18.01a Practice Exam 3, ESG Fall 2007

No books, notes or calculators.
This test should take about 50 minutes.

Poisson random variable: $P(k) = \mathrm{e}^{-m}\dfrac{m^k}{k!}$, $k = 0, 1, 2, \ldots$, mean m.

Exponential density function: $f(x) = \dfrac{\mathrm{e}^{-x/m}}{m}$, mean m.

Normal density function: (mean m, standard deviation σ): $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-(x-m)^2/2\sigma^2}$.

Standard normal density function $\phi(z)$ is the above $f(z)$ with $m = 0$, $\sigma = 1$.

Table of values for $\Phi(z)$, $Z \geq 0$, the distribution for $\phi(z)$

z:	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	
$\Phi(z)$	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159	
z:	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	
$\Phi(z)$	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713	
z:	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
$\Phi(z)$	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981	0.9987

Problem 1. A random varibable X on the interval $[0, \pi]$ has probability density with the form $f(x) = C\sin x$. Find the value of C, and the mean and standard deviation of X.

Problem 2. A textbook has 200 misprints in 300 pages. Compute the following.

a) The probability that a 9 page chapter has no misprints.

b) The probability that a 4 page chapter has at least two misprints.

Problem 3. (Like 8B/4.) Assume the mean length of time between auto accidents on the Mass Turnpike is 10 hours. Estimate the probability of no accidents for 24 hours.

Problem 4. Suppose the lifetime of a brand of lightbulb is a normal random variable X with mean 100 and standard deviation 16.

Find the probability that a given bulb lasts between 84 and 120 hours. How many bulbs in a pack of 144 would you expect to last in this range.

Problem 5. Find the sum of each of the following series.

a) $x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ldots + \dfrac{x^{2n+1}}{2n+1} + \ldots$

b) $x + 2x^2 + 3x^3 + \ldots + nx^n + \ldots$

Problem 6. State whether the following are convergent or divergent.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+1}{n^3+1}$ b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$