

The Elimination of Correlation Errors in PIV Processing

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ABSTRACT

A non-post-interrogation method of reducing sub-pixel bias errors and eliminating spurious vectors from particle image velocimetry (PIV) results is presented. Unlike methods that rely on the accuracy or similarity of neighboring vectors, errors are eliminated before correlation information is discarded using available spatial and/or temporal data. Anomalies are removed from the data set through direct element-by-element comparison of the correlation tables calculated from adjacent regions. The result is in an error correction method that is highly robust to out-of-boundary particle motion, particle overlap, inter-particle correlations, and electronic and optical imaging noise. Consequently, the sample volume size required to resolve tracer particle displacement can be reduced improving the resolution as well as the accuracy of PIV.

1. INTRODUCTION

Because Particle Image Velocimetry (PIV) is based on the statistical correlation of imaged subregions to determine local flow velocities, it is subject to inherent errors that arise from finite tracer particle numbers, sample volume size, and image resolution. These errors, in extreme cases, are relatively easy to detect as they tend to vary substantially from neighboring vectors in both magnitude and direction. Despite this, correcting these errors is often difficult as present computer algorithms lack the innate pattern recognition ability of humans. Furthermore, such errors need not present themselves in obvious manners. Velocity vectors determined by correlating finite subregions of tracer particle images are often biased to varying degrees by particles entering and exiting the sample volume in the time between exposures, correlations occurring between unmatched particle pairs, particle overlap, non-uniform particle distribution, and variations in image intensity [Keane et. al. 1992; Prasad et. al. 1992; Westerweel et. al. 1993, 1994, 1997; Raffel et. al. 1994; Lourenco et. al. 1995;

Fincham et. al. 1997]. Such errors along with errors associated with excessive velocity gradients and the finite sample volume size necessary to image a statistically meaningful number of tracer particles limit accuracy and resolution and thus, limit the usefulness of PIV.

Currently, the most widely used and accepted technique to eliminate correlation errors is to compare vectors with their neighbors to determine if they are in some statistical or physical sense inconsistent. This technique, analyzed in detail by Westerweel (1994), is based on the assumption that vectors resulting from correlation errors are far removed in magnitude and/or direction from their neighboring vectors. It assumes that the resolution of PIV data is high enough and the flow features benign enough that apparent discontinuities in the flow will not present themselves in the data and be eliminated by such a correction algorithm. In addition, an error can only be detected and replaced with an interpolated value using this approach. Even if the region represented by the vector contains sufficient information to determine the local flow velocity, that information is discarded prior to error correction. Thus, the true correlation in that region can not be determined without an attempt at reprocessing. Furthermore, it addresses only the most obvious of correlation errors and does not address the more subtle problems that severely limit sub-pixel accuracy and resolution. Thus while extremely useful, post-interrogation error correction techniques are not ideal.

Presented herein is a robust and computationally efficient method of removing errors from PIV results and resolving vectors from regions where noise in the correlation table obscures tracer particle displacement. This method, based on an element-by-element comparison of the correlation tables taken from adjacent regions, does not rely on the accuracy or similarity of neighboring vectors as does post-interrogation correction techniques. Because of the

effectiveness of this method at eliminating correlation anomalies, the size of interrogation regions can be reduced improving spatial resolution. Furthermore, recursive correlation techniques can be implemented without generating excessive spurious results.

1. CORRELATION ERRORS

PIV images are typically processed by subdivision into a regular grid of overlapping windows that bound regions of similar flow velocity; a velocity vector is then found for each window by autocorrelation or cross-correlation. Autocorrelation and cross-correlation produces a table of correlation values over a range of displacements, and the overall displacement of particles in the window is represented by a peak in this correlation table. Errors occur primarily from insufficient data whether it be a lack of imaged flow tracers or poor image quality, and/or from correlation anomalies generally resulting from unmatched tracer images within the correlated sample volume.

Insufficient Data

As the number of tracer particles within a sample volume increases, the probability of any finite region in close proximity existing with a particle set of similar intensity and pattern decreases. Thus, the probability of obtaining an accurate measure of the displacement of a set of particles using correlation increases as the number of particles increases. Keane and Adrian (1992) demonstrated this by showing that the number of spurious vectors that appear in PIV data drop dramatically as particle numbers within correlated subregions are increased up to an average of about ten particle images per region. Unfortunately, while this, in general is true, there is a limit due to finite intensity and spatial resolution capabilities of current imaging technology. Tracer particle seeding densities can be increased to a level in which the spacing between the particles contains more flow information than the particles themselves. Adding more tracers to a flow with this seeding density decreases the probability of obtaining an accurate correlation rather than increases it. Furthermore, very high seeding densities can alter the characteristics of the flow being measured and make it difficult if not impossible to adequately illuminate and image the tracer particles within a specific region of interest.

An alternative is to increase the size of the sample volume thereby increasing the number of tracer particles in the interrogation region without increasing the seeding density. This, however, results in a decrease in resolution and, as with seeding density, there is an upper limit.

Correlation Anomalies

When the difference between the movement of tracer particles in one region of a sample volume versus another is greater than the radius of a single tracer particle, than increasing the sample volume size results in an increase in correlation anomalies without increasing the peak correlation value [Keane and Adrian, 1992, Hart 1996]. There is, thus, a reduction in the correlated signal-to-noise ratio. This is due to a separation in the correlation peaks of individual particle images that is greater than the width of the correlation of a single particle image against itself. Increasing the sample volume exacerbates the problem as the relative separation in individual particle correlations from one region to another due to gradients in the flow increases as well. It, however, does reduce correlation anomalies associated with particles entering and exiting the sample volume in the time between exposures. As the sample volume is increased, a smaller fraction of particles enter and exit relative to the total number of particles that remain within the sample volume between exposures. Consequently, there is a reduction in correlation anomalies associated with this phenomenon.

In addition to excessive velocity gradients and out-of-boundary particle motion, problems associated with particle image overlap, non-uniform illumination of the sample volume, reflections off surfaces, particle coalescence, non-uniform flow seeding and image discretization all contribute to the number of correlation anomalies. These problems, although generally not as severe as velocity gradient and out-of-boundary effects, contribute to sub-pixel bias errors and are often the cause of spurious vectors.

Several processing methods have been developed that are less prone to these effects than the traditional spectral correlation methods. Okamoto et. al. (1995) developed a least energy correlation method they refer to as the *string method* that is relatively robust to large velocity gradients. Keane and Adrian (1992) and Westerweel, Dabiri and Gharib (1997) demonstrated that sub-pixel bias errors associated with correlation anomalies from particles entering and exiting the sample volume in the time between exposures could be significantly reduced by correlating with an offset interrogation window. This processing technique has the same effect as the shift and multiply technique or the much faster sparse array image correlation technique [Hart 1996] that does not rely on forced periodicity of data and this method is nearly identical to the sub-correlation technique proposed by Fincham and Spedding (1991) that also relies on offset correlation windows. These processing techniques,

although an improvement over standard spectral processing, are still subject to errors from out-of-plane particle movement, variations in image intensity, particle overlap and errors associated with seeding density.

2. CORRELATION ERROR CORRECTION

Both errors resulting from insufficient data and errors caused by correlation anomalies can be eliminated during processing, regardless of the method of correlation, simply by multiplying the correlation table generated during processing by the correlation table generated from one or more adjacent regions (Fig. 1).

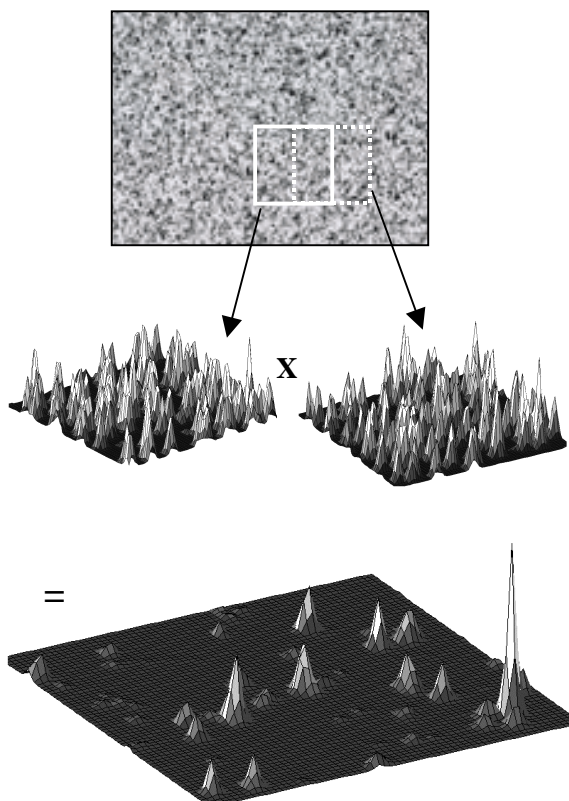


Figure 1. – Elimination of correlation anomalies by multiplying the correlation tables from adjacent regions. Correlation values that do not appear in both tables are eliminated allowing tracer particle displacement to be resolved.

This is effectively a correlation of a correlation. It is not an averaging technique. Any correlated region that does not appear in both correlation tables is eliminated from the resulting table. As the probability of exactly the same anomalies appearing in another region is very

small, correlation anomalies, regardless of their source, are eliminated from the data. Furthermore, spurious vectors due to insufficient data are eliminated as any peak in one correlation table that does not exist within the other is eliminated. Even if both correlation tables do not contain the information necessary to resolve the correct correlation peak, combined in this manner, the peak is either easily resolved or it becomes evident that neither table contains sufficient data. (This is often the result of a single particle image within the sample volume in one exposure and multiple images in another – rare in well-seeded flows.) The resulting correlation peak found in the table is weighted to the displacement of the tracer particles within the overlap of the combined regions as information within this region identically effects the correlation values in both correlation tables. Particle displacements in regions outside the overlap influence the calculated displacement but to an extent that depends on the similarity in displacement. Thus, rather than a reduction in resolution, there is an improvement that depends on the size of the overlap and the gradient of the velocity relative to the size of the sample volume.

Under extreme conditions, valid correlations can be eliminated if the displacement of the tracer particles of the combined regions relative to each other is greater than about one particle image diameter (Fig. 2).

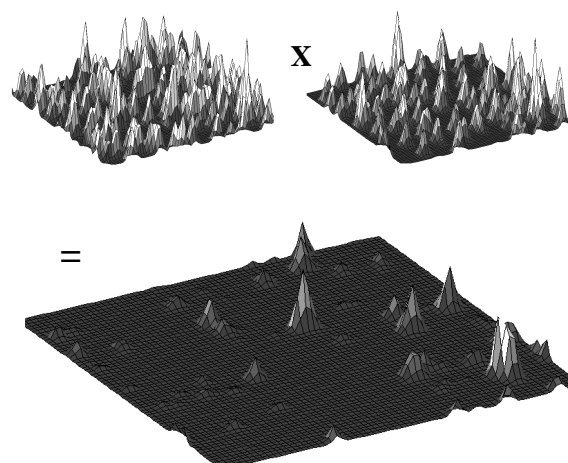


Figure 2. – Illustration of the effect of an excessive velocity gradient on the correlation signal using correlation error correction. Signal strength is significantly reduced when tables are multiplied from regions where particle displacement differs by more than one particle diameter.

It is therefore, desirable to maintain a relatively high level of overlap between regions. The level of effectiveness of this technique, however, increases as

the size of the overlapped region decreases due to a reduction in the level of shared information.

Although an extensive analysis of the optimum overlap has not been conducted, a fifty-percent overlap has been shown to effectively eliminate correlation errors (Fig. 3). As most correlation algorithms currently use a fifty-percent overlap based on Willert and Gharib's analysis (1991), there is virtually no increase in computational requirements to implement this error correction technique. Furthermore, as it requires a simple element by element multiplication between tables, it is generally easier and computationally more efficient to implement than it is to conduct post-interrogation error correction.

Unlike post-interrogation techniques that process data after most of the correlated information is discarded, this error correction technique uses all available correlation information within the interrogation regions to resolve particle displacement before the correlation peak is determined.

In addition, it does not rely on the accuracy of neighboring correlations to determine errors nor will valid correlations be eliminated if they vary substantially from neighboring regions.

3. PERFORMANCE TEST

The relative performance of correlation error correction was investigated using Monte Carlo simulations. The effects of large velocity gradients, out-of-boundary motion, and seeding density were studied.

The simulations were conducted using pairs of synthetic generated images with randomly distributed 256 grayscale Gaussian particle images 4 pixels in diameter. Correlations were conducted using FFT spectral correlation on 64 x 64 pixel interrogation regions with 10 particle images per region. Results using correlation error correction were compared with non-spurious results taken from uncorrected data.

Fig. 4 illustrates the effect of translation on bias errors by comparing uncorrected results with results corrected from fifty-percent overlapping regions. Here the standard deviation of the displacement error from one thousand interrogation regions are plotted as a function of imposed particle translational displacement normalized by the interrogation size (64 px).

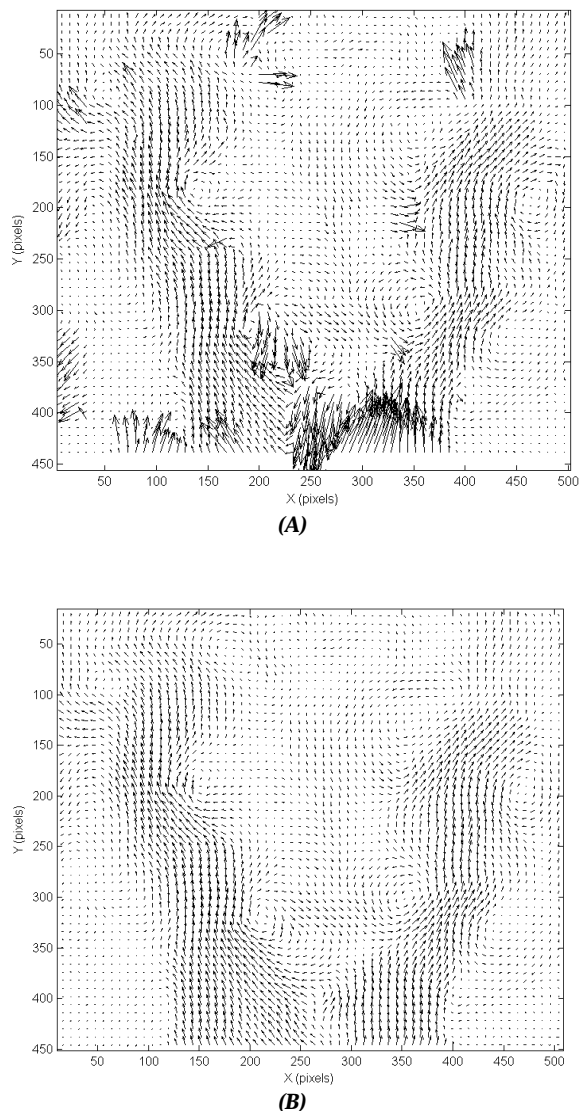


Figure 3. – Elimination of processing errors using correlation error correction. 3(A) represents uncorrected PIV results of a high Reynolds number swirling flow undergoing sudden expansion. 3(B) illustrates the same data processed using correlation error correction. The corrected vectors are true tracer particle displacements and not interpolated values.

The correlation error corrected results show a twenty-five percent improvement in resolving sub-pixel displacement. Note that the bias error increases linearly with translational displacement. This is consistent with errors conjectured by Adrian (1991) and analyzed in detail by Westerweel, Dabiri, and Gharib (1997). It is, to some extent, the result of fewer matching particle image pairs between correlated regions. In translational flow, particles are convected out of and into the interrogation region. These convected particles do not have a corresponding image

to correlate against. Consequently, an increase in translational flow has a similar effect to a reduction in seeding density, Fig. 5.

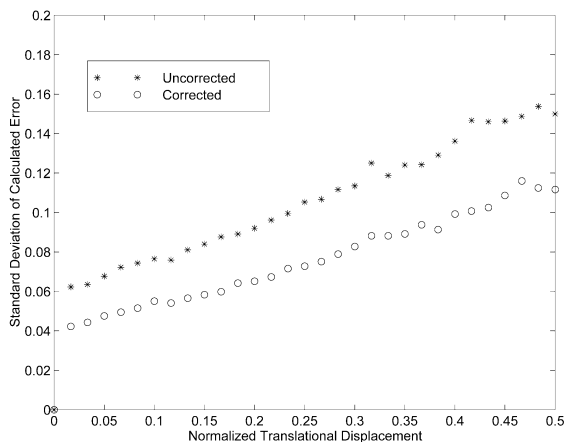


Figure 4. – Effect of translational displacement on sub-pixel accuracy. Correlation error correction improves sub-pixel accuracy by eliminating anomalies in the correlation table and strengthening the peak correlation signal.

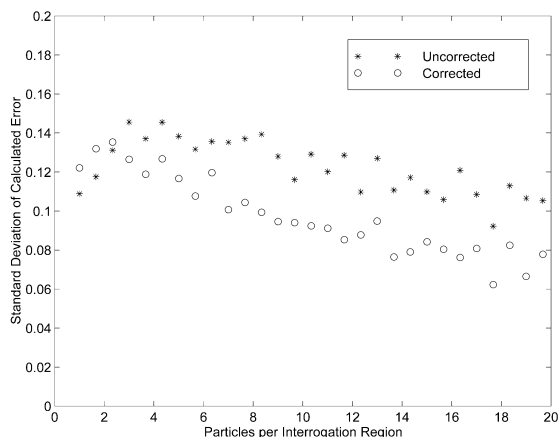


Figure 5. – Effect of seeding density on sub-pixel accuracy.

Errors associated with translational motion exist for out-of-plane particle displacement as well. It, however, is particularly limiting to PIV as it restricts measurements to regions of nearly planer flow or restricts the minimum width of the sample volume and thus, restricts spatial resolution.

As shown in Fig. 6, correlation error correction is highly effective at resolving particle displacements from regions with significant out-of-plane flow. Here the percentage of valid vectors is plotted against normalized out-of-plane translational particle displacement. The uncorrected data begins to produce excessive spurious results at fifty-percent displacement.

Using fifty-percent overlapped regions, correlation error corrected data produces few spurious results until almost 90% displacement. At 100% displacement, all particles within the interrogation region move out of the region between exposures. Thus, there exists no data to determine flow velocity regardless of the correlation algorithm.

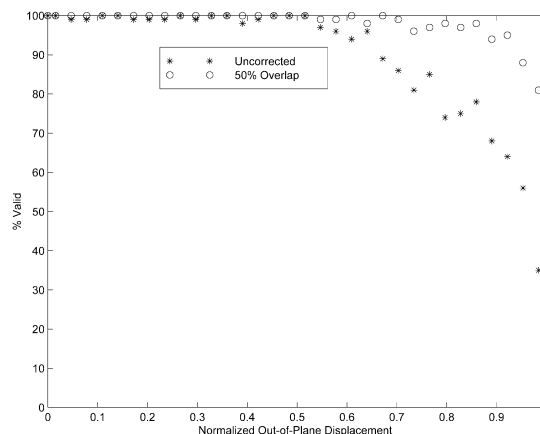


Figure 6. – Effect of out-of-plane flow on PIV processing. Correlation error correction using fifty-percent overlapped regions significantly improve the ability to resolve particle displacements.

While correlation error correction is highly effective at reducing errors associated with anomalies in the interrogation region, it is only marginally effective at resolving errors associated with large velocity gradients. Velocity gradients cause one part of an interrogation region to correlate at a different value than another region. It imparts a distortion that most correlation techniques are not able to resolve. When the distortion results in particle displacements greater than about one particle diameter, the relative correlation signal-to-noise ratio decreases significantly.

Fig. 7 shows a plot of the percentage of valid vectors as a function of normalized velocity gradient in the ij interrogation plane;

$$G_v = \frac{m|\nabla \bar{v}_{ij}|N\Delta t}{d}$$

where N is the interrogation size (64 px in this case), m is the image magnification, Δt is the time between exposures, and d is the average imaged particle diameter. As shown, correlation error correction increases the number of valid vectors calculated from regions of high velocity gradients. At this level, however, the probability of acquiring a valid correlation is unacceptably low and at lower levels, the improvement is marginal.

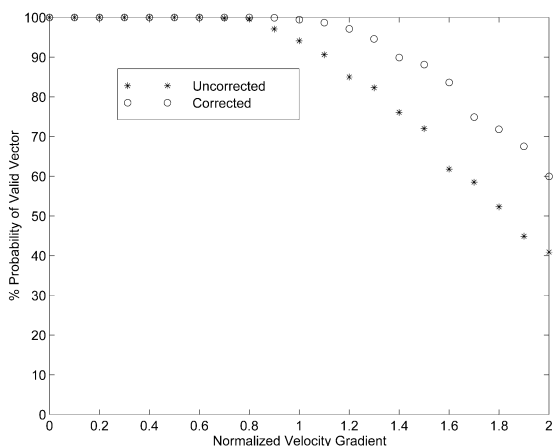


Figure 7. – Effect of velocity gradient on PIV processing. Correlation error correction improves the ability to resolve tracer particle displacements in regions that have high velocity gradients.

4. HIGH ACCURACY/ HIGH RESOLUTION PROCESSING

Because of the robustness of correlation error correction in removing spurious vectors and correlation anomalies, the size of the interrogation region needed to obtain an accurate correlation can be greatly reduced. Furthermore, recursive correlation processing can be used to iteratively arrive at local particle displacement without generating spurious results; a region is correlated, the interrogation window size is then reduced and offset by the previous result before re-correlating with the new window over a reduced region. The result of this correlation process can be quite dramatic as is illustrated in Fig. 8. Here recursive correlation along with correlation error correction is used to iteratively resolve the velocity of a swirling flow undergoing sudden expansion.

Figure 9 shows the same data resolved to 4x4 pixel interrogation regions generating 60,000 vectors each representing a 0.8mm³ region of the flow. Using sparse array image correlation, results from the entire image are processed in less than one minute (>1,000 vectors/sec) [Hart, 1996]. While it is unlikely that the original image contains tracer particle displacement information to the resolution implied by this plot, it illustrates the possibility of correlating images to the available resolution of the data without generating spurious results.

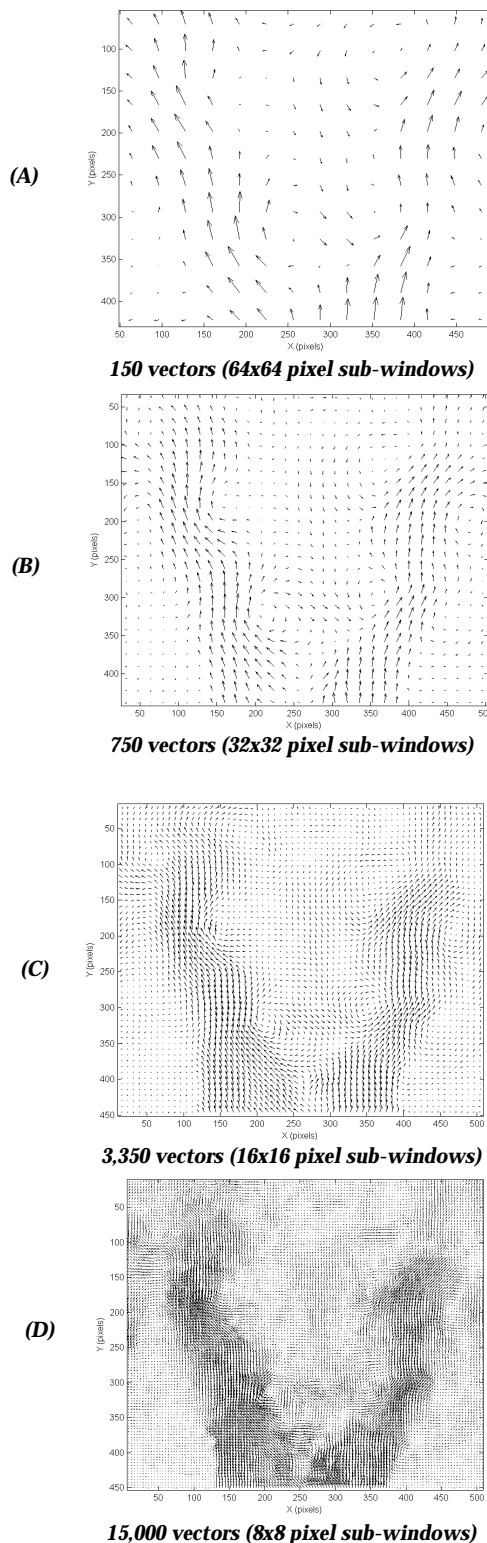


Figure 8. – Recursive processing (A to D) of a PIV image of a high Reynolds number swirling flow undergoing sudden expansion.

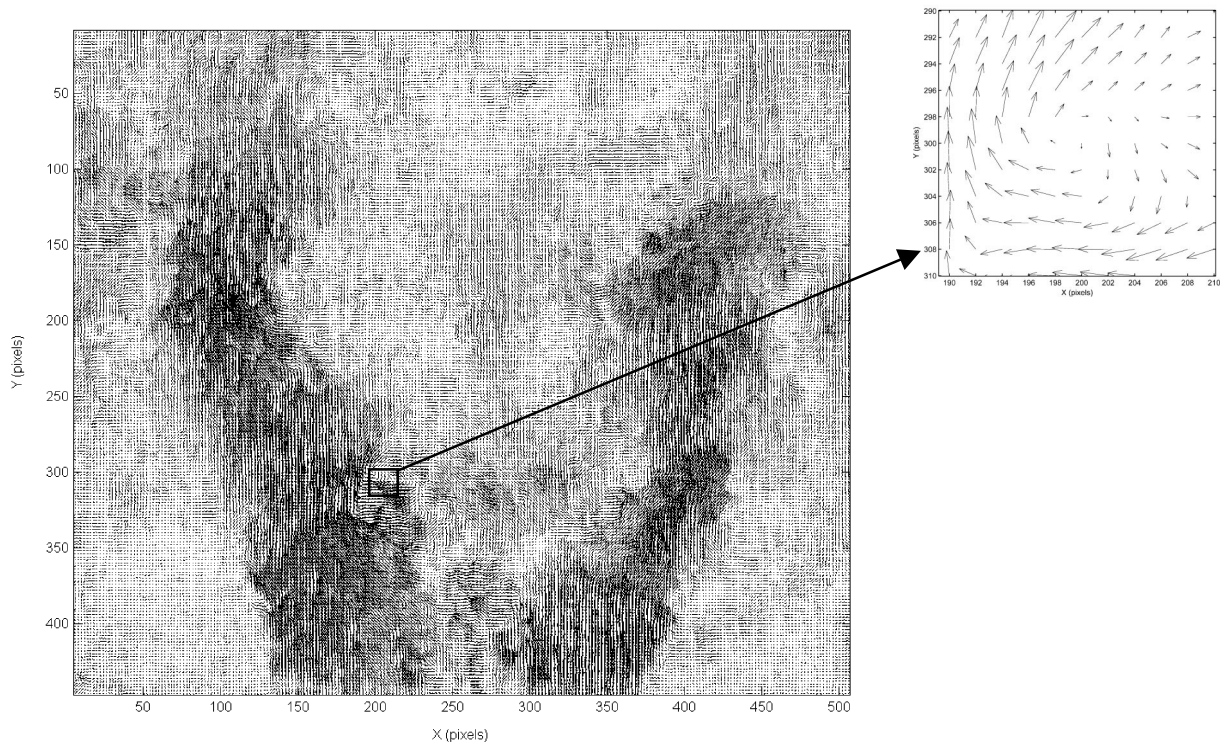


Figure 9. – 60,000 vectors calculated using recursive correlation from a 486x512 image of a high Reynolds number swirling flow undergoing sudden expansion. Each vector on average represents a single tracer particle in the flow and is the result of as many as ten sub-window correlations. These results are processed at a rate of over 1,000 vectors/sec using sparse array image correlation with correlation error correction

5. CONCLUSIONS

Stochastic correlation values can be eliminated during PIV processing by element-by-element multiplication of correlation tables calculated from adjacent regions. This correction method, which is essentially a correlation of a correlation, reduces sub-pixel bias errors and eliminates spurious vectors resulting from unmatched particle pairs, out-of-boundary motion, particle overlap, inter-particle correlations, and electronic and optical imaging noise. Particle displacement information that does not correlate equally in the combined regions is minimized. The resulting calculated displacement is weighted to the adjoining area thereby improving spatial resolution.

Correlation error correction, unlike post-interrogation methods, uses all available correlation information calculated from interrogation regions and does not rely on the accuracy or similarity of neighboring vectors. Consequently, it resolves tracer particle displacement vectors from regions that would otherwise be discarded. Thus, the correction method presented

herein reduces bias errors and eliminates spurious vectors while improving spatial resolution and vector yields.

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