18.01a Practice Exam 1, ESG Fall 2007

No books, notes or calculators.
You should be able to do this in about 65 minutes. The real test will be a shorter.
Remember to study all the material. *Not everything is on the practice exam.*

Problem 1. (8) Find the best quadratic approximation to $f(x) = \frac{e^x}{1+x}$ for $x \approx 0$.

Problem 2. (7) Find $\lim_{x\to 0} \dfrac{1 - e^{(x^2)}}{\sin^2 x}$.

Problem 3. (15)
a) (5) State the Mean Value Theorem (MVT).

b) (10) Using the MVT show that $\tan x > x$ for $0 < x < \pi/2$.

Problem 4. (10) Find the first four non-zero terms in the Taylor series around $a = 0$ for the function $\dfrac{1}{(1+x)^2}$.

Problem 5. (15) Radioactive material decays exponentially. Assuming consistent units, if you have an amount A then after a time t there will be Ae^{-kt}. (k is called the decay constant.) Suppose a new nuclear storage facility takes in radioactive material at the rate of $1 - (t-1)^2$ kg/year for its first 2 years of operation. Assume a decay constant of k and show how to write an integral for the amount of radioactive material at the end of the 2 years. (You don't have to compute the integral, but you do have to show reasoning.)

Problem 6. (10) Compute $\int_2^3 \frac{(1+\ln x)^7}{x}\,dx$.

Problem 7. (20) For this problem we have $f(x) = x^3$ and we consider the region between the graph of $f(x)$ and the y-axis for $-1 \le x \le 1$.

a) (10) Compute the volume of revolution when this region is revolved around the $y-$axis.

b) (10) Write down an integral expressing the arclength of the graph of $f(x)$. (You do not need to compute the integral.)

Problem 8. (20) Consider the function $F(x) = \int_0^x \sqrt{3 + \sin t}\,dt$. Without attempting to find an explicit formula for $F(x)$,

a) (5) Determine whether $F(x)$ is concave up or concave down in the interval $0 < x < 1$.

b) (5) Show that $F(1) \le 2$.

c) (5) Give $\int_1^2 \sqrt{3 + \sin 2t}\,dt$ in terms of $F(x)$. (Notice the factor of 2 in the sin term.)

d) (5) Let $G(x) = \int_0^{x^2} \sqrt{3 + \sin t}\,dt$. Compute $G'(x)$.

Problem 9. (15) Consider the first hump of the graph of $y = \sin x$.

a) (10) Find the average distance from this curve to the line $y = -1$.

b) (5) Find the average distance from this curve to the y-axis.