Name ______________________

Please email my grade ______

18.01a Exam 3, ESG Fall 2007

50 minutes. No books, notes or calculators.
8 pages, 7 problems

You should read through all these formulas before starting the exam

Poisson random variable: $P(k) = \mathrm{e}^{-m}\,\dfrac{m^k}{k!}$, $k = 0, 1, 2, \ldots$, mean m.

Exponential density function: $f(x) = \dfrac{\mathrm{e}^{-x/m}}{m}$, mean m.

Normal density function: (mean m, standard deviation σ): $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-(x-m)^2/2\sigma^2}$.

Standard normal density function $\phi(z)$ is the above $f(z)$ with $m = 0$, $\sigma = 1$.

Useful integral formula: $\displaystyle\int_0^\infty x^n \mathrm{e}^{-ax}\,dx = \frac{n!}{a^{n+1}}$.

Table of values for $\Phi(z)$, $Z \geq 0$, the distribution for $\phi(z)$

z :	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	
$\Phi(z)$	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159	
z :	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	
$\Phi(z)$	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713	
z :	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
$\Phi(z)$	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981	0.9987

Problem 1. (25 points)

A. (9) Does $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$ converge? *Show work or indicate reasoning.*

B. (8) Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln(n^3)}$ converge? *Show work or indicate reasoning.*

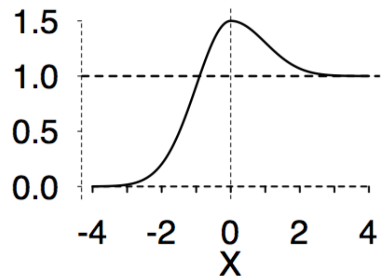
C. (8) By comparing to an integral show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1} < \frac{\pi}{2}$.

Problem 2. (10 points)
If $f(x) = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$, find a simple expression for $f(x)$, i.e., sum the series explicitly.

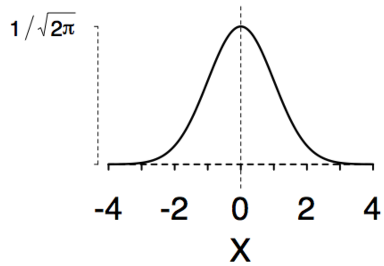
Problem 3. (15 points)
Let X be a continous random variable with range $[0,1]$ and density Ax^3. Find A, $E(X)$ and $\sigma(X)$.

Problem 4. (10 points) The following are graphs of functions.

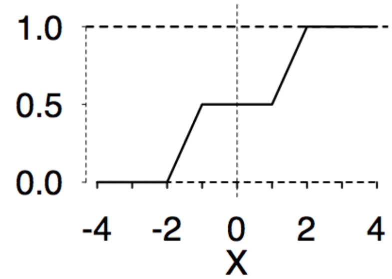
Function (i)
range=$(-\infty, \infty)$



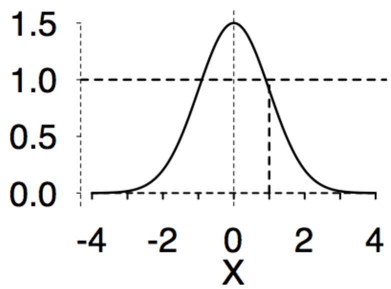
Function (ii)
range=$(-\infty, \infty)$



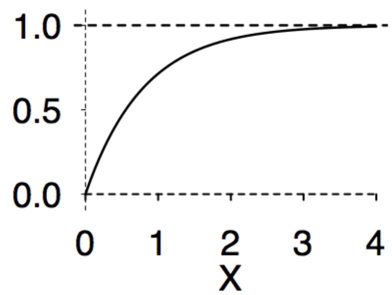
Function (iii)
range=$(-\infty, \infty)$



Function (iv)
range=$(-\infty, \infty)$



Function (v)
range=$[0, \infty)$



A. (5) For each function say whether or not it could be the cumulative distribution of a random variable in the given range. If not, give a reason.

B. (5) For each function say whether or not it could be the density of a random variable in the given range. If not, give a reason.

Problem 5. (15 points)
A cookie maker knows that on average there are 3 broken cookies per box.

A. (7) What's the probability of no broken cookies in a box?

B. (8) What's the probability of more than 5 broken cookies in 10 boxes?
Give a numerical expression for this, but do not try to simplify it.

Problem 6. (15 points)
Suppose parents tended to pick up their children an average of 5 minutes late from a daycare center. Assume the amount of time a random parent is late on any given day is an exponential random variable.

A. (5) How late is the average parent?

B. (5) What is the probability that a parent will be more than 15 minutes late?

C. (5) The daycare center decides to charge parents t^2 cents if they are t minutes late. What is the average family's daily late fee?

Problem 7. (10 points)
From a certain orchard the average weight of an apples is 400 grams with a standard deviation of 20 grams. Assuming the weight follows a normal distribution, how many apples in a box of 100 would you expect to weigh more that 420 grams?