As given in the F7 notes, the streamfunction of the flow about a circular cylinder is given by

\[ \psi = V_\infty r \sin \theta \left( 1 - \frac{R^2}{r^2} \right) + \frac{\Gamma}{2\pi} \ln r \]

where \( \Gamma \) is some arbitrary circulation. For this exercise, assume a value of \( \Gamma = 4 V_\infty R \) which might be typical on a spinning cylinder.

a) Determine the horizontal velocity \( u(y) \) along the \( y \)-axis, in the form of three terms

\[ u(y) = u_{\text{freestream}}(y) + u_{\text{vortex}}(y) + u_{\text{doublet}}(y) \]

Write out the three terms separately.

b) Plot the three separate \( u \) components as \( u/V_\infty \) versus \( y/R \) on the same plot. Make your plot go over the range \( y/R = -10 \ldots 10 \), but ignore the non-physical \( u(y) \) inside the cylinder, over the center portion \( y/R = -1 \ldots 1 \).

Optional: You may also wish to overlay the total \( u/V_\infty \) vs \( y/R \) to help answer c) and d) below.

c) Describe the relative importance of \( u_{\text{vortex}} \) and \( u_{\text{doublet}} \) to the total \( u \) for

i) small distance from the cylinder surface, and

ii) large distance from the cylinder surface.

d) Justify the statement “To an observer far away, a lifting 2D object looks like a point vortex placed in the flow”.

\[ y \]
\[ x \]