18.02a Practice Exam 4 Extra Problems

These are some extra problems. No solutions will be posted

Problem 1. Tape is unwound from a roll in a counterclockwise direction and in a manner so that the tape is always pointing straight up.
a) Give parametric equations for the curve traced out by the endpoint of the tape.

b) Assuming your parameter is θ, compute $\mathbf{r}'(\theta)$, $\dfrac{ds}{d\theta}$, $\mathbf{T}(\theta)$, $\dfrac{d\mathbf{T}}{d\theta}$ and κ.

Problem 2. If $\mathbf{r}(t) = (x(t), y(t), z(t))$ has constant length show $\frac{d\mathbf{r}}{dt} \perp \mathbf{r}(t)$.

Problem 3. This problem is on inverses of matrices and solutions to systems. You make up and solve the problem.

Problem 4.
a) Find the values of c for which there no solutions to

$$\begin{pmatrix} 1 & 2 & c \\ 1 & 3 & c \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}.$$

b) Find all values of c for which there are no solutions to

$$\begin{pmatrix} 1 & c & 2 & c \\ 2 & c & 1 & c \\ c & 2 & 2 & 1 \\ c & 5 & 1 & 7 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 5. A point P moves in space so its position vector is given by
$\overrightarrow{\mathbf{OP}} = \mathbf{r}(t) = \cos t\mathbf{i} + \sqrt{2}\sin t\mathbf{j} + \cos t\mathbf{k}$.
a) Find $\dfrac{d\mathbf{r}}{dt}$, $\dfrac{ds}{dt}$, $\mathbf{T}(t)$, $\dfrac{d\mathbf{T}}{dt}$, κ, R, C (center of curvature) and $\mathbf{N}$.
b) Show the point moves in a plane.

Problem 6. a) This problem is about distances from points to planes or points to lines. You make some up and solve them. For a challenge, find the distance between two lines in space.

b) This problem is about the intersection of lines and planes or two planes. Write down the equations of some lines and planes and figure out where they intersect.

Problem 7. Give the coordinates of 5 points in the plane that are the vertices of a regular pentagon.