Name ________________________

Please email my grade ______

18.01a Exam 2, ESG Fall 2007

50 minutes. No books, notes or calculators.
6 pages, 7 problems

Problem 1. (30 points)
Evaluate the following integrals.

A.) (10) $\displaystyle\int \frac{x}{(x+1)(x-2)}\,dx$

B.) (10) $\displaystyle\int_0^2 \frac{x^2}{\sqrt{4-x^2}}\,dx$

C.) (10) $\displaystyle\int \sin^{-1} x\,dx$ (hint: use parts)

Problem 2. (10 points)

Use the trapezoidal rule with $n = 4$ to approximate $\displaystyle\int_1^3 \frac{1}{1+x^2}\,dx$.

You can leave the expression you get as a sum of fractions. You don't need to get a common denominator.

Problem 3. (15 points)
A. (10) Use an integral to give an approximate value for $S = \sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{400}$. Show your work and state clearly what approximation rule you are using.

B. (5) Is your approximation in part (a) an over or an underestimate?

Problem 4. (15 points)
Say whether the following integrals converge or diverge. (Give reasons.)

A. (8) $\displaystyle\int_2^\infty \frac{x^2}{x^4 + 2x^2 - 1}\, dx$

B. (7) $\displaystyle\int_1^\infty \frac{e^{2x}}{x^3}\, dx$

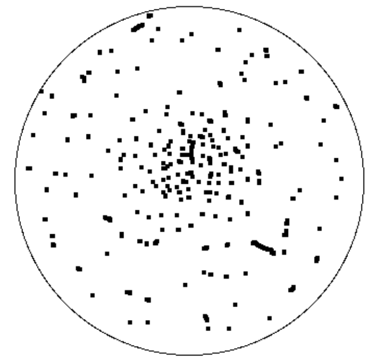
Problem 5. (10 points)

A. (5) Why is the integral in problem 1C improper?

B. (5) Does it converge or diverge?

Problem 6. (10 points)

An infinite circular dartboard has many dart holes, whose density $\delta(r)$ (in holes/sq. cm) depends on the distance r from the center, and is given by $\delta(r) = \dfrac{1}{1+r^p}$. A crude sketch is provided at right.



For which values of p does the dart board have finitely many holes? (Hint: divide the circular board area into thin concentric rings, and find the number of holes in each ring.)

Problem 7. (10 points)
Derive a reduction formula for $\int x(\ln x)^n \, dx$ which expresses the integral in terms of a similar integral involving a lower power of $\ln x$.