

First-Order DC Electric Motor Model

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Nomenclature

Ω	rotation rate	v	terminal voltage
Q_m	torque	\mathcal{R}	resistance
P_{shaft}	shaft power	i	current
η_m	efficiency	i_o	zero-torque current
K_V	speed constant		
K_Q	torque constant		

1 Motor Model

1.1 Fundamental relations

The behavior of DC electric motor is described by the equivalent circuit model shown in Figure 1.

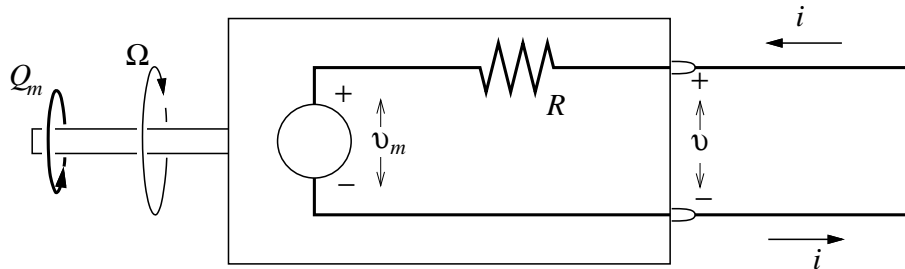


Figure 1: Equivalent circuit for a DC electric motor.

1.1.1 Resistance model

The resistance \mathcal{R} of the motor is assumed to be constant.

1.1.2 Torque model

The shaft torque Q_m is assumed proportional to the current i via the torque constant K_Q , minus a friction-related current i_o .

$$Q_m(i) = (i - i_o)/K_Q \quad (1)$$

1.1.3 Voltage model

The internal back-EMF v_m is assumed proportional to the rotation rate Ω via the motor speed constant K_V .

$$v_m(\Omega) = \Omega/K_V \quad (2)$$

The motor terminal voltage is then obtained by adding on the resistive voltage drop.

$$v(i, \Omega) = v_m(\Omega) + i \mathcal{R} = \Omega/K_V + i \mathcal{R} \quad (3)$$

1.2 Derived relations

The model equations above are now manipulated to give the current, torque, power, and efficiency, all as functions of the motor speed and terminal voltage. First, equation (3) is used to obtain the current function.

$$i(\Omega, v) = \left(v - \frac{\Omega}{K_V} \right) \frac{1}{\mathcal{R}} \quad (4)$$

The remaining motor variables follow immediately.

$$Q_m(\Omega, v) = [i(\Omega, v) - i_o] \frac{1}{K_Q} = \left[\left(v - \frac{\Omega}{K_V} \right) \frac{1}{\mathcal{R}} - i_o \right] \frac{1}{K_Q} \quad (5)$$

$$P_{\text{shaft}}(\Omega, v) = Q_m \Omega \quad (6)$$

$$\eta_m(\Omega, v) = \frac{P_{\text{shaft}}}{i v} = \left(1 - \frac{i_o}{i} \right) \frac{K_V}{K_Q} \frac{1}{1 + i \mathcal{R} K_V / \Omega} \quad (7)$$

In the limiting case of zero friction ($i_o = 0$) and zero resistive losses ($\mathcal{R} = 0$), the efficiency (7) becomes

$$\eta_m = \frac{K_V}{K_Q} \quad (\text{zero losses}) \quad (8)$$

Hence, energy conservation requires that the torque constant K_Q must be equal to the speed constant K_V . The equations here assume K_V is in rad/s/Volt, and K_Q is in the equivalent units of Amp/Nm. However, K_V is usually given in RPM/Volt.

2 Motor Parameter Measurement

The motor operation functions (4) – (7) depend on the “motor constants” \mathcal{R} , i_o , K_V , K_Q . These can be obtained by benchtop measurements, together with simple data fitting.

2.1 Motor resistance

The motor resistance \mathcal{R} can be measured directly with a milli-ohmmeter. Alternatively, it can be determined using a power supply, an ammeter, and a voltmeter. A representative range of currents i is sent through the motor by applying a suitable voltages v to the motor terminals, while the shaft is held to prevent rotation. The resistance is then computed using Ohm’s Law.

$$\mathcal{R} = v/i \quad (9)$$

On a commutated motor this will likely vary with shaft position, in which case the various \mathcal{R} values need to be averaged over different shaft positions.

2.2 Zero-load current

With the motor shaft free to turn, a voltage v is applied to the motor such that the RPM is comparable to the expected operating RPM. The resulting zero-load current i_o is then measured.

2.3 Speed constant

Using the previously-obtained resistance \mathcal{R} , the motor back-EMF voltage v_m can be computed for the freely-spinning case, using the measured zero-load v , i_o data.

$$v_m = v - i_o \mathcal{R} \quad (10)$$

The speed constant is then computed from (2).

$$K_v = \Omega/v_m \quad (11)$$

2.4 Torque constant

The torque constant can be simply assumed to be the same as K_v .

$$K_Q = K_v \quad (12)$$

Alternatively, it can be obtained from motor torque data if this is available. Ideally, the motor is operated at different loads (the zero-load tests are not usable here), and over a range of speeds by applying different voltages. According to the torque model (1), the Q_m data versus $i - i_o$ should be a straight line passing through the origin. The slope of the best-fit line to the data then gives $1/K_Q$.

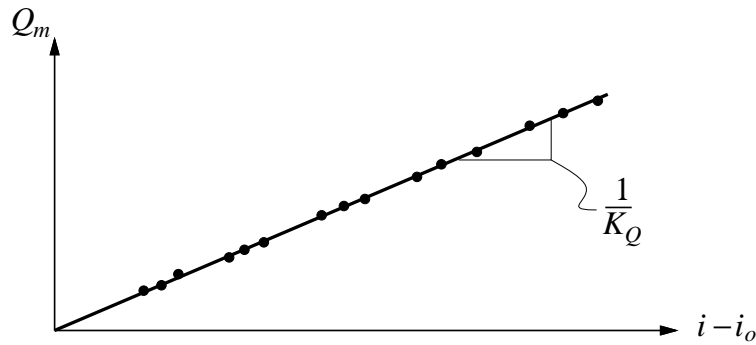


Figure 2: Q_m data versus $i - i_o$, with curve fit to determine K_Q .

As can be seen from (8), any resulting discrepancy between this measured K_Q and K_v will give a nonunity efficiency even if all the loss quantities \mathcal{R} , i_o are set to zero. Hence, a nonunity K_v/K_Q ratio indicates the degree of imperfection of the present motor model. However, simply allowing K_Q to be different from K_v will at least partially account for the model imperfections, since the efficiency is then likely to be predicted more accurately.