DC Motor / Propeller Matching

3 Mar 05

Lab 5 Lecture Notes

Nomenclature

T	prop thrust	ρ	air density
Q	prop torque	μ	air viscosity
Q_m	motor torque	Re	Reynolds number
P_{shaft}	shaft power	V	flight velocity
P_{elec}	electrical power	Ω	motor and propeller rotation rate
C_T	thrust coefficient based on tip speed	v	motor terminal voltage
C_P	power coefficient based on tip speed	$\mathcal{R}$	motor resistance
λ	advance ratio	i	motor current
η_p	propeller efficiency	K_{v}	motor speed constant
η_m	motor efficiency	$()_o$	quantity at zero load

Motor Parameters

The behavior of an electric motor is fairly accurately described by the equivalent circuit model shown in Figure 1. The internal back-EMF v_m is proportional to the rotation rate Ω via the motor speed constant K_{v}. Applying the usual circuit equations, together with the conservation of energy, gives the following motor parameters as functions of the motor current i and motor terminal voltage v:

$$Q_m(i) = (i - i_o)/K_{\text{v}} \tag{1}$$

$$\Omega(i,v) = (v - i\mathcal{R})K_{\text{v}} \tag{2}$$

$$P_{\text{shaft}}(i,v) = Q_m\,\Omega = (i - i_o)(v - i\mathcal{R}) \tag{3}$$

$$P_{\text{elec}}(i,v) = v\,i \tag{4}$$

$$\eta_m(i,v) = P_{\text{shaft}}/P_{\text{elec}} = (1 - i_o/i)(1 - i\mathcal{R}/v) \tag{5}$$

These relations depend on the "motor constants" $\mathcal{R}$, i_o, K_{v}, which can be measured via simple benchtop experiments. The equations here assume K_{v} is in rad/s/Volt, although it's usually given in RPM/Volt.

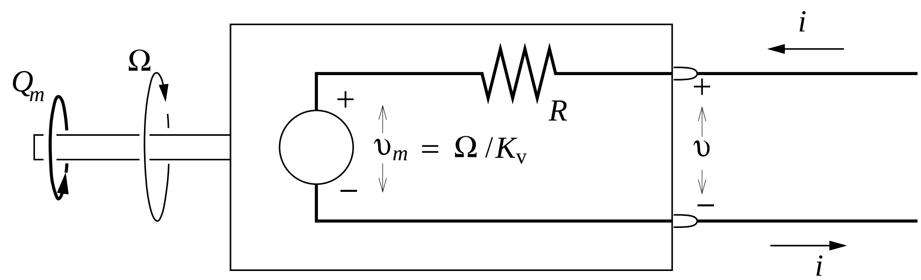

Figure 1: Equivalent circuit for a brushed DC electric motor.

For the purpose of matching the motor to a load, such as a propeller, we first manipulate relation (2) into a function for the current,

$$i(\Omega, v) = \left(v - \frac{\Omega}{K_{\text{v}}}\right)\frac{1}{\mathcal{R}} \tag{6}$$

which is then substituted into all the other righthand sides to give the following functions of motor speed and voltage:

$$Q_m(\Omega, v) = \left[\left(v - \frac{\Omega}{K_\mathrm{v}}\right)\frac{1}{\mathcal{R}} - i_o\right]\frac{1}{K_\mathrm{v}} \tag{7}$$

$$P_\mathrm{shaft}(\Omega, v) = \left[\left(v - \frac{\Omega}{K_\mathrm{v}}\right)\frac{1}{\mathcal{R}} - i_o\right]\frac{\Omega}{K_\mathrm{v}} \tag{8}$$

$$\eta_m(\Omega, v) = \left[1 - \frac{i_o\mathcal{R}}{v - \Omega/K_\mathrm{v}}\right]\frac{\Omega}{v\,K_\mathrm{v}} \tag{9}$$

These functions are sketched in Figure 2 versus motor speed, for three applied voltages.

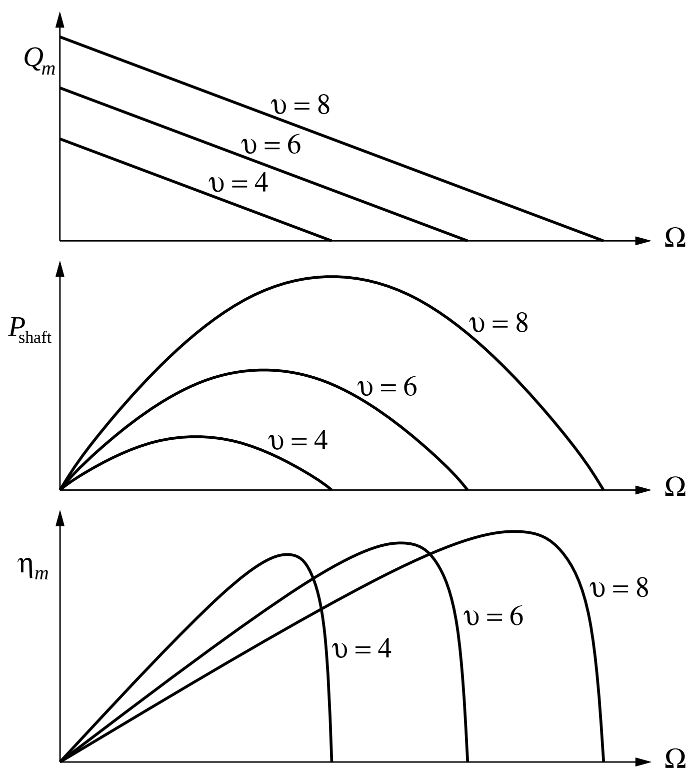

Figure 2: Motor output variables versus motor speed and applied voltage.

Propeller Parameters

A propeller is characterized by the thrust and power coefficients, which depend primarily on the advance ratio λ, the blade Reynolds number Re, and on the prop geometry.

$$C_T = C_T(\lambda, Re, \text{geometry}) \tag{10}$$

$$C_P = C_P(\lambda, Re, \text{geometry}) \tag{11}$$

$$\lambda = \frac{V}{\Omega R} \tag{12}$$

$$Re = \frac{\rho \Omega R\, c_\mathrm{ave}}{\mu} \tag{13}$$

Once the propeller geometry is specified and the C_T, C_P, η_p functions generated by measurement or analysis, the dimensional thrust and torque can be computed for any other V and Ω by dimensionalizing the coefficients.

$$\lambda(\Omega, V) = \frac{V}{\Omega R} \tag{14}$$

$$T(\Omega, V) = \tfrac{1}{2}\rho(\Omega R)^2\, \pi R^2\, C_T = \tfrac{1}{2}\rho V^2\, \pi R^2\, \frac{C_T(\lambda, Re)}{\lambda^2} \tag{15}$$

$$Q(\Omega, V) = \tfrac{1}{2}\rho(\Omega R)^2\, \pi R^3\, C_P = \tfrac{1}{2}\rho V^2\, \pi R^3\, \frac{C_P(\lambda, Re)}{\lambda^2} \tag{16}$$

Figure 3 shows the resulting T and Q variation for a propeller operating at two fixed V values (presumably set by the aircraft trim), as functions of the rotational speed Ω (presumably varied by the effective motor voltage). Strictly speaking, the correct Re which increases with Ω should be used to evaluate the C_T and C_P functions, although its effect is usually weak and a typical constant Re is often assumed.

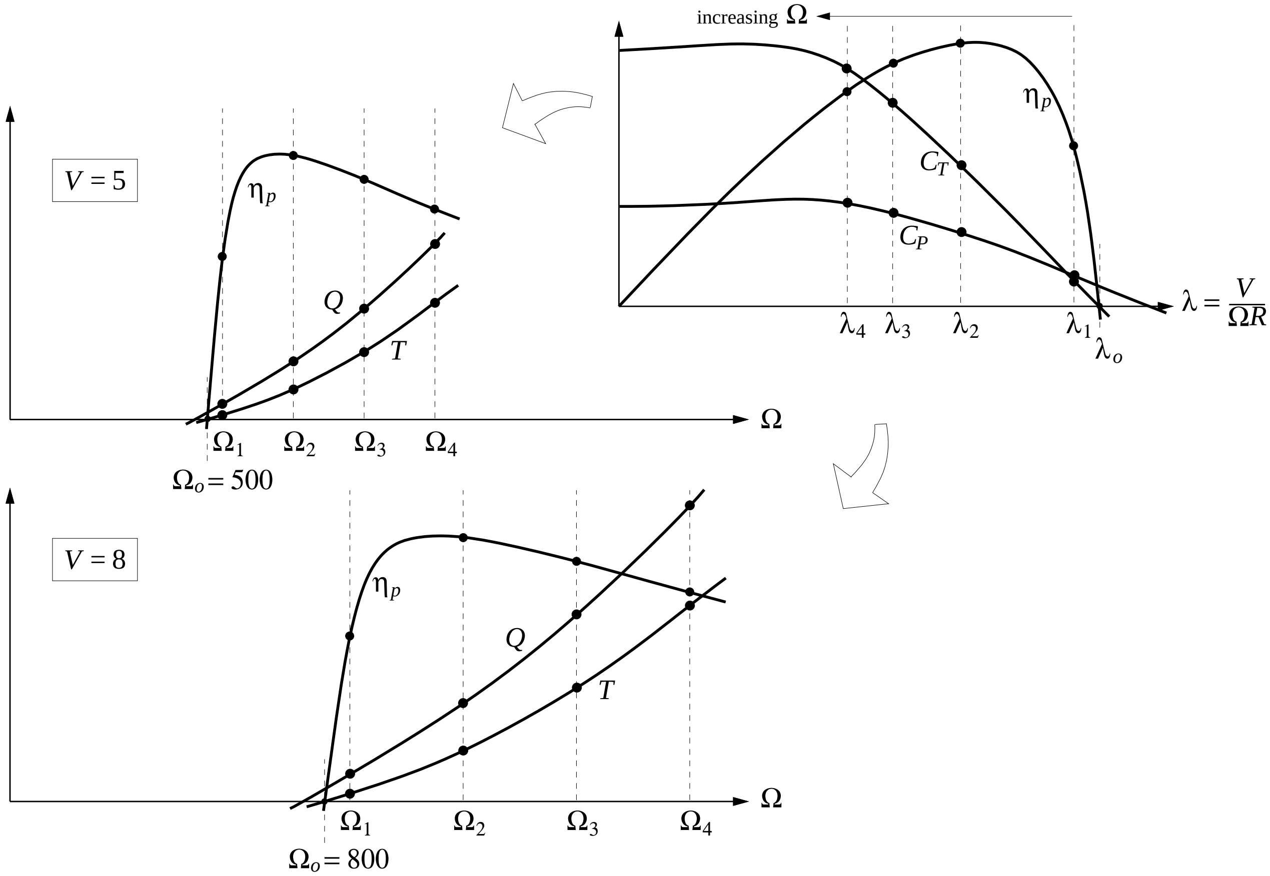

Figure 3: Propeller T, Q, η_p versus Ω for two flight speeds $V = 5, 8$, generated using common C_T, C_P, η_p curves. Zero-thrust speeds Ω_o occur at the same $V/\Omega_o R = \lambda_o$ values.

Motor/Prop Matching

The equilibrium operating speed Ω of the motor/prop combination occurs when the torques are equal:

$$Q_m(\Omega, v) = Q(\Omega, V) \tag{17}$$

A common situation is the need to determine all the prop and motor operating parameters which result from a specified flight speed V and applied motor voltage v. This is shown graphically in Figure 4, where the torque-matching condition (17) is applied to first determine the motor speed Ω. All the other prop and motor parameters can then be determined from the propeller and motor characteristic curves.

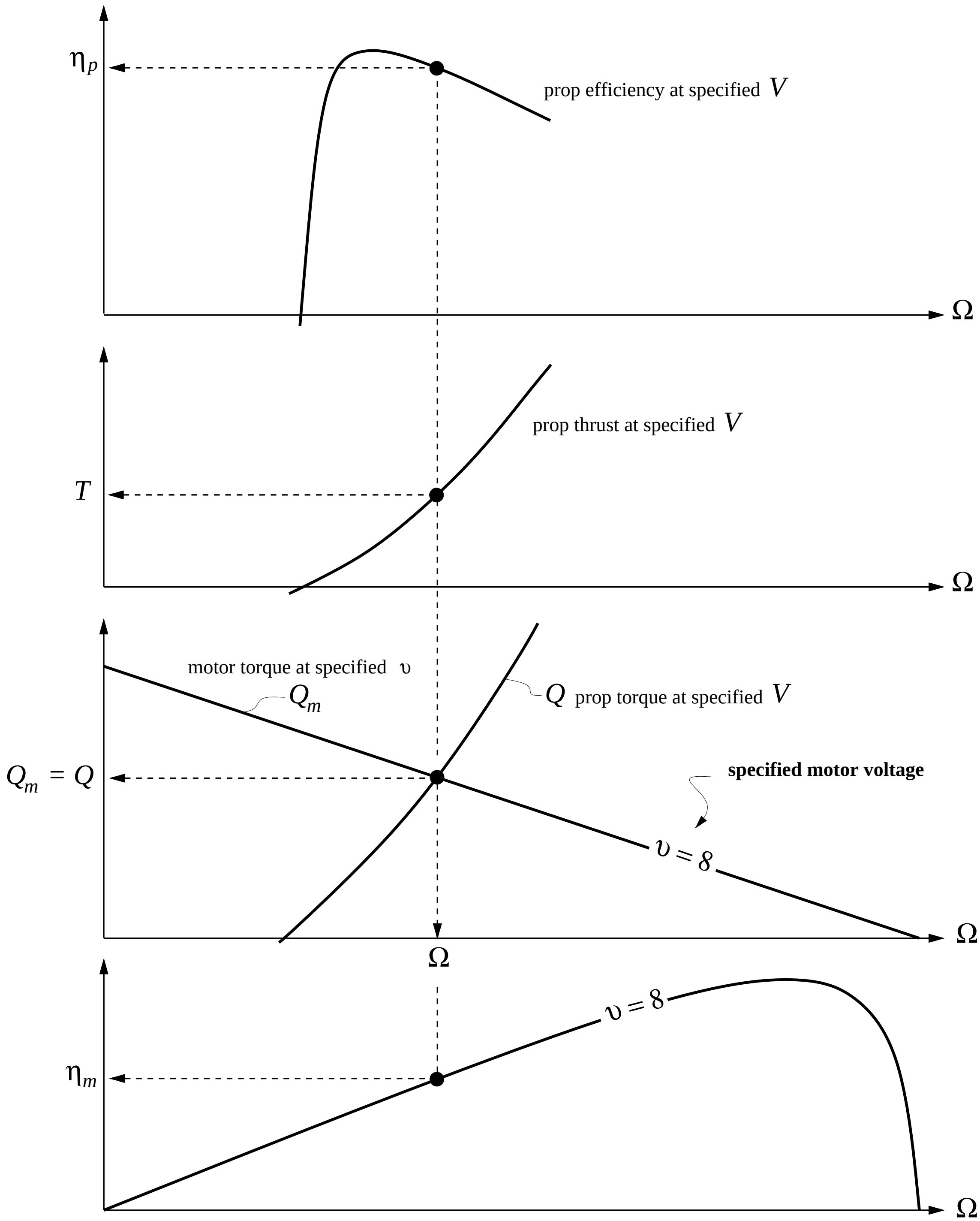

Figure 4: Prop and motor parameters obtained from a specified flight speed V and a specified motor voltage v. The motor speed Ω is read from the intersection of of the prop and motor torque curves, and all the other prop and motor parameters follow.

Another situation is the need to determine the motor voltage which will give a required thrust T at some specified flight speed V (e.g. to sustain level flight). This procedure is shown graphically in Figure 5. Again, the torque-matching condition (17) is used, but this time to read off the required voltage, suitably interpolated from the computed $Q_m(\Omega, v)$ lines.

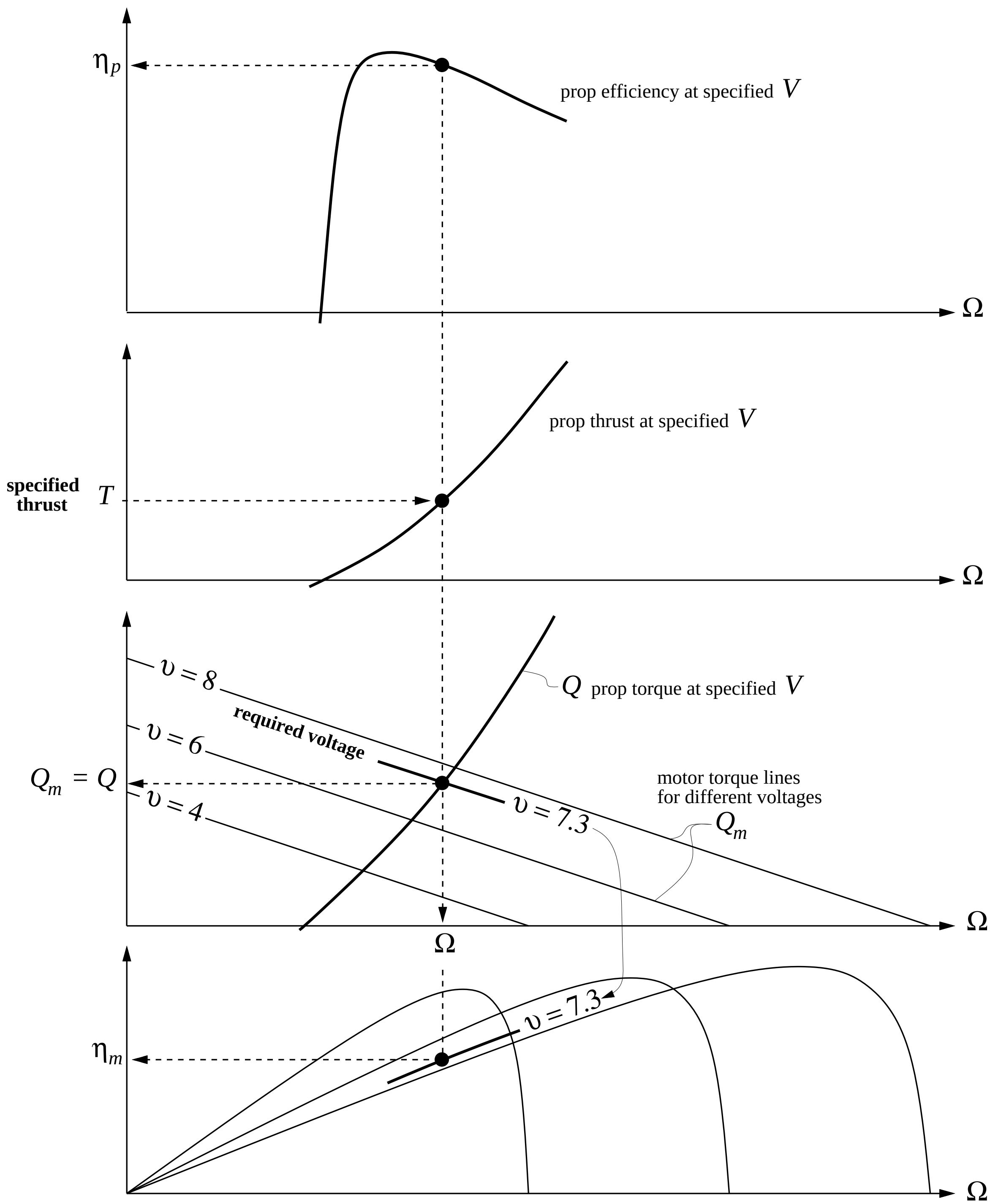

Figure 5: Prop and motor parameters obtained from a specified flight speed V and a required thrust T. The required motor voltage v is read from the intersection of the prop and motor torque curves, and all the other motor parameters follow.

An alternative to graphical interpolation is obtained if we take equation (7), which defines the $Q_m(\Omega, v)$ lines in Figure 5, and invert it, also replacing Q_m with Q.

$$v(\Omega, Q) \;=\; (K_\text{v}\, Q \,+\, i_o)\,\mathcal{R} \,+\, \frac{\Omega}{K_\text{v}} \tag{18}$$

Equation (18) gives the motor voltage v in terms of the propeller speed Ω and torque Q, both of which follow directly from the specified thrust T.

Well-Matched Systems

A power supplier and a power sink (the motor and the propeller in this case), are said to have a *good impedance match* if they are both operating at close to their peak efficiency. This means that their efficiency curves must both have their peaks at roughly the same speed Ω, and at the required thrust. Figure 6 shows good and poor match cases. A common approach to get a better impedance match in propeller systems is with *gearing*, which allows shifting the efficiency peak of the motor/gearbox system to a lower output-shaft Ω, where the prop wants to operate. But the gearing has drawbacks in its own losses, plus more weight, more cost, and possibly less reliability. Hence, a careful tradeoff is called for.

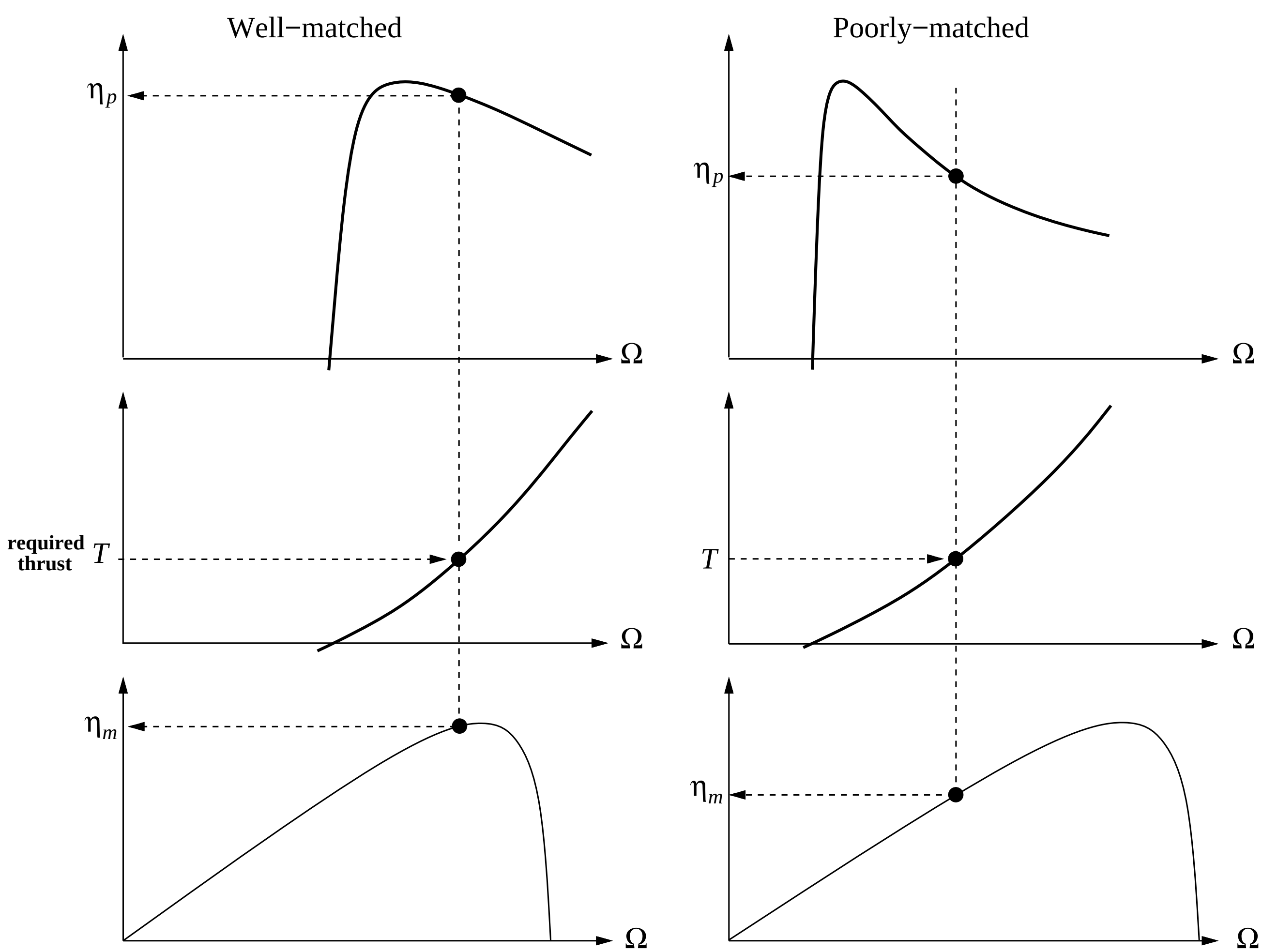

Figure 6: Well-matched and poorly matched motor and propeller pairs.