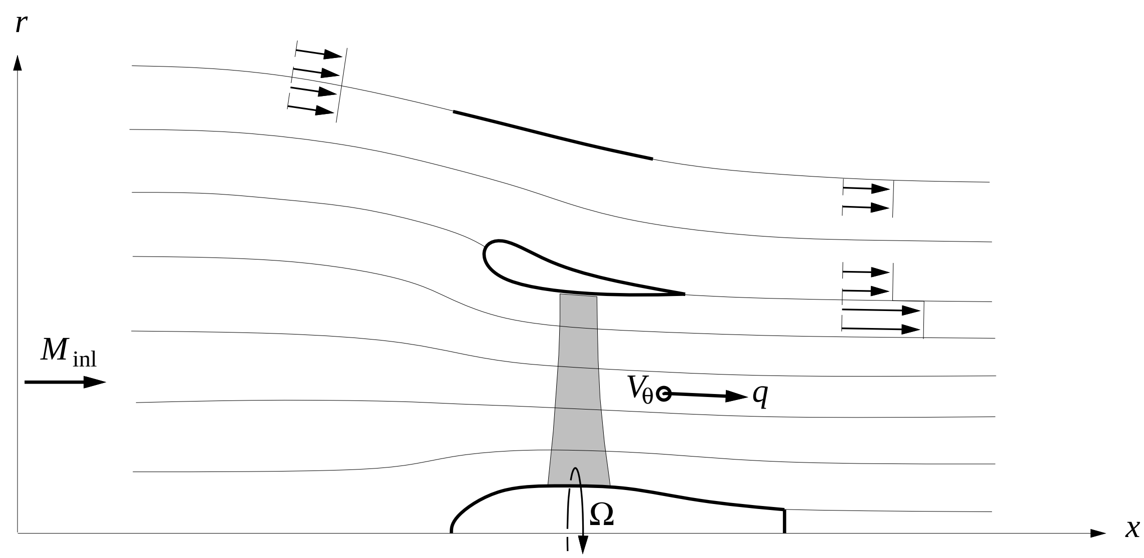
Computational Modeling of Compressible Swirling Flows for Design and Analysis of Rotors and Turbomachinery

(MTFLOW 2.0)

Mark Drela 6 March 09

Assumed Axisymmetric Flowfield

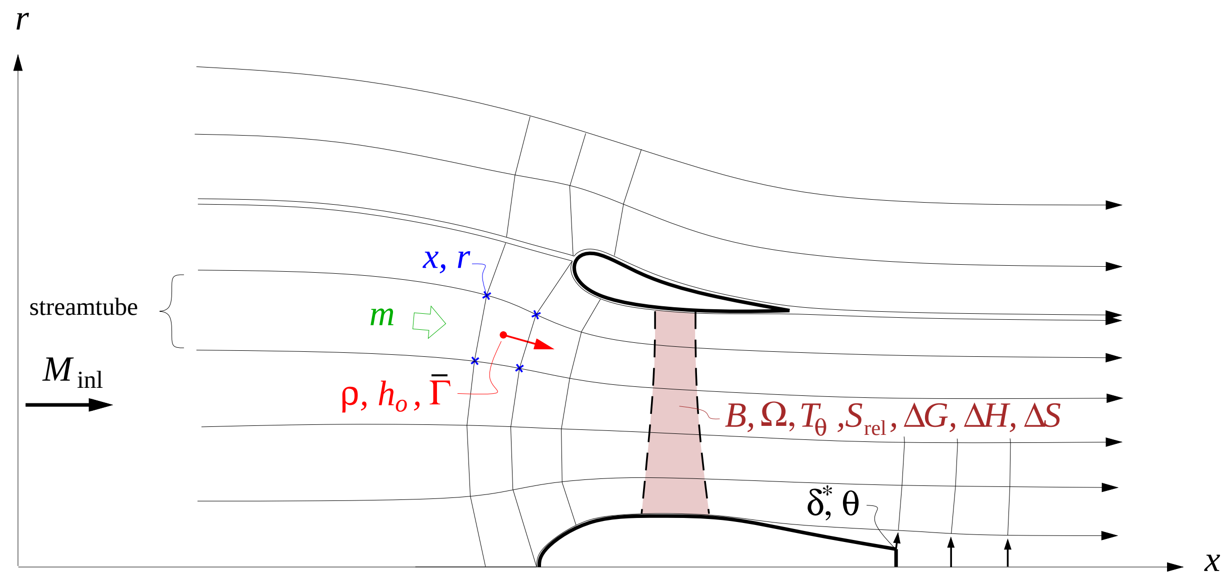


- Arbitrary circumferential bodies
- Prescribed forcing-field regions
- Blade/swirl interaction model

Governing Equations

- Axisymmetric Euler equations with source terms
- Integral boundary layer equations for viscous option
- Inlet/Oulet BCs
- Solid-wall BCs
- Prescribed-pressure BCs (free jet surface, inverse calcs)
- Infinite-flow farfield BCs

Streamline-Based Discretization



- Euler equations discretized on streamline grid
- Axisymmetric form of MSES, with source terms
- Primary flowfield variables: $x, r, \rho, h_o, \bar{\Gamma}\, ;\; m\, ;\; \delta^*, \theta$
- Prescribed field parameters: $B, \Omega, T_\theta, \mathcal{S}_{\rm rel}, \Delta G, \Delta H, \Delta S$

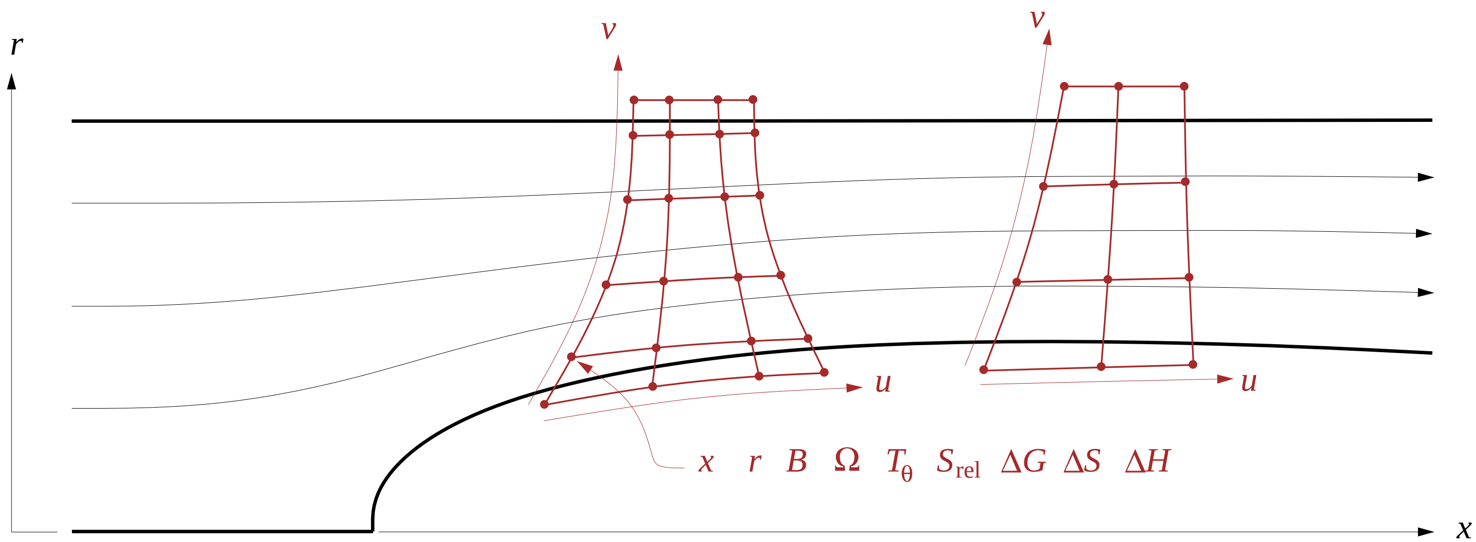
Field Parameters

B	Number of blades in a blade row
Ω	Rotation rate of a blade row
T_θ	Circumferential blade thickness.
$\mathcal{S}_{\rm rel}$	Blade slope ($= \tan\beta$) in the m'–θ plane.
ΔG	Swirl change due to blade row loading.
ΔH	Total enthalpy change due to heating or cooling
ΔS	Entropy change due to adiabatic loss

Field Parameters for Ideal Components

	ΔG	ΔH	ΔS
Stator ($\Omega = 0$):	$+/-$		
Compressor:	$\mathrm{sign}(\Omega)$		
Combustor:		$+$	
Adiabatic Turbine:	$-\mathrm{sign}(\Omega)$		
Cooled Turbine:	$-\mathrm{sign}(\Omega)$	$-$	
Throttle:			$+$

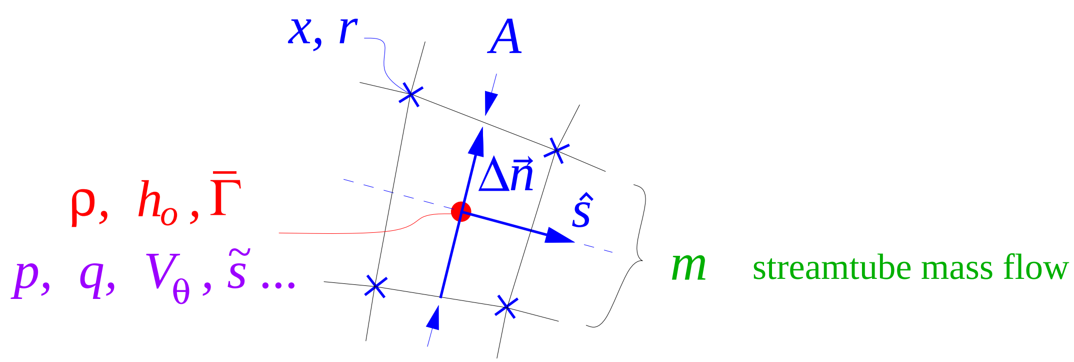
Field-Parameter Parameterization



- All field parameters specified as bicubic splines in u, v
- $x(u,v), r(u,v)$ numerically inverted to give $u(x,r), v(x,r)$
- Then

$$
\begin{aligned}
T_{\theta(x,r)} &= T_\theta\big(\,u(x,r)\,,\; v(x,r)\,\big) \\
\mathcal{S}_{\mathrm{rel}(x,r)} &= \mathcal{S}_{\mathrm{rel}}\big(\,u(x,r)\,,\; v(x,r)\,\big) \\
&\;\vdots
\end{aligned}
$$

Secondary Variables – Kinematic

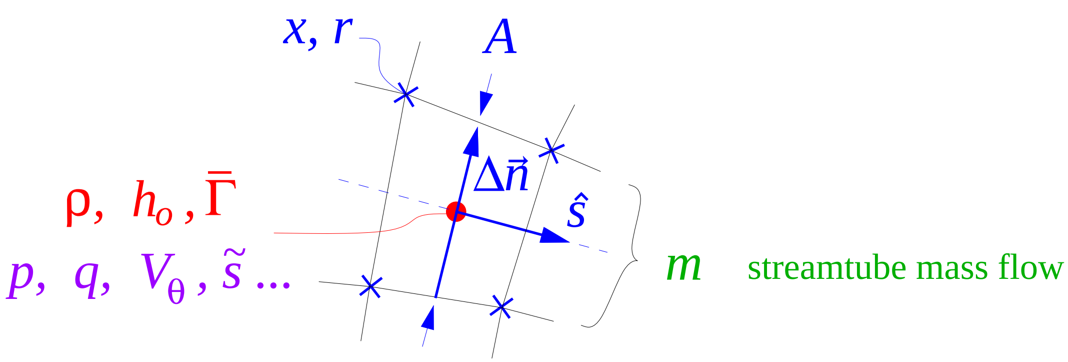


$$A = (\hat{s} \times \Delta\vec{n}) \cdot \hat{\theta}$$ streamtube normal height

$$q = \frac{m}{\rho A\,(2\pi r - BT_\theta)}$$ meridional speed $(= \sqrt{V_x^2 + V_r^2})$

$$V_\theta = \frac{\bar{\Gamma}}{r}$$ tangential speed

Secondary Variables – Thermodynamic



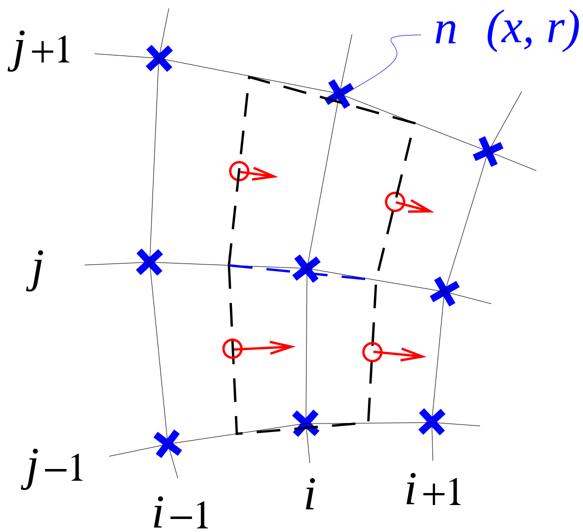
$$h = h_o - \frac{1}{2}\left(q^2 + V_\theta^2\right)$$ static enthalpy

$$p = \frac{\gamma - 1}{\gamma}\,\rho\,h$$ static pressure

$$\tilde{s} = \ln\left[\frac{(h/h_{\rm inl})^{\gamma/(\gamma-1)}}{p/p_{\rm inl}}\right]$$ entropy

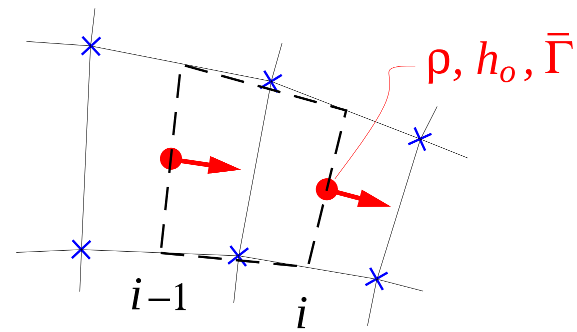
Equation Stencils

Elliptic



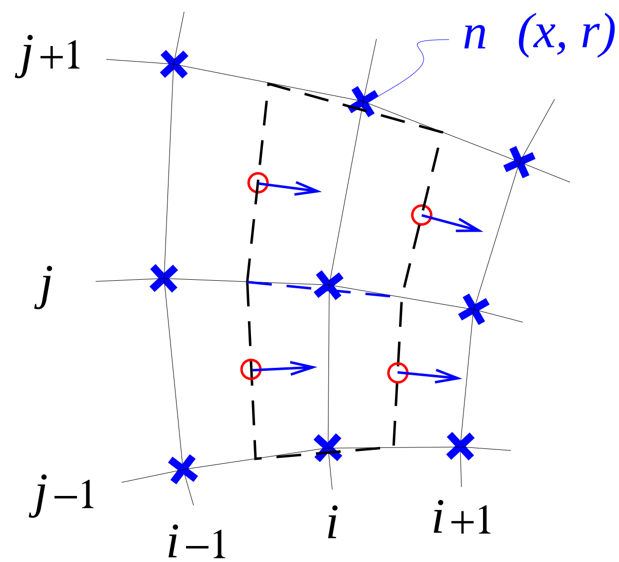
n-momentum

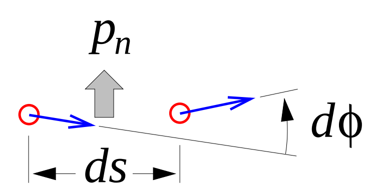
Convective



s-momentum
total enthalpy
tangential momentum

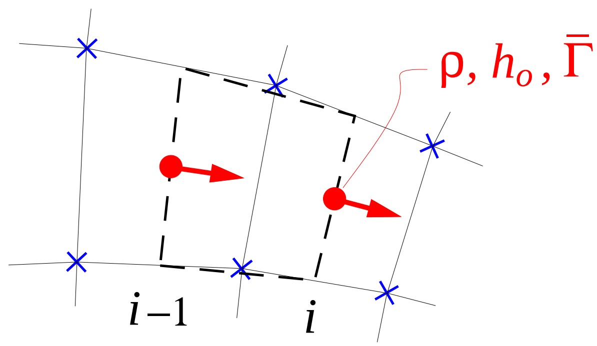
Normal-Momentum Equation

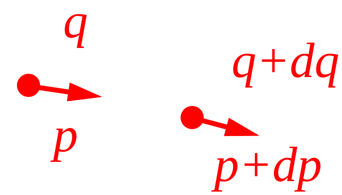




$$R_n \equiv \left[\rho\, q^2\, d\phi \;+\; p_n\, ds\right]_{j+1/2} - \left[\rho\, q^2\, d\phi \;+\; p_n\, ds\right]_{j-1/2}$$

Streamwise-Momentum Equation

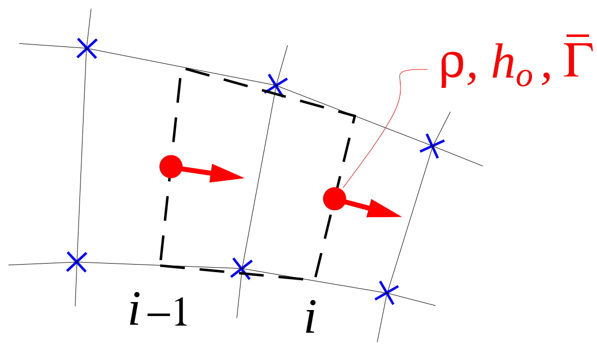


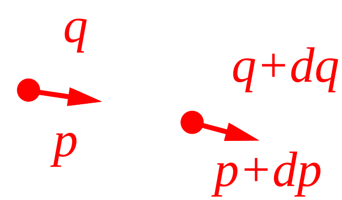


$$R_s \equiv dp + \rho\, q\, dq + \rho V_\theta\, dV_\theta + p\, d(\Delta S) - \rho\Omega\, d\bar{\Gamma}$$

or: $$R_s \equiv -p\, d\tilde{s} + p\, d(\Delta S) + \rho\, d(\Delta H)$$

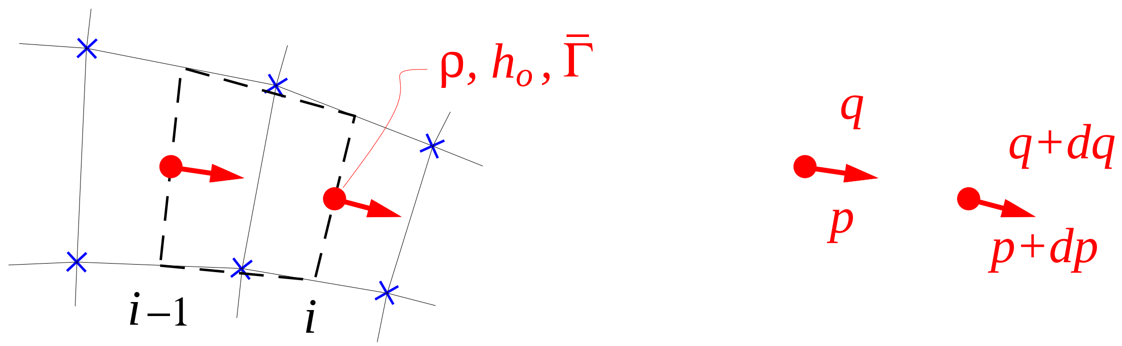
Total-Enthalpy Equation





$$R_h \;\equiv\; dh_o \;-\; d(\Delta H) \;-\; \Omega\, d\bar{\Gamma}$$

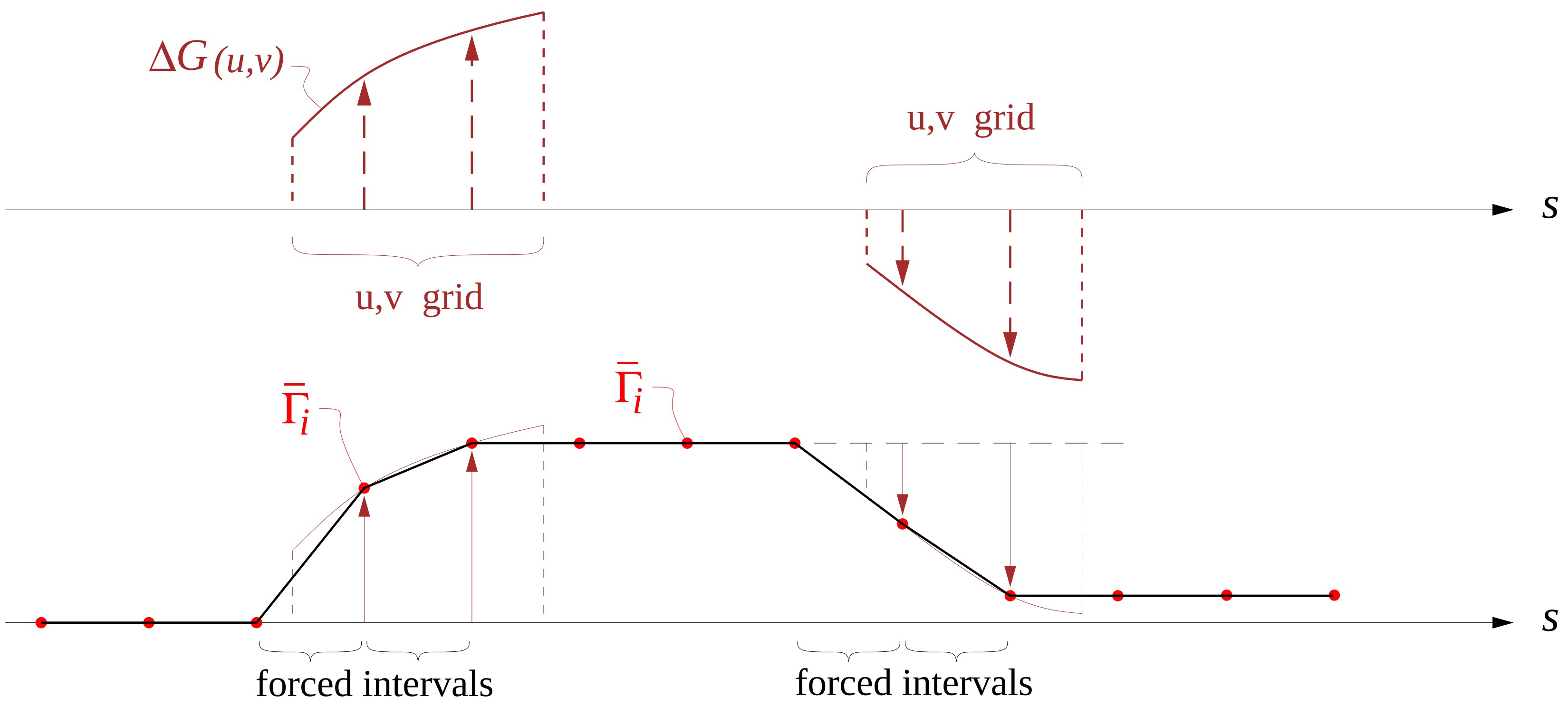
Tangential-Momentum Equation



If loading prescribed (design):

$$R_\theta \equiv d\bar{\Gamma} - d(\Delta G)$$

Cumulative Forcing



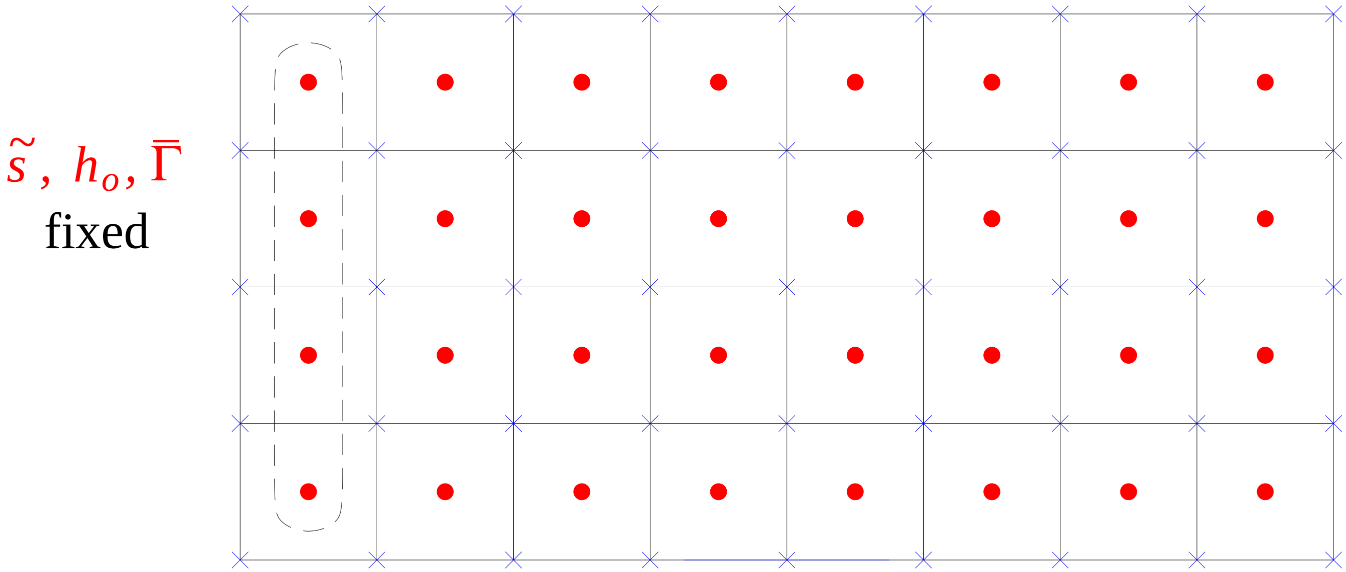
- A forced interval has downstream node inside a u, v grid
- Causes forcing to accumulate downstream

Convective Equations w/o Forcing or Shocks

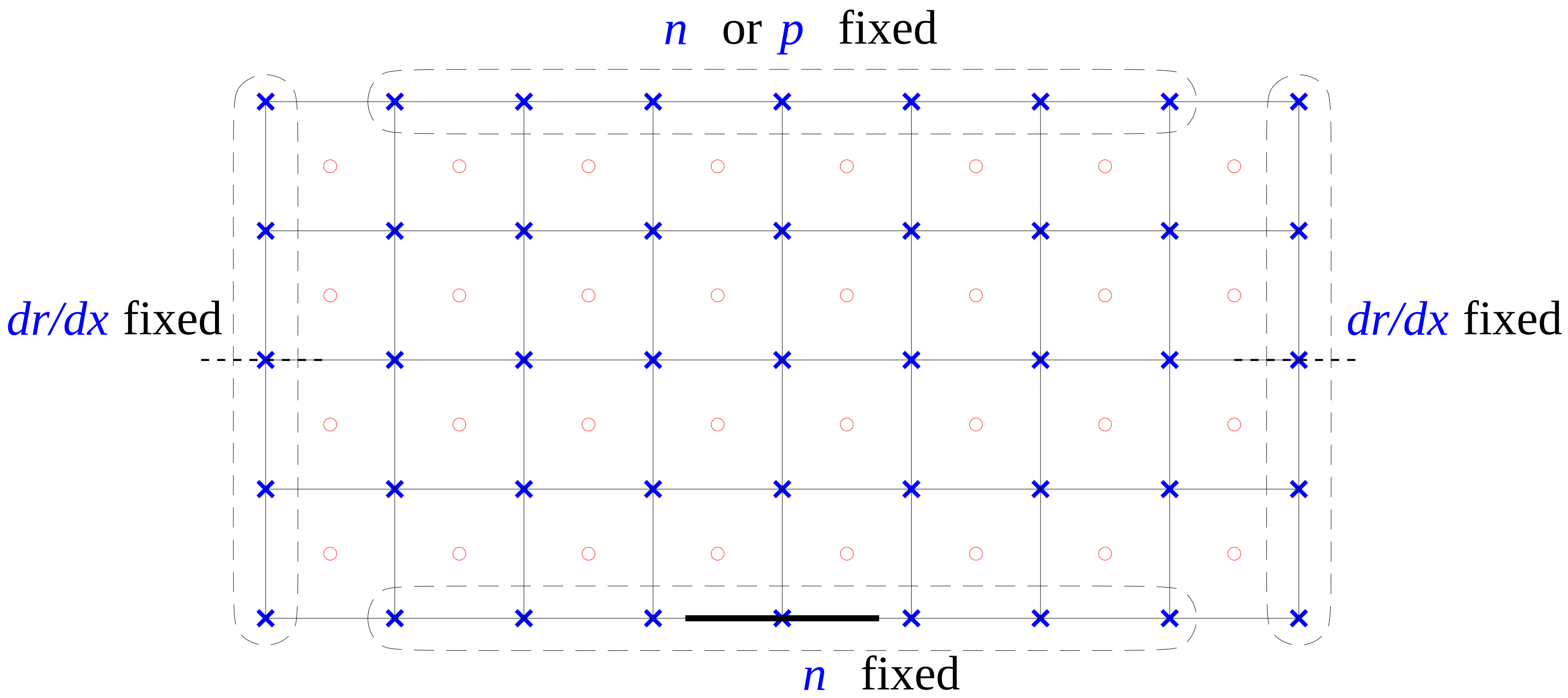
$$
\begin{aligned}
R_s &= d\tilde{s} & \rightarrow \quad \tilde{s} &= \text{const.} \\
R_h &= dh_o & \rightarrow \quad h_o &= \text{const.} \\
R_\theta &= d\bar{\Gamma} & \rightarrow \quad \bar{\Gamma} &= \text{const.}
\end{aligned}
$$

- Exactly conserve total pressure, total enthalpy, angular momentum

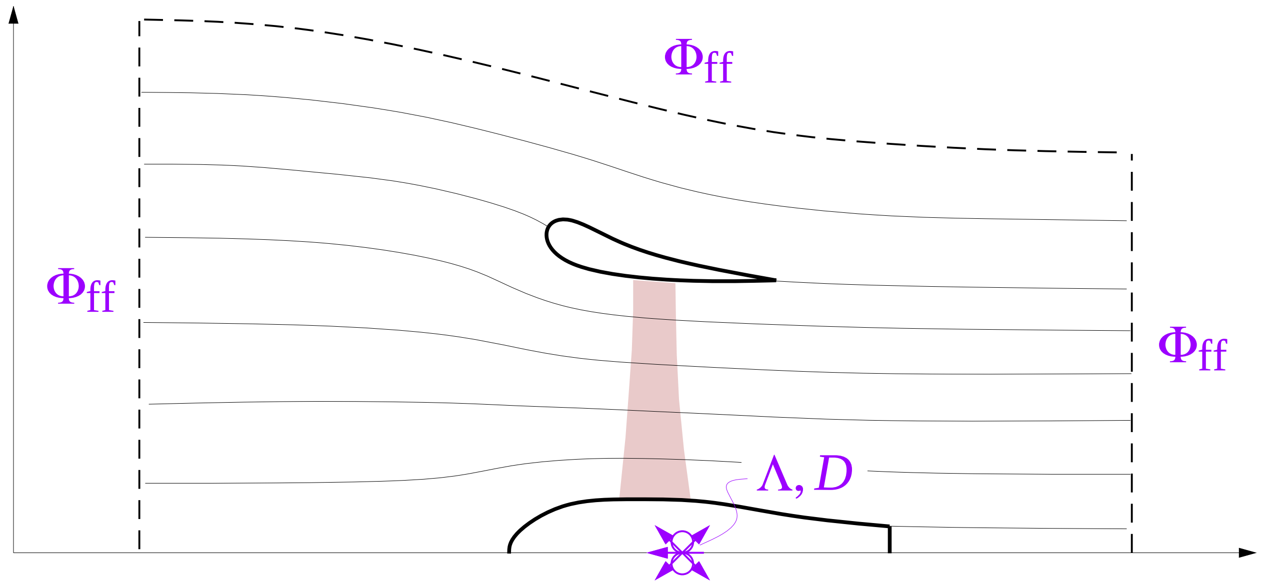
Convective Equations' BCs



Elliptic n-Momentum Equation BCs



Farfield Source+Doublet Model for External Flows



Farfield velocity potential:

$$
\begin{aligned}
\Phi_{ff} &= V_\infty x + \frac{\Lambda}{4\pi} \frac{-1}{\left(\bar{x}^2 + \bar{r}^2\right)^{1/2}} + \frac{D}{4\pi} \frac{\bar{x}}{\left(\bar{x}^2 + \bar{r}^2\right)^{3/2}} \\
\bar{x} &= x - x_s \\
\bar{r} &= r \sqrt{1 - M_\infty^2}
\end{aligned}
$$

Farfield Source+Doublet Model for External Flows

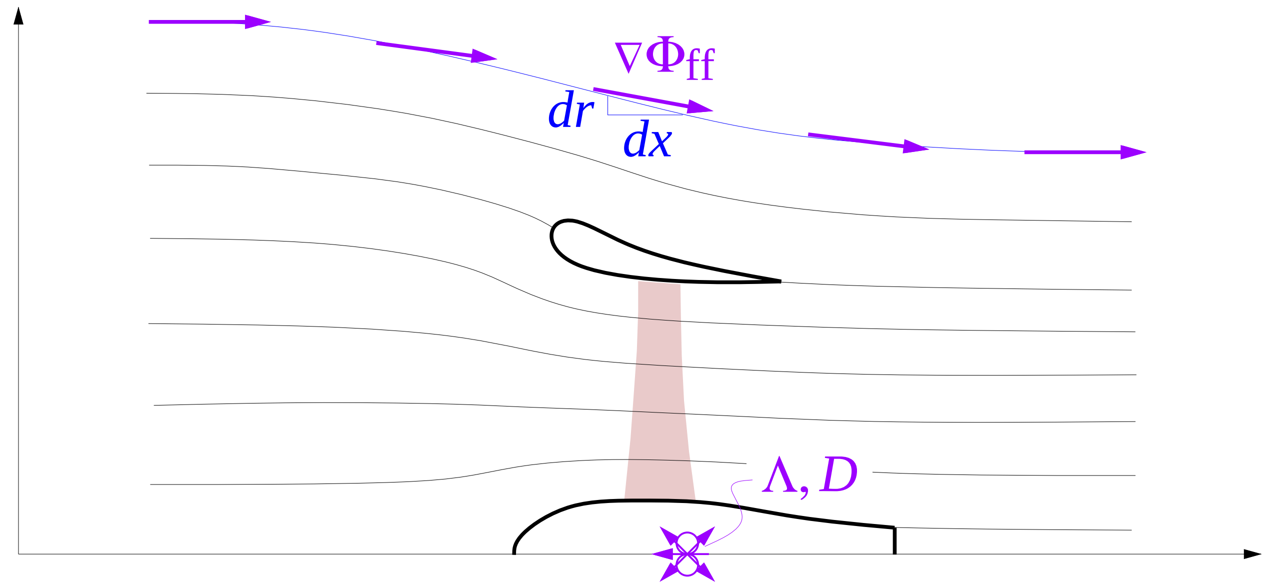
Outer-streamline n-momentum BC:

$$R_{nBC} \;=\; p \;-\; p_{o_\infty}\left(1 - \frac{1}{2h_{o_\infty}}\left|\nabla\Phi_{\text{ff}}\right|^2\right)^{\gamma/(\gamma-1)}$$

Inlet/outlet n-momentum BC:

$$R_{nBC} \;=\; \frac{\partial\Phi_{\text{ff}}}{\partial x}\,dr \;-\; \frac{\partial\Phi_{\text{ff}}}{\partial r}\,dx$$

Farfield Singularity Strengths

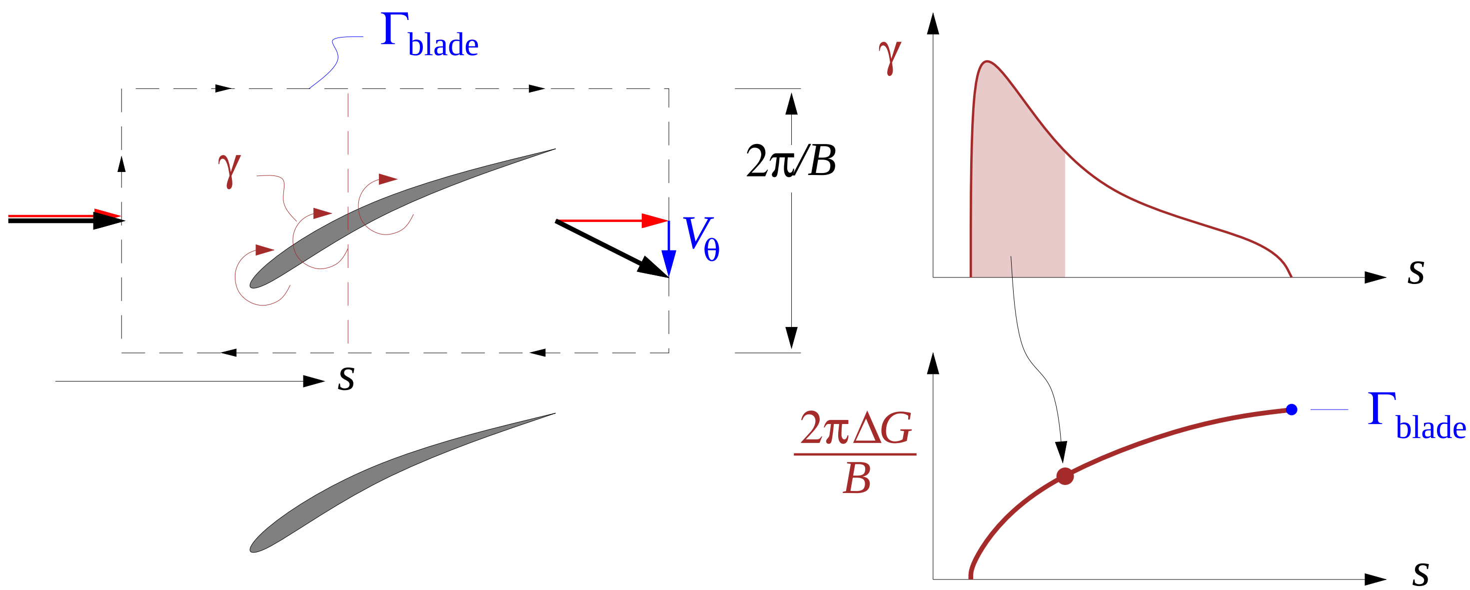


Minimize mismatch of $\nabla\Phi_{\rm ff}$ and outer streamline:

$$I \;=\; \int \frac{1}{2}\left|\frac{\partial \Phi_{\rm ff}}{\partial x}\,dr \;-\; \frac{\partial \Phi_{\rm ff}}{\partial r}\,dx\right|^2$$

$$R_\Lambda \;\equiv\; \frac{\partial I}{\partial \Lambda} \quad , \qquad R_D \;\equiv\; \frac{\partial I}{\partial D}$$

Swirl Development in Blade Row



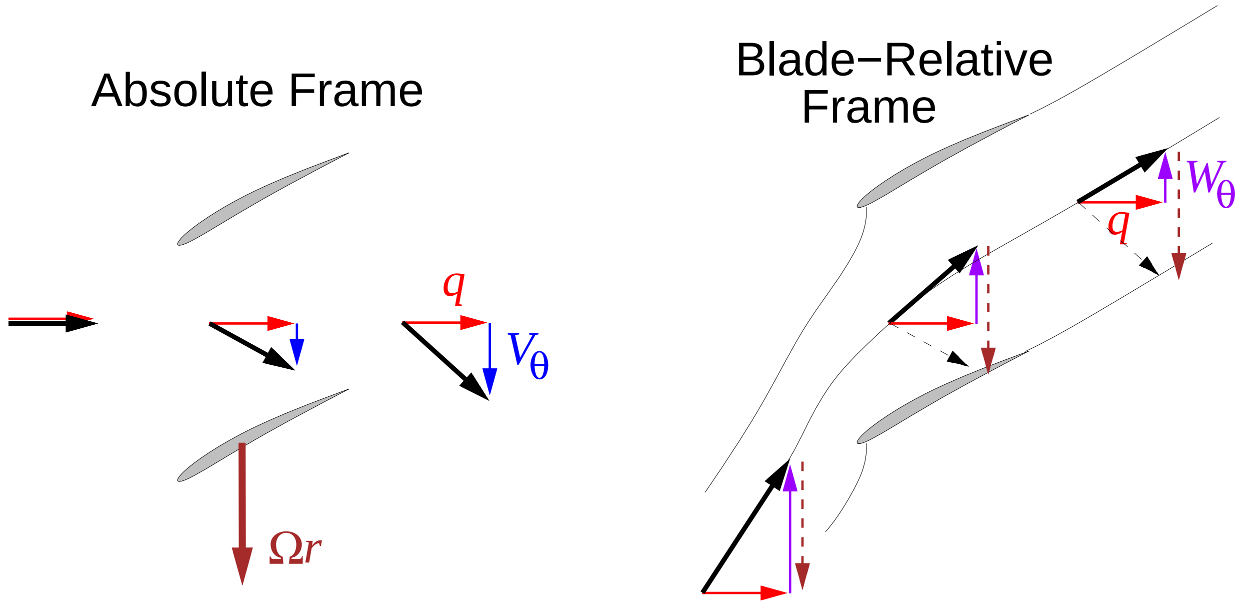
Swirl rise is accumulated blade vortex sheet strength γ:

$$2\pi\,\Delta G(s) \;=\; B \int^{s} \gamma \, ds \;=\; \int_0^{2\pi/B} B\, V_\theta \, r \, d\theta \;=\; 2\pi\,\bar{\Gamma}(s)$$

Total swirl change is blade circulation:

$$2\pi \Delta G(c) \;=\; 2\pi \bar{\Gamma}(c) \;=\; B \Gamma_{\rm blade}$$

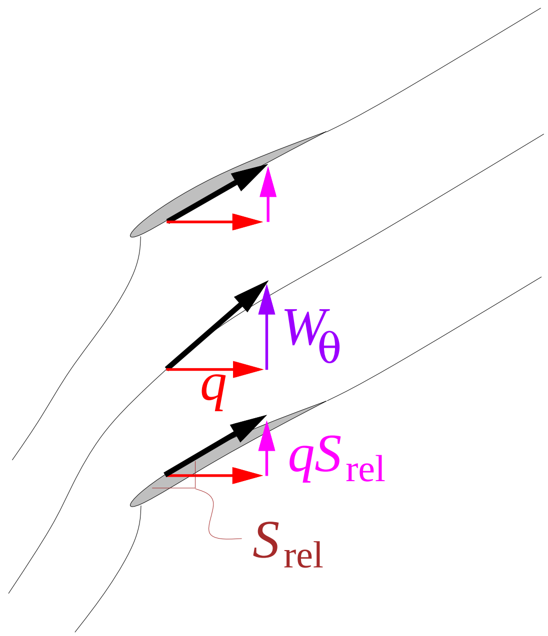
Swirl Development in Blade Row



θ-averaged tangential velocity in blade-relative frame:

$$W_\theta \;=\; V_\theta - \Omega r \;=\; \left(\bar{\Gamma} - \Omega r^2\right)/r$$

Swirl Development in Blade Row



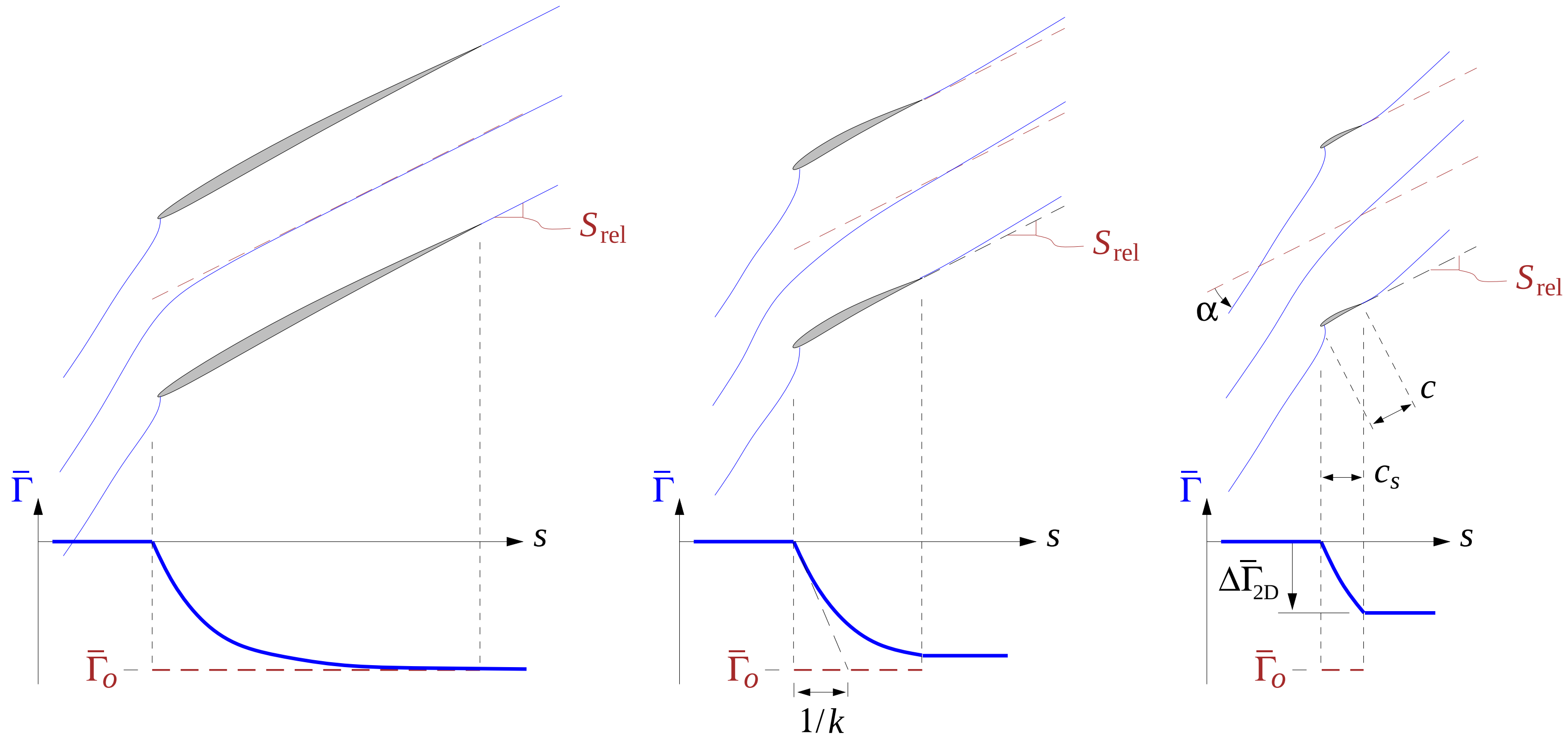
$$\bar{\Gamma} \equiv \frac{1}{2\pi}\int_0^{2\pi} rV_\theta \, d\theta$$

$$\bar{\Gamma} = (W_\theta + \Omega r)\, r$$

$$\bar{\Gamma}_o \equiv (q\mathcal{S}_{\mathrm{rel}} + \Omega r)\, r$$

- θ-averaged $\bar{\Gamma}$ is not the same as flow-tangency implied swirl $\bar{\Gamma}_o$, since $W_\theta \neq q\mathcal{S}_{\mathrm{rel}}$, but ...
- Want $W_\theta \rightarrow q\mathcal{S}_{\mathrm{rel}}$ in high-solidity limit
- Want 2D airfoil lift in low-solidity limit

First-Order Swirl-Evolution Model



$$\frac{d\bar{\Gamma}}{ds} = k\left(\bar{\Gamma}_o - \bar{\Gamma}\right)$$

Choosing the lag constant $k = B/2r$ gives correct stage loading in high and low-solidity limits

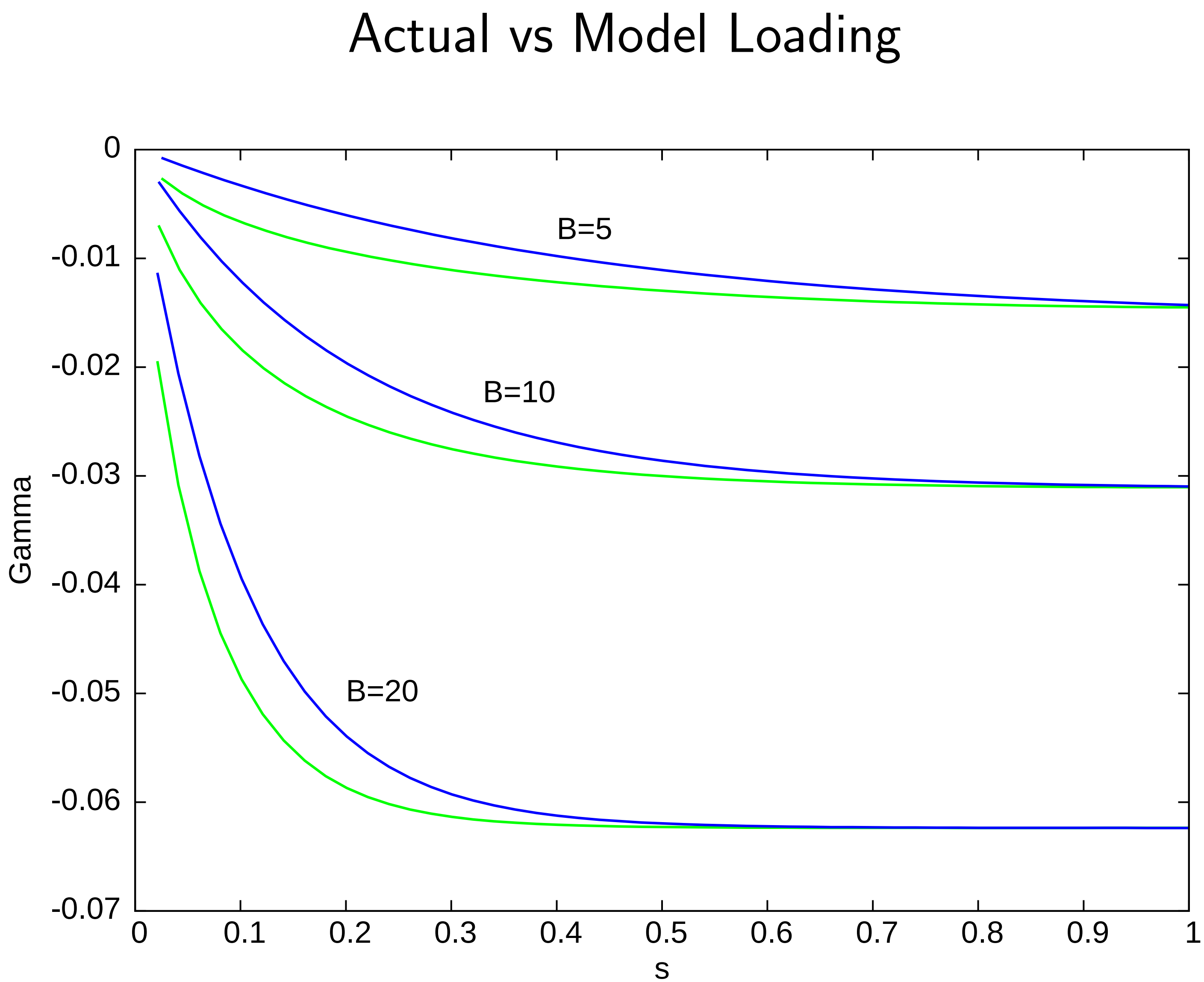
First-Order Swirl-Evolution Model

High-solidity limit (zero deviation)

$$\bar{\Gamma} \; = \; \bar{\Gamma}_o$$

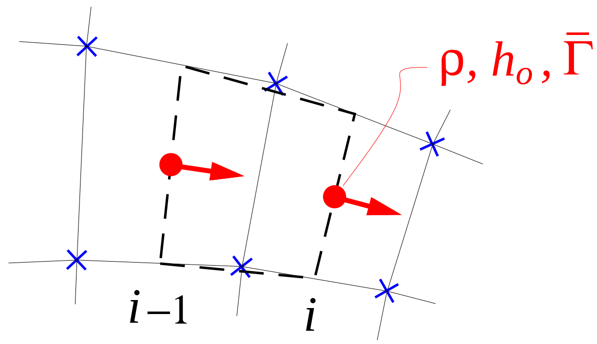
Low-solidity limit (2D airfoil lift)

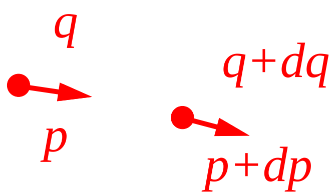
$$c_\ell \; \equiv \; \frac{-2\pi \, \Delta\bar{\Gamma}_{\rm 2D}}{B \, c W/2} \; \simeq \; 2\pi \, \alpha$$



Actual vs Model Loading
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
Gamma
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
s
B=5
B=10
B=20

Tangential-Momentum Equation



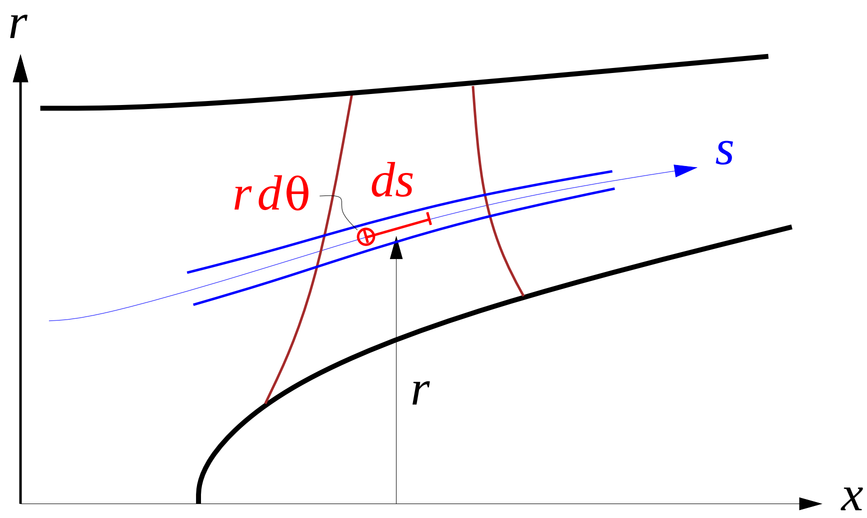


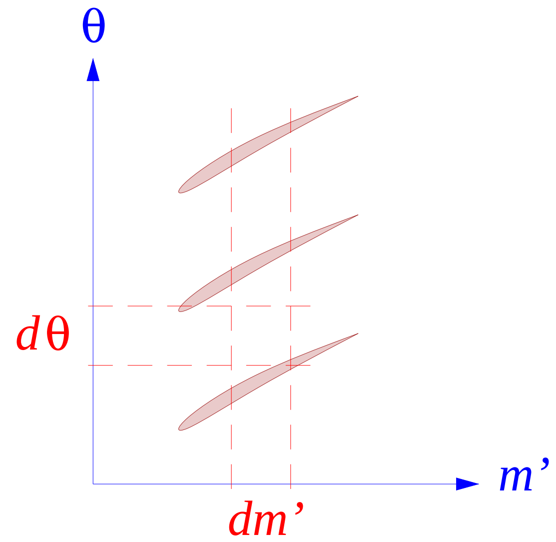
If geometry prescribed (analysis):

$$R_\theta \;\equiv\; d\bar{\Gamma} \;+\; k\left(\bar{\Gamma} \;-\; \bar{\Gamma}_o\right) ds$$

$$\bar{\Gamma}_o \;=\; \left(q\,\mathcal{S}_{\rm rel} \;+\; \Omega r\right) r$$

Blade-to-blade Coordinates m'–θ





$$\theta = \int d\theta = \int \frac{r\,d\theta}{r}$$

$$m' = \int \frac{ds}{r} = \int \frac{\sqrt{dx^2 + dr^2}}{r}$$

Tangential-Momentum Options

	Design	Analysis
Specified:	ΔG	$\mathcal{S}_{\rm rel}$
Result:	$\bar{\Gamma}$	$\bar{\Gamma}$

For Design case, blade camberline $m'(s), \theta(s)$ is generated from unused swirl-evolution equation

$$\mathcal{S}_{\rm rel} = \frac{d\theta}{dm'} = \frac{r\,d\theta}{ds} = \frac{1}{q}\left(\frac{2}{B}\frac{d\bar{\Gamma}}{ds} + \frac{\bar{\Gamma}}{r} - \Omega r\right)$$

$$m'(s) = \int^s \frac{1}{r}\,ds \quad , \quad \theta(s) = \int^s \mathcal{S}_{\rm rel}\frac{1}{r}\,ds$$

Higher-Fidelity Blade Load Modeling

- 1st-order swirl-evolution model is approximate
- Cascade solver in m'–θ (MISES) needed to give actual loading from geometry
- Implemented via Axisymmetric/Cascade solver iteration