Name ______________________

18.01a Exam 1, ESG Fall 2007

50 minutes. No books, notes or calculators.
6 pages, 8 problems

Problem 1. (10 points)
Using the best quadratic approximation to $(1+x)^{1/3}$ for $x \approx 0$ approximate $(1.1)^{1/3}$.
(You can leave fractions unsimplified, e.g., 1 + 2/3 + 7/12.)

Problem 2. (10 points)
A.) (7) By substituting into the Taylor series around $x = 0$ for $\cos x$ find the first three terms of the Taylor series around $x = 0$ for $f(x) = \cos\sqrt{x}, \quad x \geq 0$.

B.) (3) If these three terms are used to calculate $\cos(\sqrt{.0001})$, the error will be around 10^{-n}, where n is what positive interger? (indicate reasoning)

Problem 3. (15 points) Compute each of the following.

A.) (8) $\lim_{x \to 0} \dfrac{\cos(x)\sin(x)}{x(x^2+1)}$.

B.) (7) $\lim_{x \to \infty} x^2 \mathrm{e}^{-x}$.

Problem 4. (20 points) Consider the region bounded by $y = \sin x$, $x = 0$, $x = \pi$ and $y = -1$. Set up an integral (*but don't compute it*) for each of the following.

A.) (5) The area of the region.

B.) (5) The volume of revolution given by rotating the region around the y-axis.

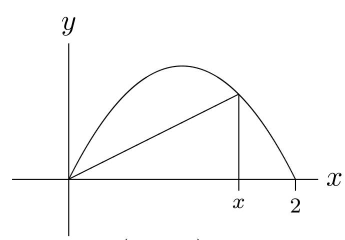
C.) (5) The volume of revolution given by rotating the region around the line $y = -1$.

D.) (5) The arclength of the curve $y = \sin x$ between $x = 0$ and $x = \pi$.

Problem 5. (10 points) Compute $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos^7 x}\, dx.$

Problem 6. (10 points)

For the graph of $y = x(2-x)$ over the interval $[0,2]$ triangles are inscribed as shown. If the point x is chosen randomly in the interval $[0,2]$ find the average value of their area.
Set up but do not compute the integral.



$y = x(2-x)$

Problem 7. (10 points)
Let A be the region bounded by the y-axis, the line $y = 9$ and the graph of $y = x^2$. A cup is made by revolving A around the y-axis. The cup is filled with a variable density fluid and then spun around the y-axis. The centrifugal action causes the fluid to separate by density with the more dense liquid towards the outside of the cup. Measurements show that the density varies according to the formula $\delta(x) = \sqrt{x}\text{kg/m}^3$, where x represents the distance to the y-axis.

Find the total mass of fluid in the cup.
(Your final answer can be left in a messy form, e.g. $10\,(\frac{12}{7} \cdot 3^{3/2} - \frac{3}{4} \cdot 3^{7/2})$.)

Problem 8. (15 points) Let $F(x) = \int_2^x \frac{1}{\ln t}\, dt$.

A.) (5) For what values of x is $F(x)$ defined? (Indicate reason.)

B.) (5) Evaluate $\lim_{x\to\infty} \frac{\ln x}{F(x)}$. (You may assume that $F(x) \to \infty$ as $x \to \infty$.)

C.) (5) Express $\int_1^2 \frac{1}{\ln 2t}\, dt$ in terms of values of $F(x)$.