Power Balance in Aerodynamic Flows

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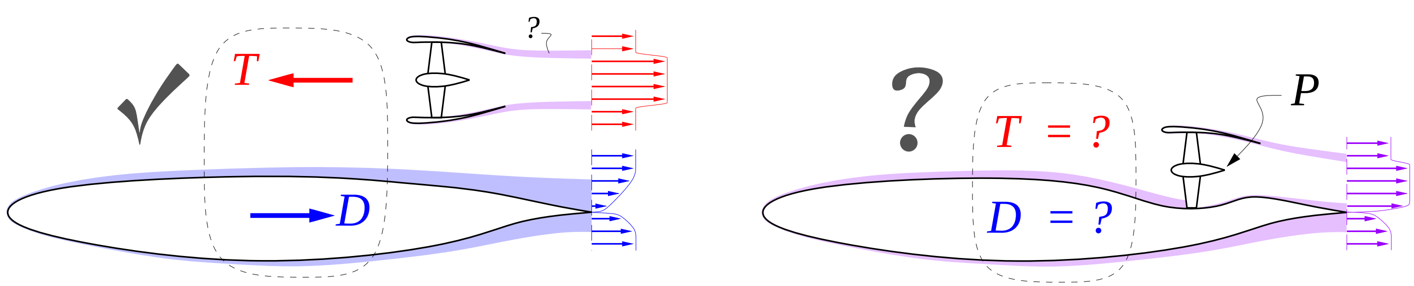
Objectives

- Quantify Mechanical Energy sources and sinks in flowfield
- Use for aerodynamic analyses, in lieu of Thrust and Drag
- Framework for configuration comparisons and evaluations

Motivation — I

- Thrust and Drag accounting is just a means to an end: Range, or Flight Power, or Fuel Consumption
- Thrust and Drag are ambiguous for integrated propulsion
- Standard performance relations become inapplicable, e.g.

$$R = \frac{V}{TSFC}\,\frac{C_L}{C_D}\,\ln\left(\frac{W_0}{W_e}\right)$$



$\Rightarrow$ Will sidestep ambiguity by relating Power to flowfield directly

Motivation — II

Want to identify ...

- flowfield power sinks (what's causing the loss/fuel burn?)
- power sink locations (where's the loss?)
- opportunities of power savings (can I reduce the loss?)

and quantify ...

- aero performance of disparate aircraft configurations and concepts
- effectiveness of BL ingestion and flow control systems
- parasite and interference drags in conventional force analysis
- etc.

Assumed Governing Equations

Mass:

$$\nabla \cdot \left(\rho \vec{V}\right) = 0$$

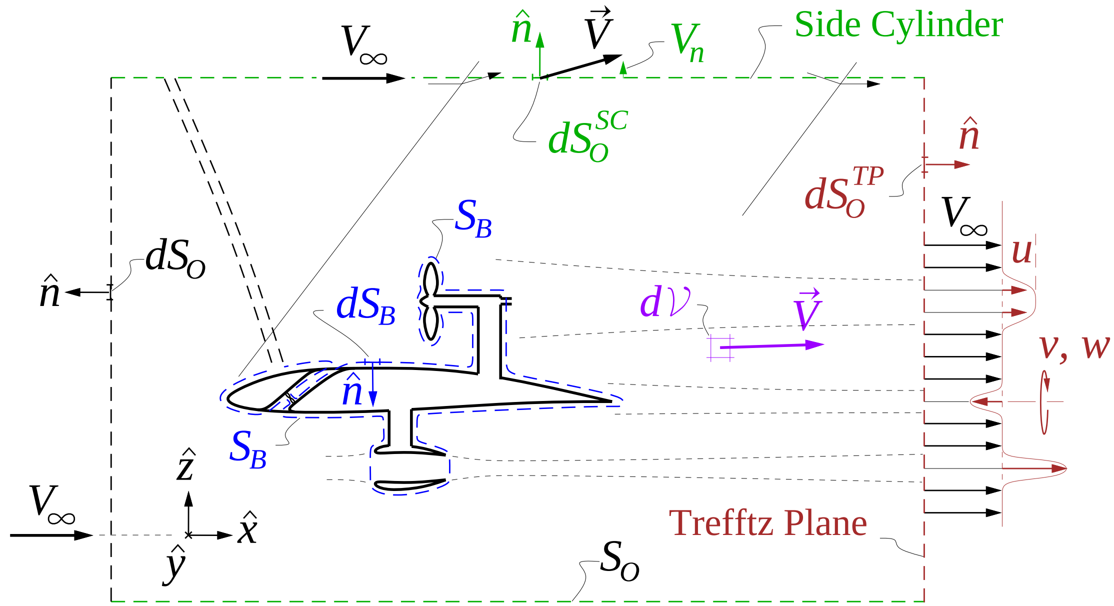
Momentum:

$$\rho \vec{V} \cdot \nabla \vec{V} = -\nabla p + \nabla \cdot \bar{\bar{\tau}}$$

Forming $\left\{\overrightarrow{\text{momentum}}\right\} \cdot \vec{V}$ gives Kinetic Energy equation:

$$\rho \vec{V} \cdot \nabla \left(\tfrac{1}{2} V^2\right) = -\nabla p \cdot \vec{V} + \left(\nabla \cdot \bar{\bar{\tau}}\right) \cdot \vec{V}$$

Control Volume



- Outer boundary $\mathcal{S}_O$ has ...
  - Trefftz Plane $\perp$ to freestream
  - Side Cylinder $\parallel$ to freestream
- Inner body boundary $\mathcal{S}_B$ can ...
  - cover propulsor blading to include shaft power
  - cover internal ducting to include flow losses, pump power

Integral Kinetic Energy Equation

$$\iiint \left\{ \rho\vec{V} \cdot \nabla \left(\tfrac{1}{2}V^2\right) \;=\; -\,\nabla p \cdot \vec{V} \;+\; (\nabla \cdot \bar{\bar{\tau}}) \cdot \vec{V} \right\} d\mathcal{V}$$

$$\vdots$$

$$\boxed{P_S \;+\; P_K \;+\; P_V \;=\; W\dot{h} \;+\; \dot{E}_\mathrm{a} \;+\; \dot{E}_\mathrm{v} \;+\; \dot{E}_\mathrm{p} \;+\; \dot{E}_\mathrm{w} \;+\; \Phi}$$

- Exact decomposition into physically-clear components
- Primary use:
  $\Rightarrow$ Obtain total power (LHS) by evaluating all RHS terms

Integral KE Equation — Energy inflow or production

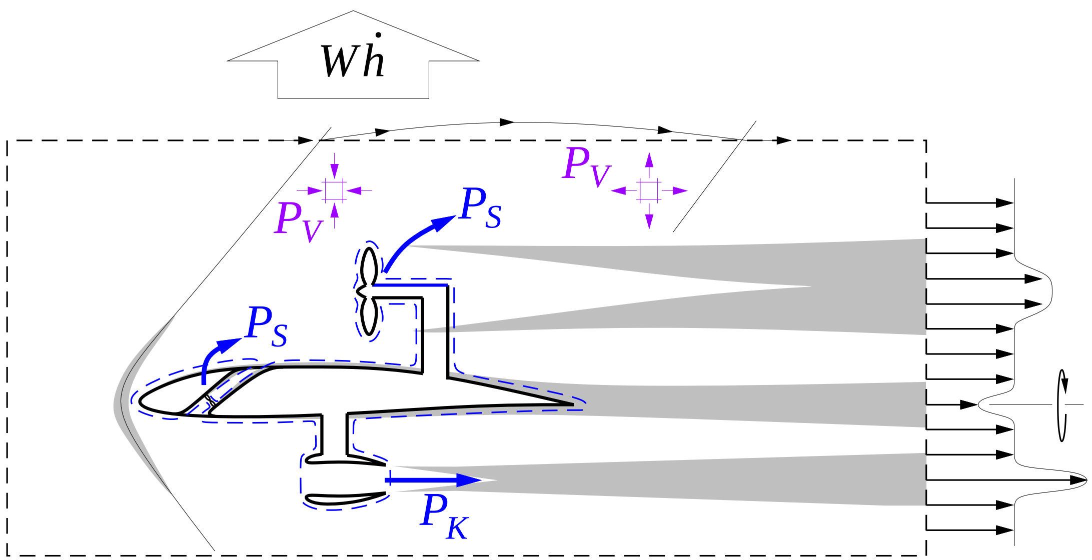
$$P_S + P_K + P_V = W\dot{h} + \dot{E}_{\rm a} + \dot{E}_{\rm v} + \dot{E}_{\rm p} + \dot{E}_{\rm w} + \Phi$$

Shaft power: $P_S = \oiint (-p\hat{n} + \vec{\tau}) \cdot \vec{V} \, d\mathcal{S}_B$

K.E. inflow: $P_K = \oiint -\left[p - p_\infty + \frac{1}{2}\rho\left(V^2 - V_\infty^2\right)\right] \vec{V} \cdot \hat{n} \, d\mathcal{S}_B$

P dV power: $P_V = \iiint (p - p_\infty) \, \nabla \cdot \vec{V} \, d\mathcal{V}$

Potential energy rate: $W\dot{h}$



Integral KE Equation — Energy flow out of CV

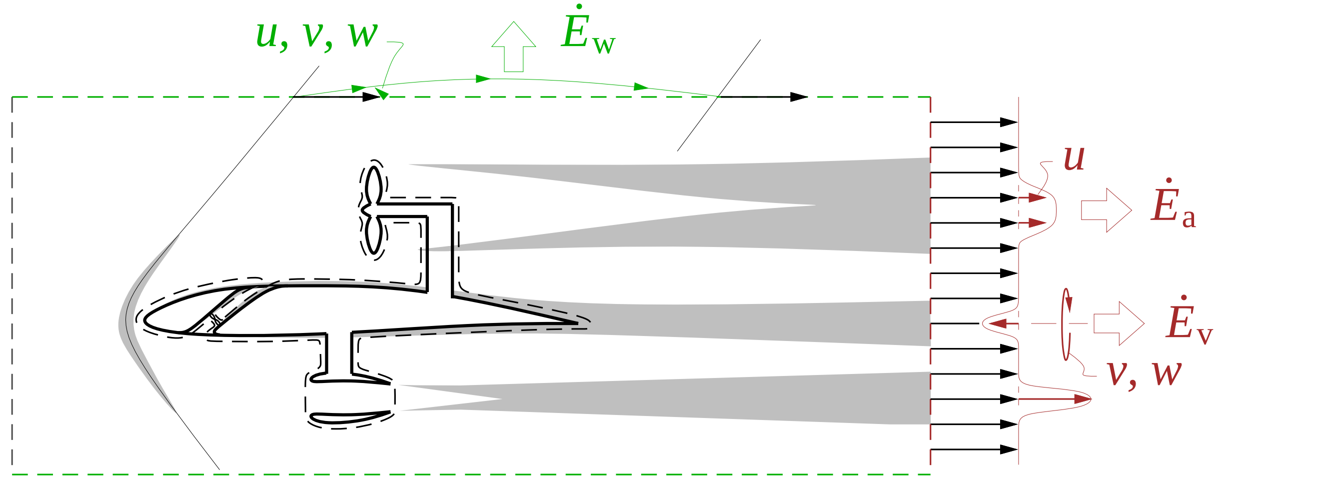
$$P_S + P_K + P_V = W\dot{h} + \dot{E}_a + \dot{E}_v + \dot{E}_p + \dot{E}_w + \Phi$$

Axial K.E. outflow: $\dot{E}_a = \iint \frac{1}{2}\rho\, u^2\, (V_\infty + u)\; d\mathcal{S}_O^{TP}$

Vortex K.E. outflow: $\dot{E}_v = \iint \frac{1}{2}\rho\, (v^2 + w^2)\, (V_\infty + u)\; d\mathcal{S}_O^{TP}$

Pressure work: $\dot{E}_p = \iint (p - p_\infty)\, u\; d\mathcal{S}_O^{TP}$

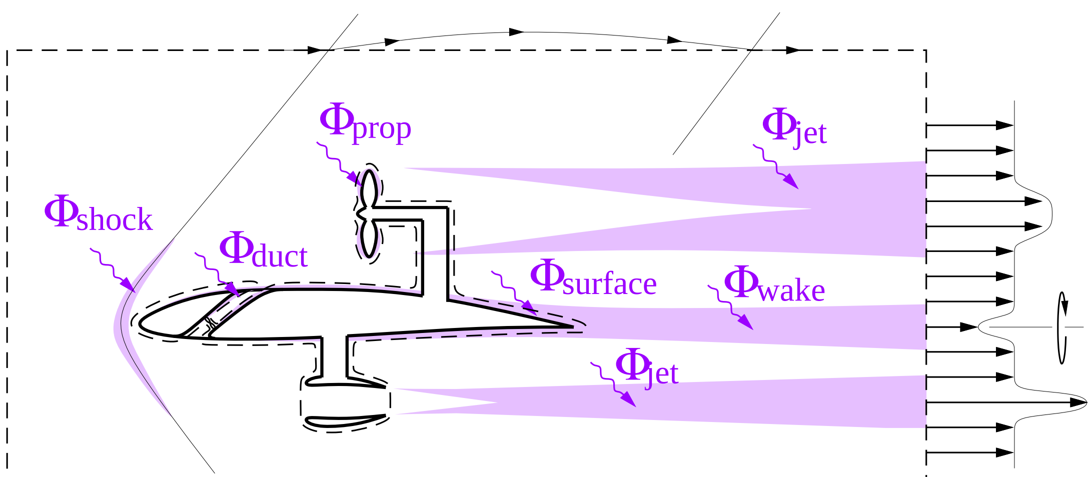
Wave outflow: $\dot{E}_w = \iint \left[p - p_\infty + \frac{1}{2}\rho\left(u^2 + v^2 + w^2\right)\right] V_n\, d\mathcal{S}_O^{SC}$



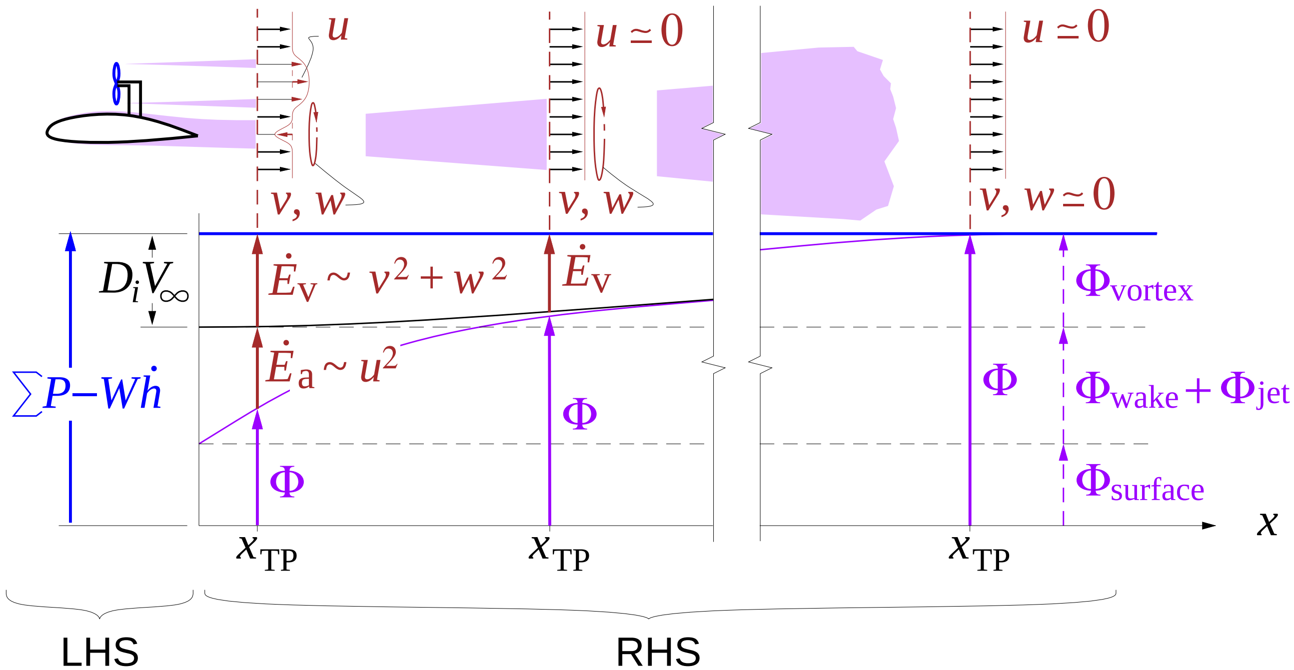
Integral KE Equation — Energy lost inside CV

$$P_S + P_K + P_V = W\dot{h} + \dot{E}_{\mathrm{a}} + \dot{E}_{\mathrm{v}} + \dot{E}_{\mathrm{p}} + \dot{E}_{\mathrm{w}} + \Phi$$

Viscous dissipation: $\Phi = \iiint \left(\bar{\bar{\tau}} \cdot \nabla\right) \cdot \vec{V} \, d\mathcal{V}$



Outflow/Dissipation term balance



- Same lost power (LHS) is obtained for any chosen RHS Trefftz Plane location
- $\dot{E}$ outflow terms account for dissipation outside of CV

Loss Calculation and Estimation

$\dot{E}$, Φ components often available from other theories/methods

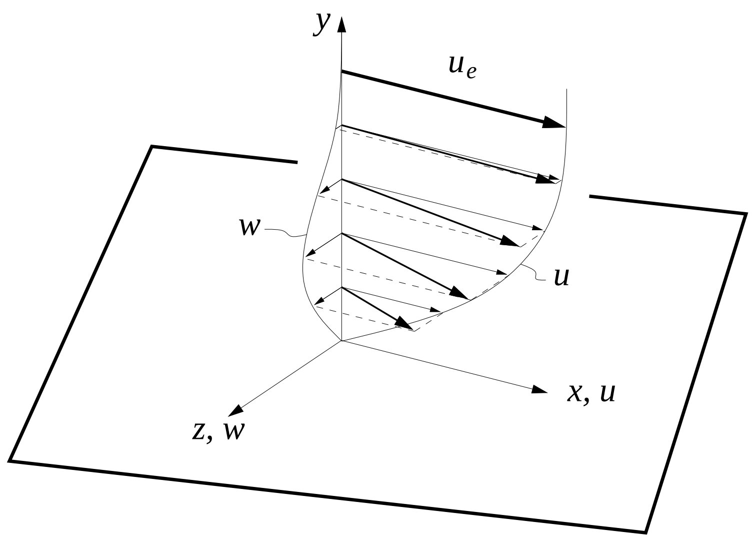
$$\dot{E}_{\rm v} \;=\; \Phi_{\rm vortex} \;=\; D_i \, V_\infty \;\;=\; \frac{L^2}{\frac{1}{2}\rho V_\infty^2 \, \pi \, b^2 \, e} \, V_\infty$$

$$\dot{E}_{\rm w} \;=\; D_w \, V_\infty \;\geq\; \frac{\sin\Lambda}{\sqrt{1 - M_\perp^2}} \, \frac{L^2}{\frac{1}{2}\rho V_\infty^2 \, \pi \, b^2} \, V_\infty$$

$$\Phi_{\rm jet} \;=\; P_S \;(1 - \eta_{\rm froude}) \;=\; P_S \, \frac{\sqrt{T_c + 1} \;-\; 1}{\sqrt{T_c + 1} \;+\; 1}$$

$$\Phi_{\rm prop} \;=\; P_S \;(1 - \eta_{\rm profile}) \;=\; P_S \left(1 - \frac{1 - (c_d/c_\ell)\tan\phi}{1 + (c_d/c_\ell)/\tan\phi} \right)$$

Dissipation in Boundary Layers and Wakes



$$\Phi_{\mathrm{surface,wake}} = \iiint (\bar{\bar{\tau}} \cdot \nabla) \cdot \vec{V} \, d\mathcal{V}$$

$$\simeq \iiint \left(\tau_{xy} \frac{\partial u}{\partial y} + \tau_{zy} \frac{\partial w}{\partial y} \right) dx \, dy \, dz \quad \text{(for 3D shear layer)}$$

$$\simeq \iiint \tau_{xy} \frac{\partial u}{\partial y} \, dx \, dy \, dz \quad \text{(for 2D shear layer)}$$

Dissipation Coefficient

It's convenient to work with a dissipation coefficient $C_{\mathcal{D}}$:

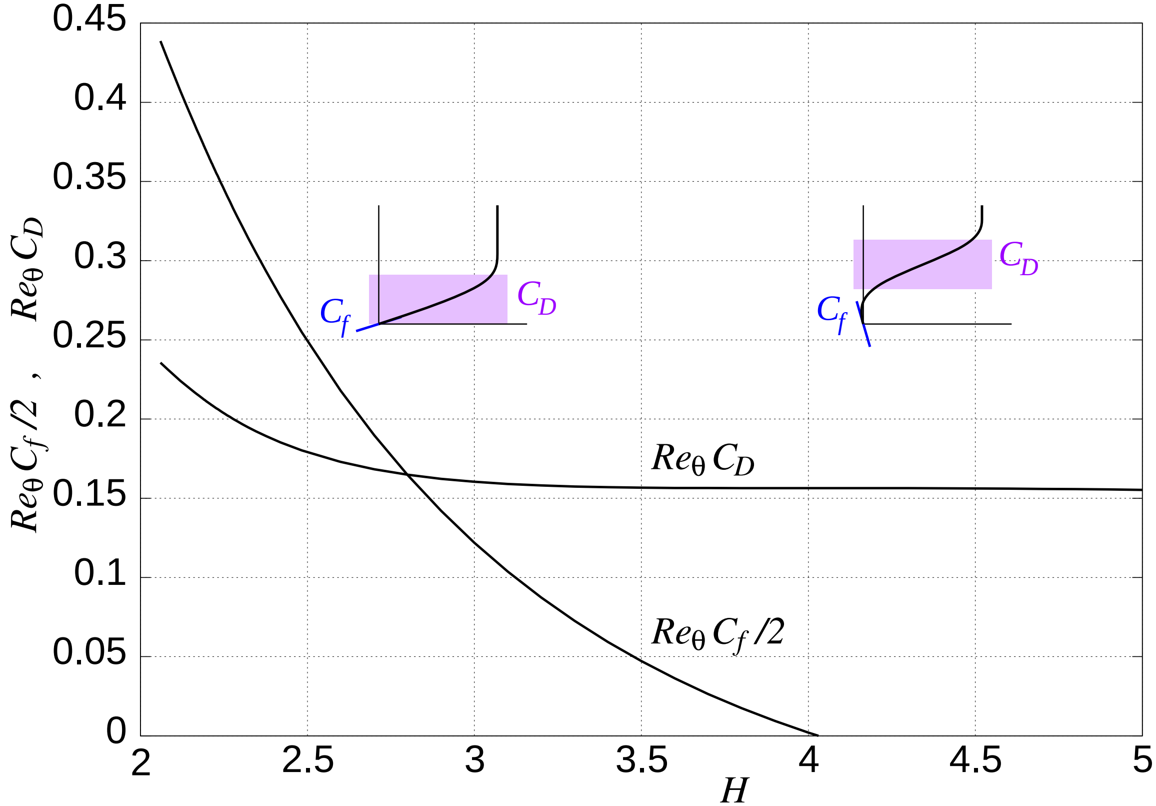
$$\begin{aligned} \Phi_{\mathrm{surface}} &= \iint \left[\int_0^\delta \tau_{xy} \frac{\partial u}{\partial y} \, dy \right] dx \, dz \\ &= \iint \; \rho_e u_e^3 \, C_{\mathcal{D}} \;\; dx \, dz \end{aligned}$$

Analogous to skin friction coefficient C_f:

$$\begin{aligned} D_f &= \iint \; [\; \tau_{xy} \;]_{y=0} \, dx \, dz \\ &= \iint \tfrac{1}{2} \rho_e u_e^2 \, C_f \;\; dx \, dz \end{aligned}$$

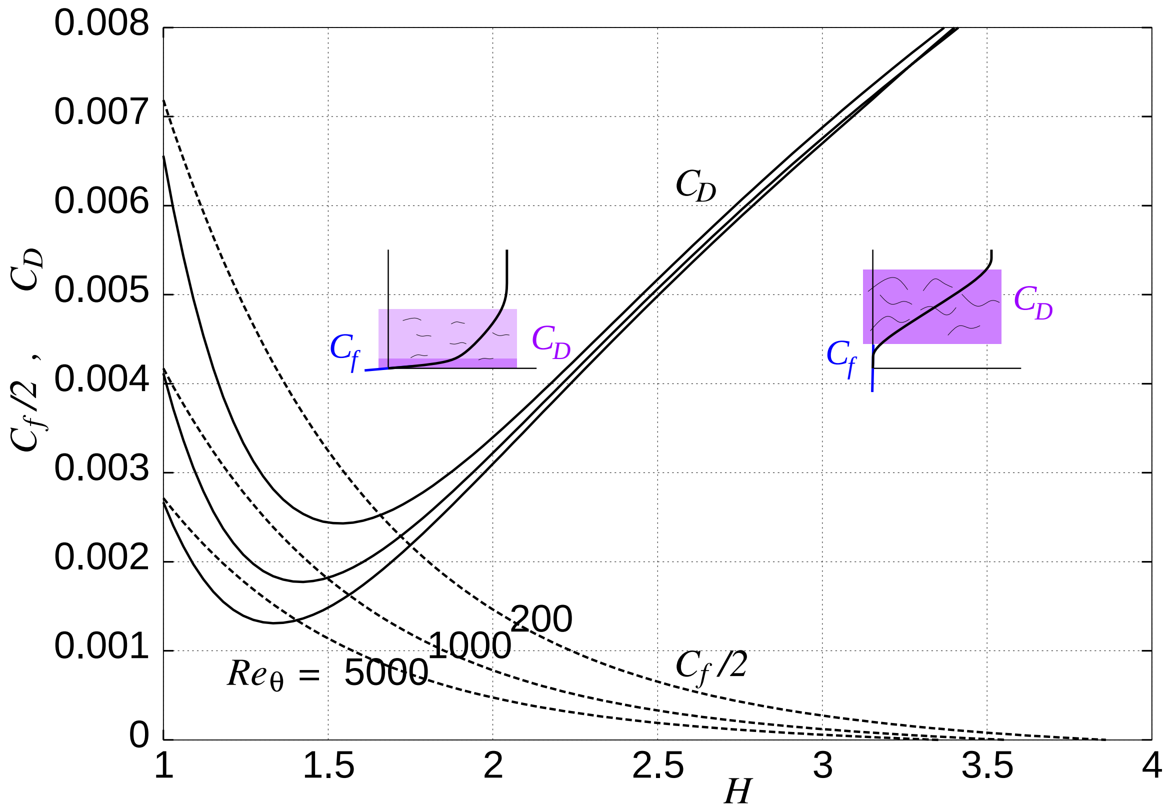
- $C_{\mathcal{D}}$ and Φ capture all drag-producing loss mechanisms
  C_f and D_f still leave out the pressure-drag contribution
- $C_{\mathcal{D}}$ and Φ are scalars – no drag-force direction issues
- $C_{\mathcal{D}}$ is strictly positive – no force-cancellation problems

$C_{\mathcal{D}}$ and C_f in Shear Layers – Laminar



Laminar $C_{\mathcal{D}}$ is very insensitive to pressure gradient — dissipation region just moves on/off the surface

$C_{\mathcal{D}}$ and C_f in Shear Layers – Turbulent



Increased mixing increases $C_{\mathcal{D}}$ in adverse pressure gradients

Note: A turbulent free shear layer has $C_{\mathcal{D}} \simeq 0.02$ (!)

Section Loss Quantification via BL and Wake Thicknesses

Dissipation via dissipation coefficient:

$$\int d\Phi \;=\; \int_0^x \rho_e u_e^3\, C_{\mathcal{D}}\, dx$$

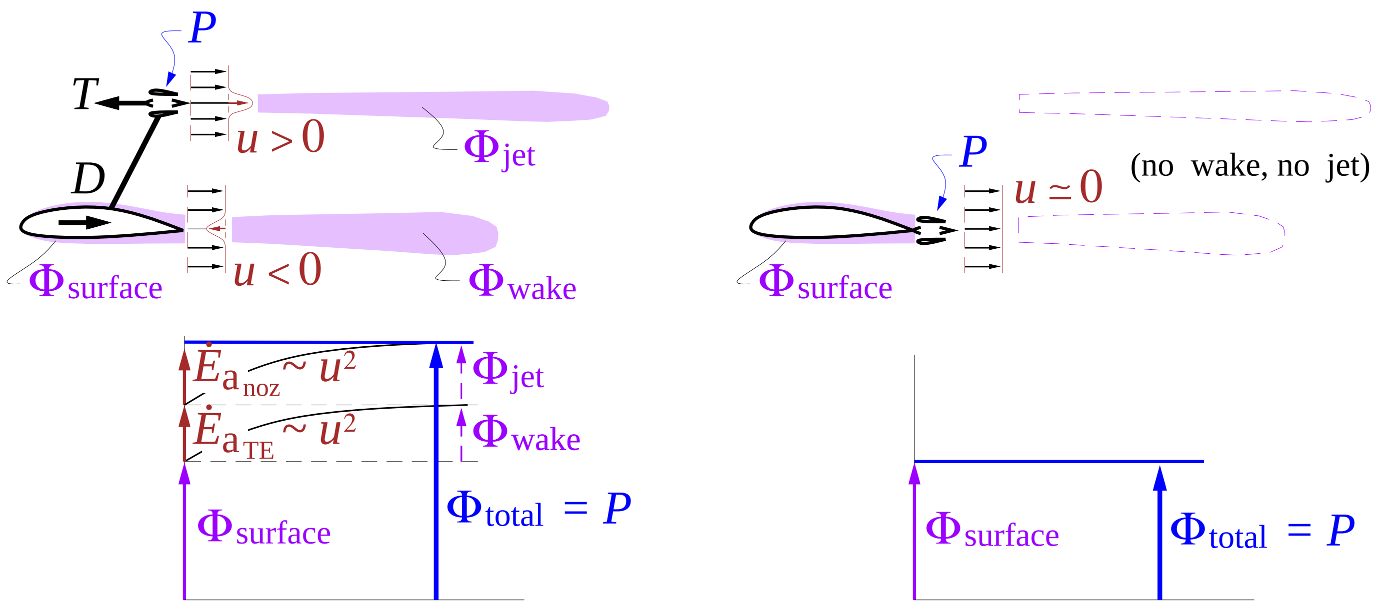
Dissipation via K.E. thickness:

$$\int d\Phi \;=\; \tfrac{1}{2}\rho_e u_e^3\, \theta^* \;=\; \int_0^{y_e} \tfrac{1}{2}\left(u_e^2 - u^2\right)\, \rho u \; dy$$

K.E. outflow via K.E. and momentum thickness:

$$\dot{E}_{\mathrm{a}} \;=\; \tfrac{1}{2}\rho_e u_e^3\, \delta_K \;=\; \int_0^{y_e} \tfrac{1}{2}\left(u_e - u\right)^2\, \rho u \; dy \;=\; \tfrac{1}{2}\rho_e u_e^3 \theta^* \left(\frac{2}{H^*} - 1\right)$$

Power Balance in Simple Cases – Propelled body



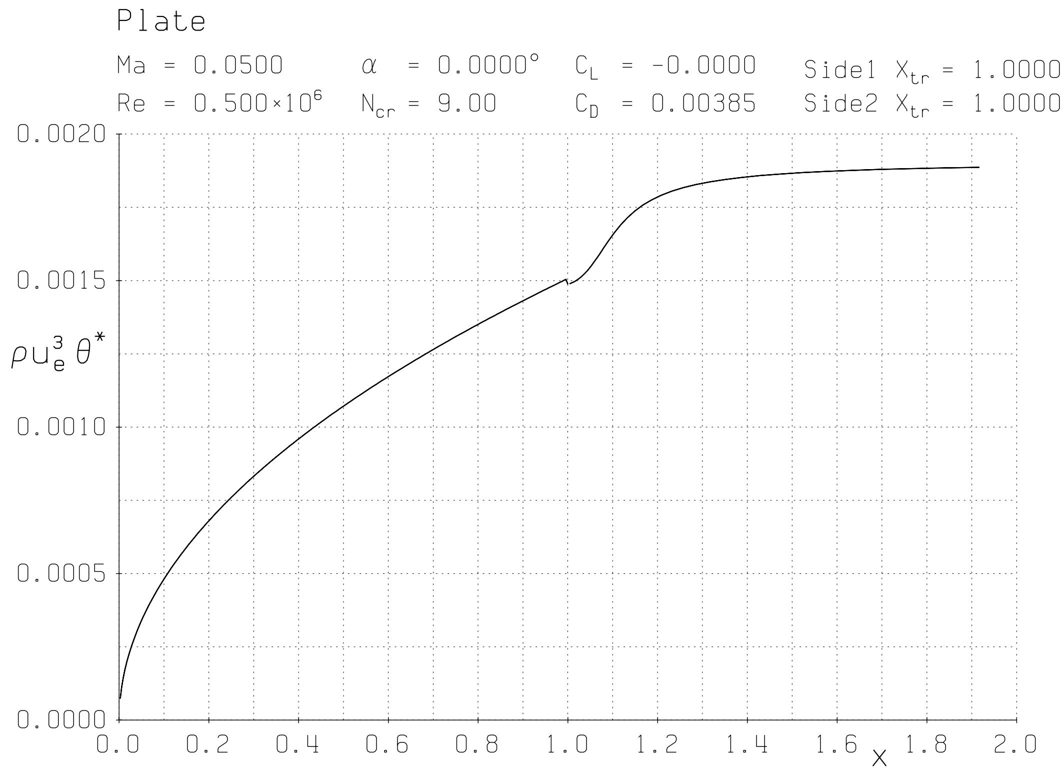
Isolated propulsor: $P \;=\; \Phi_{\mathrm{total}} \;=\; \Phi_{\mathrm{surface}} + \dot{E}_{a_{\mathrm{TE}}} + \dot{E}_{a_{\mathrm{noz}}}$

Ingesting propulsor: $P \;=\; \Phi_{\mathrm{total}} \;=\; \Phi_{\mathrm{surface}}$

Wake ingestion benefit is via elimination of wake and jet dissipation (or equivalently, elimination of K.E. outflow)

Power Balance in Simple Cases – Laminar flat plate

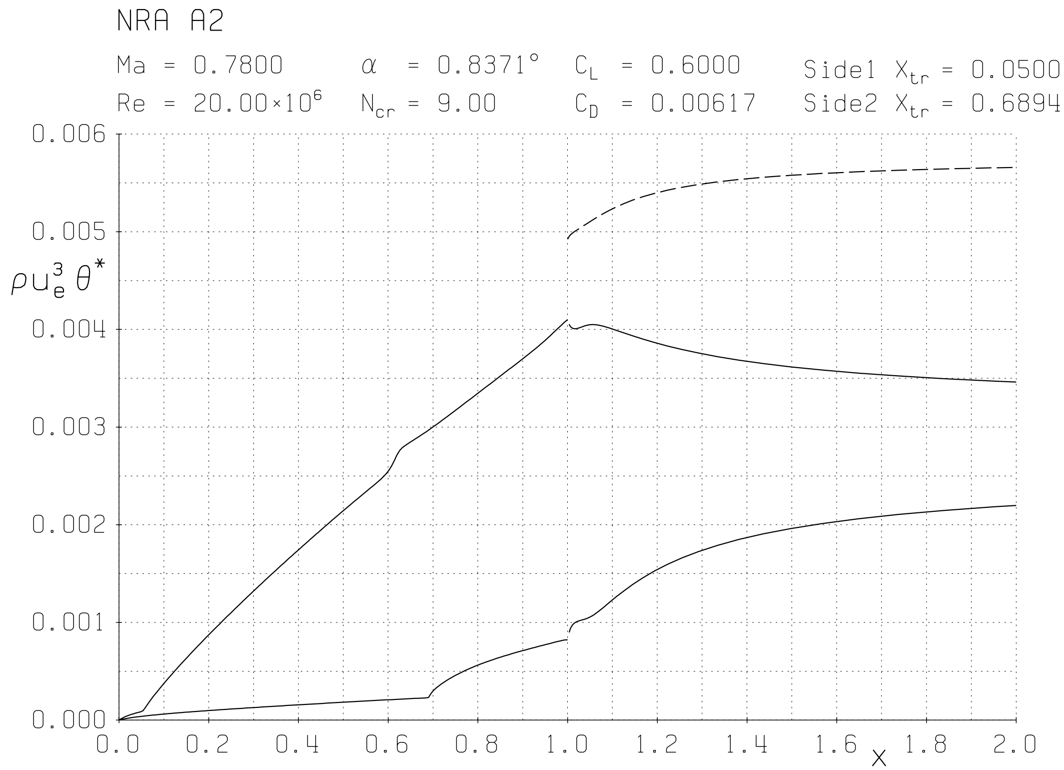
K.E. defect $\frac{1}{2}\rho_e u_e^3 \theta^*(x) = \int_0^x d\Phi$ along surface and wake



Potential wake-ingestion power savings is 21%
(possibly more from reduced $\Phi_{\rm jet}$)

Power Balance in Simple Cases – Transonic Airfoil

K.E. defect $\frac{1}{2}\rho_e u_e^3 \theta^*(x) = \int_0^x d\Phi$ along surface and wake



Potential wake-ingestion power savings is 13%
(possibly more from reduced $\Phi_{\rm jet}$)

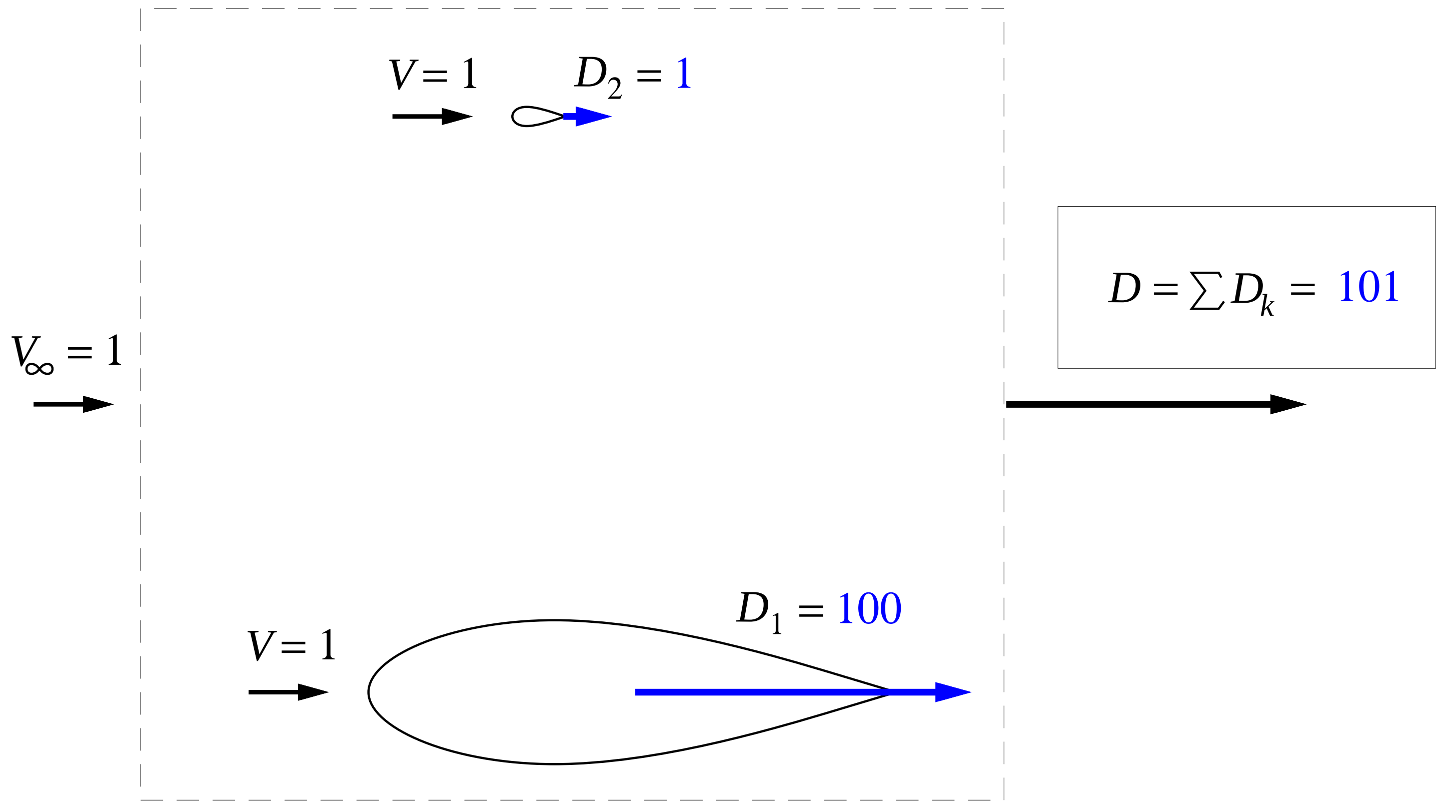
Dependence on Flow Velocity

Surface BL: $\Phi \sim \int \rho_e u_e^3 \, dx$

Free shear layer: $\Phi \sim \int \rho (\Delta u)^3 \, dx$

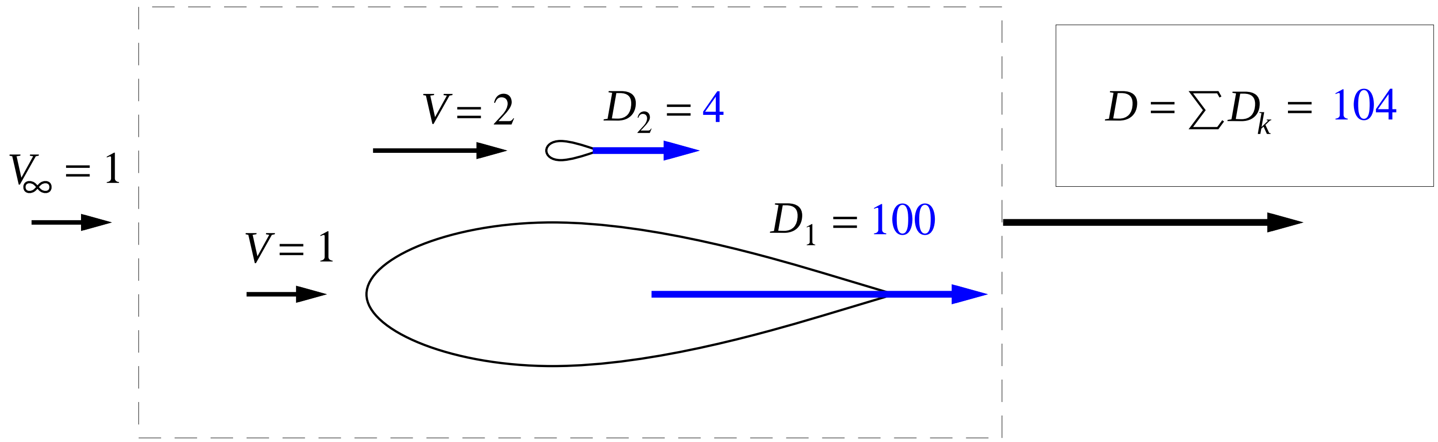
- High-speed regions are very costly (C_p spikes, etc)
- Low-speed regions are innocuous (cove separation, etc)
- Local excrescense drag should be scaled with u_e^3, not u_e^2

Drag Build-Up via Force Summation — Isolated Bodies



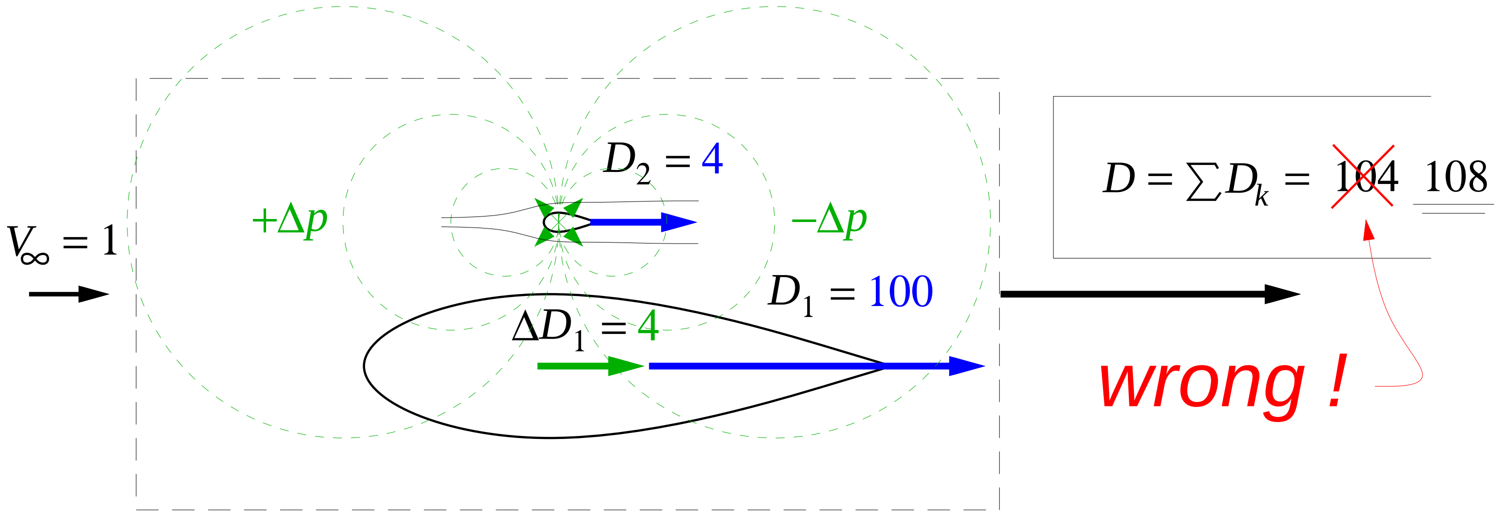
Drag Build-Up via Force Summation — Nearby Bodies

Assuming $D_k \sim V^2$ ...



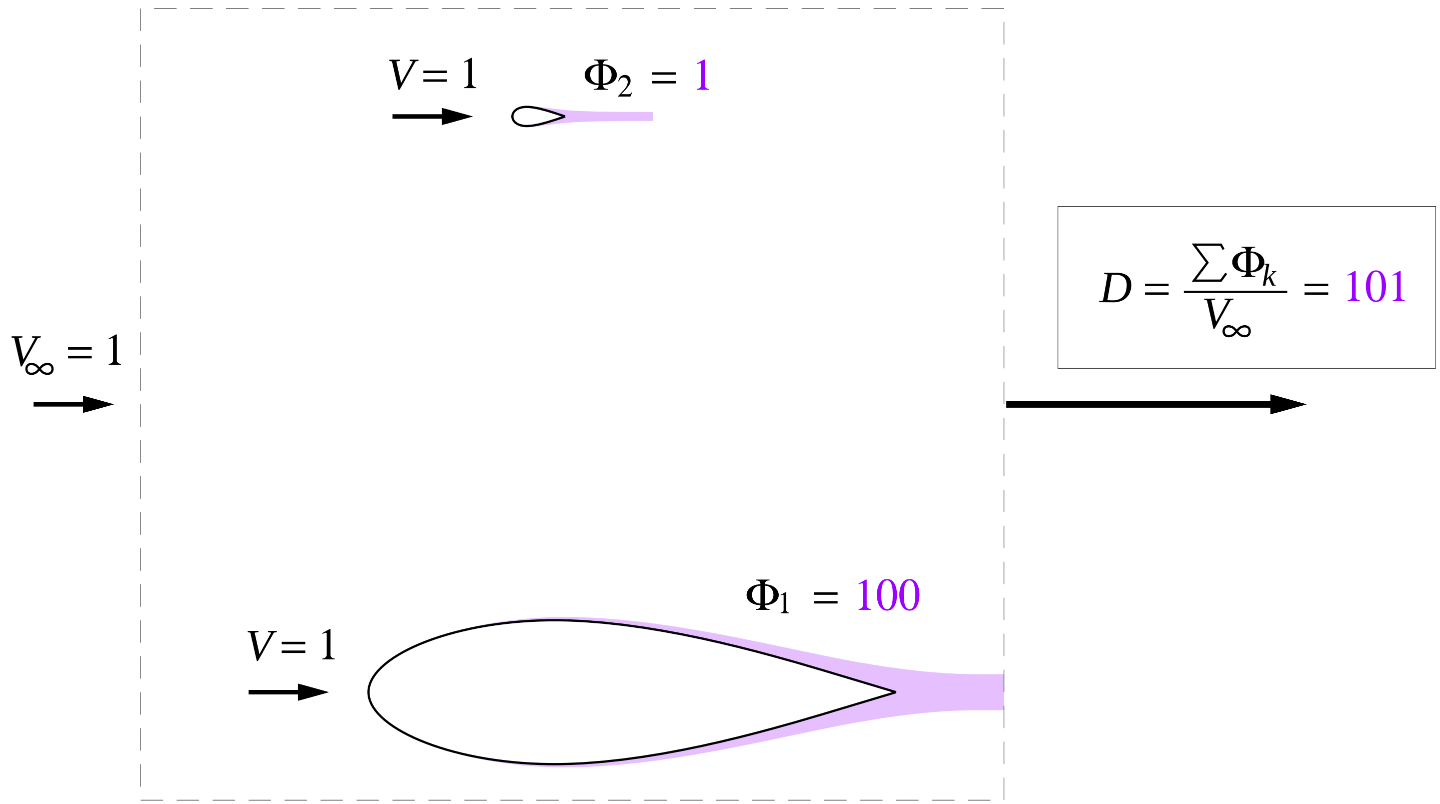
Drag Build-Up via Force Summation — Nearby Bodies

Assuming $D_k \sim V^2$ ...



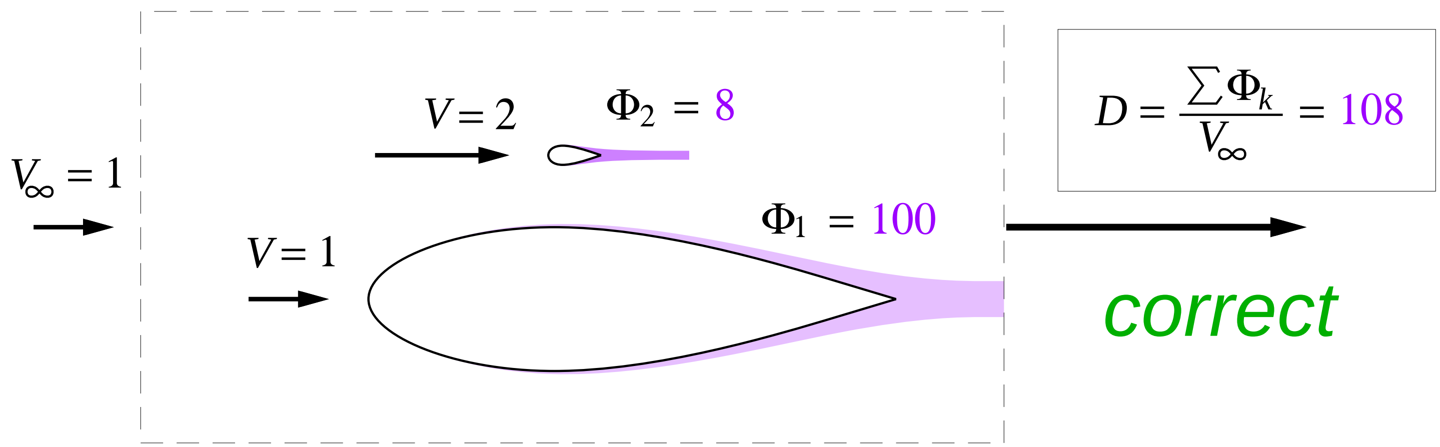
Also present is buoyancy drag ΔD_1 on large body,
due to small body's source-disturbance pressure field Δp.

Drag Build-Up via Dissipation Summation — Isolated Bodies



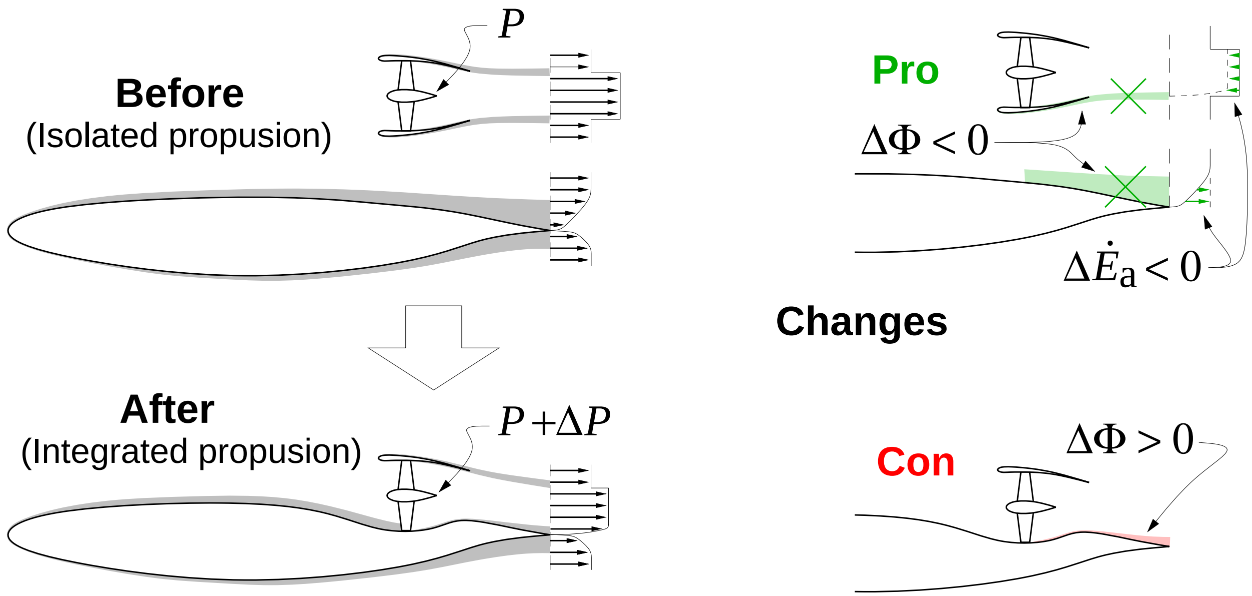
Drag Build-Up via Dissipation Summation — Nearby Bodies

Assuming $\Phi_k \sim V^3$ ...



Dissipation Build-Up captures "mystery interference drag", which cannot be estimated via simple Drag Build-Up.

Evaluation of Integrated-Propulsion Benefits



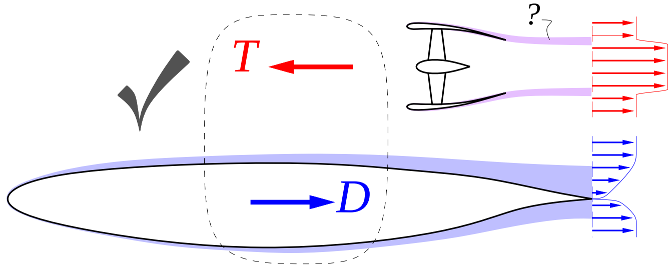
Net propulsive power change is sum of all Pro,Con changes:

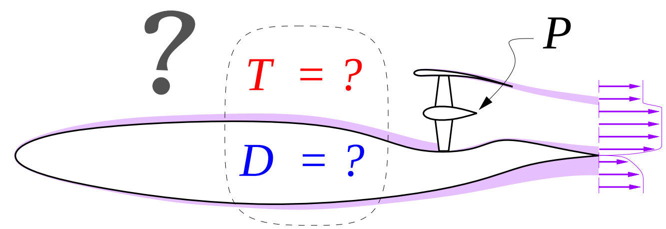
$$\Delta P \; = \; \sum \Delta \dot{E} \; + \; \sum \Delta \Phi$$

Motivation — I (again)

Usual Range Equation is ambiguous for integrated propulsion:

$$R = \frac{V}{TSFC}\,\frac{C_L}{C_D}\,\ln\left(\frac{W_0}{W_e}\right)$$

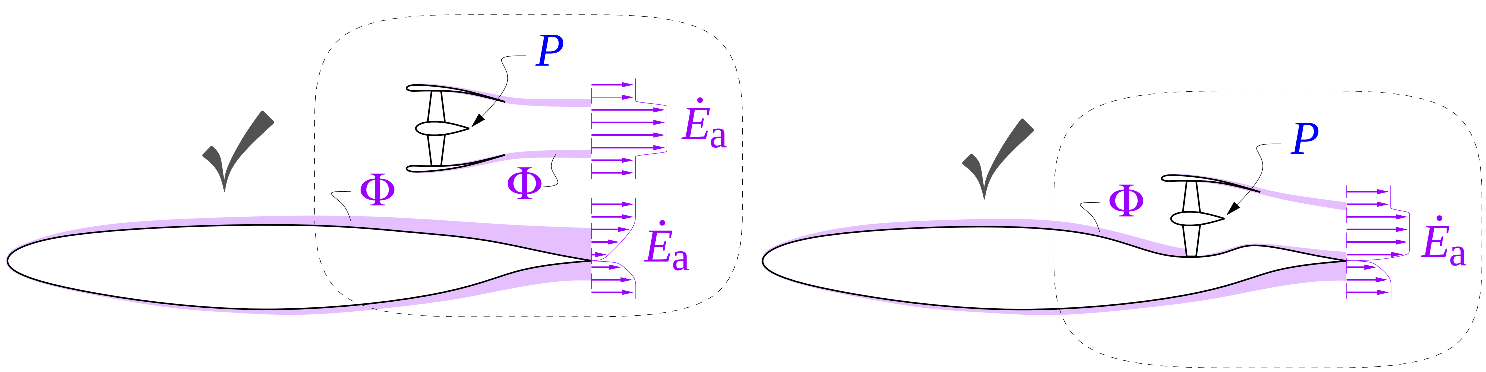




Motivation — I (resolved)

Power-balance form of Range Equation is unambiguous, for any configuration:

$$R = \frac{1}{PSFC} \, \frac{C_L}{C_E + C_\Phi} \, \ln\left(\frac{W_0}{W_e}\right)$$



$$PSFC \equiv \frac{\dot{W}_{\rm fuel}}{\sum P} \,, \qquad C_E \equiv \frac{\sum \dot{E}}{\frac{1}{2}\rho_\infty V_\infty^3 \, S} \,, \qquad C_\Phi \equiv \frac{\sum \Phi}{\frac{1}{2}\rho_\infty V_\infty^3 \, S}$$

Conclusions

- Flight power calculation without drag,thrust definition
- Quantities which determine flight power clearly identified
- Compatible with traditional force-based analyses
  - can frequently use established drag estimation methods
  - can use existing propulsive efficiency estimates
- Formulation is exact
  - does not require isolation of wakes or plumes
  - no small-disturbance or low-speed approximations are used
- Is more reliable for “interference drag” situations