18.01a Practice Exam 2, ESG Fall 2007

No books, notes or calculators.
This will take significantly more than 50 minutes. The real test will be much shorter.

Problem 1. Integrate each of the following.

a) $\displaystyle\int \frac{7-x}{(x-1)(x^2+1)}\,dx$ b) $\displaystyle\int \frac{3x^3+6x^2+2x+2}{x^2+2x}\,dx$

c) $\displaystyle\int \sin^{-1} x\,dx$ d) $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$

e) $\displaystyle\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx$

Problem 2.
a) Use Simpson's rule with $n = 2$ to approximate $\displaystyle\int_0^{\pi} \sqrt{\cos^2\theta + 4\sin^2\theta}\,d\theta$.

b) Use the trapezoidal rule to approximate $\displaystyle\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{N}$.
Is this an underestimate or an overestimate?

Problem 3.
a) Compute $\displaystyle\int x^2 \ln x\,dx$.

b) Compute $\displaystyle\int \sec^{-1} x\,dx$.

c) Find a reduction formula for $\displaystyle\int \sin^n x\,dx$ and use it to compute $\displaystyle\int_0^{\pi/2} \sin^6 x\,dx$.

Problem 4. Compute $\displaystyle\int_1^{\infty} \frac{\ln x}{x^2}\,dx$.
Do this formally, writing the improper integral as a limit.

Problem 5. State whether the following are convergent or divergent.

a) $\displaystyle\int_0^{\infty} \frac{x}{\sqrt{x^3+1}}\,dx$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+1}{n^3+1}$ c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

Problem 6. Show $\displaystyle\int_0^{\infty} \frac{dt}{1+t^4} < 4/3$.

Problem 7. Let R be the region under the graph of $y = \frac{1}{x^p}$ and over $1 \le x < \infty$. Consider the volume and surface of revolution of this region around the x-axis.

a) For which values of p is the volume finite and the surface area infinite.

b) If I choose one such p and fill the volume with a finite amount of paint it would cover the inside surface. Since the surface has infinite area this would seem to be impossible. Discuss this situation.