c(s) = \frac{13\,(s+0.5)}{s(s+20)}$$

such that the overall open-loop transfer function is

$$G_c(s)G(s) = \frac{3882\,(s+0.5)(s+0.4)}{s(s+20)\left(s+10\right)\left(s^2+0.4987s+1.7064\right)}$$

The controlled response has an overshoot of 5 degrees, and a settling time of about 5 seconds. Both of these results are fairly good, so we have reached our goal of designing a good controller.