18.02a Practice Exam 4, ESG Fall 2007

No books, notes or calculators. This should take about 80 minutes. The actual test will be shorter –designed to take 50 minutes.

Problem 1. The square with vertices $A = (1,1)$, $B = (1,-1)$, $C = (-1,-1)$ and $D = (-1,1)$ in the xy plane is the base of a pyramid. The point P at the apex of the pyramid is on the z-axis at a height 2.

a) Give the components of the vectors $\overrightarrow{\mathbf{PA}}$ and $\overrightarrow{\mathbf{PB}}$.

b) Find the angle on one of the faces at the apex.

c) Find the area of any one of the four faces touching the apex.

Problem 2. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

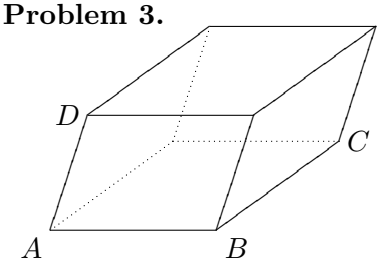
a) Find A^{-1}.

b) Use part (a) to solve $x + z = 1$, $3x + 2y + z = 0$, $x + y + 2z = 4$.

c) For what c will the system of equations $x + z = 0$, $3x + 2y + z = 0$, $x + y + cz = 0$ have a non-zero solution?

d) For the value of c found in part (c) find a non-zero solution to the system.

Problem 3.



If $A = (1,2,3)$, $B = (2,2,4)$, $C = (5,4,5)$, $D = (2,3,5)$ compute the volume of the parallelpiped shown.

Problem 4.

a) Write the equation of the plane containing the three points $(1,1,1)$, $(1,2,1)$, $(2,2,3)$.

b) Find the distance from the point $(0,0,3)$ to the plane in part (a).

Problem 5. Find the intersection of the line $(x,y,z) = (2,3,0) + t(1,3,5)$ and the plane $2x - 3y + z = 7$.

Problem 6.

a) Write the curve $y = \sin x$ in parametric form $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$.

b) For your answer in part (a) find: $\dfrac{d\mathbf{r}}{dt}$, $\dfrac{ds}{dt}$, $\mathbf{T}(t)$, $\dfrac{d\mathbf{T}}{ds}$, κ. (Note: some of the derivatives are messier than our typical problem –work carefully.)

Problem 7. Let P be a point halfway along a radius of a circle of radius a. Use vector methods to write the parametric equations for the curve traced out by P as the circle rolls along the x-axis. Assume the circle starts with both its center and P on the y-axis.