18.02a Practice Midterm Questions, Fall 2007 Solutions

Problem 1.

a) Distance $= \mathbf{PQ} \cdot \dfrac{\mathbf{N}}{|\mathbf{N}|} = \langle -19, 1, 1\rangle \cdot \dfrac{\langle 1,2,3\rangle}{\sqrt{14}} = \dfrac{14}{\sqrt{14}} = \boxed{\sqrt{14}.}$

b) $\boxed{\text{Line: } P + t\mathbf{N} = (20+t, 2t, 3t) \;\Leftrightarrow\; x = 20+t;\; y = 2t;\; z = 3t.}$

c) Put equation for line into equation of plane:

$(20+t) + 4t + 9t = 6 \;\Rightarrow\; 14t = -14 \;\Rightarrow\; t = -1. \;\Rightarrow\; \boxed{R = (19, -2, -3).}$

d) Let $\theta = \angle PQR$: $|QP||QR|\cos\theta = \mathbf{QP}\cdot\mathbf{QR} = \langle 19,-1,-1\rangle \cdot \langle 18,-3,-4\rangle = 349 \;\Rightarrow$

$\boxed{\cos\theta = \dfrac{349}{\sqrt{363}\sqrt{349}} \approx 0.98 \;\Rightarrow\; \theta \approx .2\,radians \approx 11^\circ.}$

e) $\mathbf{RP} = \langle 1,2,3\rangle \;\Rightarrow\; |\mathbf{RP}| = \sqrt{14}$. (Same as part (a).)

f) Area $= \frac{1}{2}|\mathbf{PQ}\times\mathbf{PR}|$. $\mathbf{PQ}\times\mathbf{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -19 & 1 & 1 \\ -1 & -2 & -3 \end{vmatrix} = -\mathbf{i} - 58\mathbf{j} + 39\mathbf{k}$

$\Rightarrow$ area $= \frac{1}{2}\sqrt{1 + 58^2 + 39^2} = \boxed{\frac{1}{2}\sqrt{4886} = 34.9.}$

Problem 2.

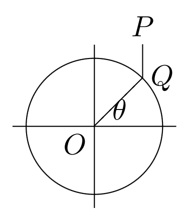
Parametrize by θ: $\mathbf{r}(\theta) = \mathbf{OP} = \mathbf{OQ} + \mathbf{QP}$.

$\mathbf{OQ} = \langle 2\cos\theta,\, 2\sin\theta\rangle$,

$\mathbf{QP}$ is vertical of length $2\theta \;\Rightarrow\; \mathbf{QP} = \langle 0,\, 2\theta\rangle$.

$\Rightarrow \boxed{\mathbf{r}(\theta) = \langle 2\cos\theta,\, 2\sin\theta + 2\theta\rangle.}$

Physically this makes sense for $0 \le \theta \le \pi$.



Problem 3. a) $\mathbf{v}(t) = \langle 4\cos t,\, -5\sin t,\, 3\cos t\rangle$. $\quad \dfrac{ds}{dt} = |\mathbf{v}| = 5$.

$\mathbf{T} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \langle \frac{4}{5}\cos t,\, -\sin t,\, \frac{3}{5}\cos t\rangle$.

$\mathbf{a} = \langle -4\sin t,\, -5\cos t,\, -3\sin t\rangle \;\Rightarrow\; \mathbf{a}\times\mathbf{v} = \langle -15, 0, 20\rangle \;\Rightarrow\; \kappa = \dfrac{|\mathbf{a}\times\mathbf{v}|}{|\mathbf{v}|^3} = \boxed{\dfrac{1}{5}.}$

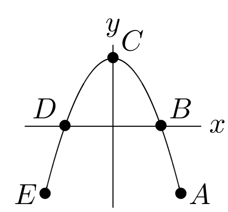
b) $\mathbf{r}\cdot\langle 3,0,-4\rangle = 12\sin t - 12\sin t = 0 \;\Rightarrow$ they're perpendicular.

This means P moves in a plane with normal $\langle 3,0,-4\rangle$.

Problem 4. $x = \sin t$, $y = \cos 2t = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t \quad\Rightarrow\; y = 1 - 2x^2$.

Coordinates:

$A{=}(1,-1)$, $B{=}(\sqrt{2}/2, 0)$, $C{=}(0,1)$, $D{=}(-\sqrt{2}/2, 0)$, $E{=}(-1,-1)$.



Path:

Pos.:	$C \to$	$B \to$	$A \to$	$B \to$	$C \to$	$D \to$	$E \to$	$D \to$	C (repeat)
time:	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

(continued)

Problem 5.

a) Have to show $B \cdot A_2 = I$:

$$\frac{1}{4}\begin{pmatrix} 3 & 1 & -2 \\ -5 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3+3-2 & 0+2-2 & 3+1-4 \\ -5+3+2 & 0+2+2 & -5+1+4 \\ 1-3+2 & 0-2+2 & 1-1+4 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} = I.$$

b) The system is $A_2\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} \Rightarrow \mathbf{x} = A_2^{-1}\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \boxed{\frac{1}{4}\begin{pmatrix} -5 \\ 3 \\ 9 \end{pmatrix}}.$

c) There are non-zero solutions to the homogeneous solution when the determinant of the coefficient matrix is 0. $\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & c \end{vmatrix} = 0 \Rightarrow 2c = 0 \Rightarrow \boxed{c = 0.}$

d) $\vec{\mathbf{A}} = (1,0,1)$, $\vec{\mathbf{B}} = (3,2,1)$ and $\vec{\mathbf{C}} = (1,1,0)$ are the rows of the matrix.
Determinant equal 0 means they all lie in a plane (volume of parallelpiped = 0)

$\Rightarrow$ Want perp. to plane: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}.$

Answer: $(-2, 2, 2)$ (or any multiple of this).

e) $A_1 = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Determinant = 2.

Minors: $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ -2 & -2 & 2 \end{pmatrix}$, Cofactors: $\begin{pmatrix} 1 & -2 & 1 \\ 1 & 0 & -1 \\ -2 & 2 & 2 \end{pmatrix}$, Adjoint: $\begin{pmatrix} 1 & 1 & -2 \\ -2 & 0 & 2 \\ 1 & -1 & 2 \end{pmatrix}$,

Inverse: $\boxed{\frac{1}{2}\begin{pmatrix} 1 & 1 & -2 \\ -2 & 0 & 2 \\ 1 & -1 & 2 \end{pmatrix}.}$

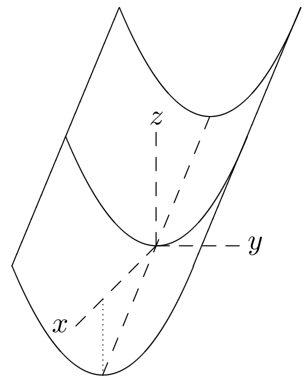
Problem 6.

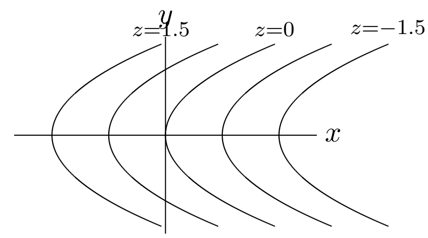
a) Let $w = x^3 + y^3 z$, then the normal $= \nabla w = \langle 3x^2,\, 3y^2 z,\, y^3 \rangle$
$\Rightarrow$ at $(1,1,2)$ the normal $= \boxed{\langle 3, 6, 1 \rangle.}$

b) The graph of $z = f(x,y)$ is the same as the level surface:
$g(x,y,z) = f(x,y) - z = 0.$
Direct computation: $\nabla g = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \rangle.$
Since ∇g is perpendicular to the level surface the result follows.

(continued)

Problem 7.
Level curves:
$z = c = y^2 - x \;\Rightarrow\; x = y^2 - c$
$=$ sideways parabolas.
Graph:
Fix $y = 0 \;\Rightarrow\;$ line $z = -x$.
Fix $x = c \;\Rightarrow\;$ parabola $z = y^2 - c$
Looks like a half-cylinder sloping up along the line $z = -x$.
The vertical cross-sections parallel to the yz-plane are parabolas.





Problem 8.
a) Chain rule: $\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \boldsymbol{\nabla} f \cdot \mathbf{r}'$.

b) Along a level curve w is constant $\Rightarrow$ $\frac{dw}{dt} = 0$.
Part (a) $\Rightarrow$ $\boldsymbol{\nabla} f \cdot \mathbf{r}' = 0 \;\Rightarrow\; \boldsymbol{\nabla} f \perp \mathbf{r}'$.
$\mathbf{r}'$ is tangent to the curve $\Rightarrow$ $\boldsymbol{\nabla} f \perp$ curve.

Problem 9. a) $\frac{\partial w}{\partial x} = y\cos(xy)$ and $\frac{\partial w}{\partial y} = x\cos(xy)$ $\Rightarrow$ $x\frac{\partial w}{\partial x} - y\frac{\partial w}{\partial y} = xy\cos(xy) - yx\cos(xy) = 0$.

b) (Same as part (a) but replace $\cos(xy)$ by $\frac{df}{du}$.
Chain rule: $\frac{\partial w}{\partial x} = \frac{df}{du}y$ and $\frac{\partial w}{\partial y} = \frac{df}{du}x$ $\Rightarrow$ $x\frac{\partial w}{\partial x} - y\frac{\partial w}{\partial y} = xy\frac{df}{du} - yx\frac{df}{du} = 0$.

Problem 10.
a) i) Apply chain rule to w: $\left(\frac{\partial w}{\partial z}\right)_x = 2x\left(\frac{\partial x}{\partial z}\right)_x + 2y\left(\frac{\partial y}{\partial z}\right)_x + 2z\left(\frac{\partial z}{\partial z}\right)_x$.

Easily: $\left(\frac{\partial x}{\partial z}\right)_x = 0$ and $\left(\frac{\partial z}{\partial z}\right)_x = 1$.

Implicit diff.: $0 = \left(\frac{\partial x}{\partial z}\right)_x = f_y\left(\frac{\partial y}{\partial z}\right)_x + f_z\left(\frac{\partial z}{\partial z}\right)_x = z\left(\frac{\partial y}{\partial z}\right)_x + y = z\left(\frac{\partial y}{\partial z}\right)_x + y$

$\Rightarrow$ $\left(\frac{\partial y}{\partial z}\right)_x = -\frac{y}{z}$. $\Rightarrow$ $\boxed{\left(\frac{\partial w}{\partial z}\right)_x = -2\frac{y^2}{z} + 2z.}$

ii) Differentials: $dw = 2x\,dx + 2y\,dy + 2z\,dz$; $dx = z\,dy + y\,dz$.
Independent variables are z, x so want to remove dy from formula for dw.
Solve the second equation for dy: $dy = \frac{1}{z}\,dx - \frac{y}{z}\,dz$.
Substitute: $dw = 2x\,dx + 2y(\frac{1}{z}\,dx - \frac{y}{z}\,dz) + 2z\,dz$.
Collect terms: $dw = (2x + 2y\frac{1}{z})\,dx + (-2y\frac{y}{z} + 2z)\,dz$.
$\Rightarrow$ $\boxed{\left(\frac{\partial w}{\partial z}\right)_x = -2\frac{y^2}{z} + 2z.}$ (Also get $\left(\frac{\partial w}{\partial x}\right)_z = 2x + 2y\frac{1}{z}$.)

(continued)

b) i) Apply chain rule to w: $\left(\frac{\partial w}{\partial z}\right)_x = 2x\left(\frac{\partial x}{\partial z}\right)_x + 2y\left(\frac{\partial y}{\partial z}\right)_x + 2z\left(\frac{\partial z}{\partial z}\right)_x$.

Easily: $\left(\frac{\partial x}{\partial z}\right)_x = 0$ and $\left(\frac{\partial z}{\partial z}\right)_x = 1$.

Implicit differentiation: $0 = \left(\frac{\partial x}{\partial z}\right)_x = f_y\left(\frac{\partial y}{\partial z}\right)_x + f_z\left(\frac{\partial z}{\partial z}\right)_x = f_y\left(\frac{\partial y}{\partial z}\right)_x + f_z$

$\Rightarrow \left(\frac{\partial y}{\partial z}\right)_x = -\frac{f_z}{f_y}$. $\Rightarrow$ $\boxed{\left(\frac{\partial w}{\partial z}\right)_x = -2y\left(\frac{f_z}{f_y}\right) + 2z.}$

ii) Differentials: $dw = 2x\,dx + 2y\,dy + 2z\,dz$; $\quad dx = f_y\,dy + f_z\,dz$.

Independent variables are z, x so want to remove dy from formula for dw.

Solve the second equation for dy: $\quad dy = \frac{1}{f_y}\,dx - \frac{f_z}{f_y}\,dz$.

Substitute: $\quad dw = 2x\,dx + 2y\left(\frac{1}{f_y}\,dx - \frac{f_z}{f_y}\,dz\right) + 2z\,dz$.

Collect terms: $\quad dw = \left(2x + 2y\frac{1}{f_y}\right)dx + \left(-2y\frac{f_z}{f_y} + 2z\right)dz$.

$\Rightarrow$ $\boxed{\left(\frac{\partial w}{\partial z}\right)_x = -2y\frac{f_z}{f_y} + 2z.}$ (Also get $\left(\frac{\partial w}{\partial x}\right)_z = 2x + 2y\frac{1}{f_y}$.)

Problem 11.

a) Chain rule:
$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} \qquad w_u = w_x x_u + w_y y_u$$
or
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} \qquad w_v = w_x x_v + w_y y_v$$

In matrix form: $(w_u, w_v) = (w_x, w_y)\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}$.

In full glory: $\left(\left(\frac{\partial w}{\partial u}\right)_v, \left(\frac{\partial w}{\partial v}\right)_u\right) = \left(\left(\frac{\partial w}{\partial x}\right)_y, \left(\frac{\partial w}{\partial y}\right)_x\right)\begin{pmatrix} \left(\frac{\partial x}{\partial u}\right)_v & \left(\frac{\partial x}{\partial v}\right)_u \\ \left(\frac{\partial y}{\partial u}\right)_v & \left(\frac{\partial y}{\partial v}\right)_u \end{pmatrix}$.

b) $x = r\cos\theta$, $y = r\sin\theta \Rightarrow (w_r, w_\theta) = (w_x, w_y)\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix}$.

c) We have, $x = x$ and $y = x\tan\theta$.

Here, it's best to set up the correspondence: $x, y \leftrightarrow x, y$ and $u, v \leftrightarrow x, \theta$.

All we have to do is substitute for u, v in the (fully glorious) equation of part (a):

$$\left(\left(\frac{\partial w}{\partial x}\right)_\theta, \left(\frac{\partial w}{\partial \theta}\right)_x\right) = \left(\left(\frac{\partial w}{\partial x}\right)_y, \left(\frac{\partial w}{\partial y}\right)_x\right)\begin{pmatrix} \left(\frac{\partial x}{\partial x}\right)_\theta & \left(\frac{\partial x}{\partial \theta}\right)_x \\ \left(\frac{\partial y}{\partial x}\right)_\theta & \left(\frac{\partial y}{\partial \theta}\right)_x \end{pmatrix}$$

$$= \left(\left(\frac{\partial w}{\partial x}\right)_y, \left(\frac{\partial w}{\partial y}\right)_x\right)\begin{pmatrix} 1 & 0 \\ \tan\theta & x\sec^2\theta \end{pmatrix}.$$

Problem 12. a) $\nabla w = \langle 3x^2y + 1/y,\ x^3 - x/y^2\rangle \Rightarrow$ $\boxed{\nabla w|_P = \langle 13, 6\rangle.}$

b) $\frac{dw}{ds} = \nabla w \cdot \frac{\langle 1, 3\rangle}{\sqrt{10}} = \boxed{31/\sqrt{10}.}$

c) Need $\widehat{\mathbf{u}}$ such that $\nabla w \cdot \widehat{\mathbf{u}} = 0$: Take $\widehat{\mathbf{u}}$ in the direction of $\boxed{\langle -6, 13\rangle.}$

d) $\Delta w \approx 13\Delta x + 6\Delta y = 13(.1) + 6(-.1) = .7$.

$w(P) = 10 \Rightarrow w(2.1, 0.9) \approx \boxed{10.7.}$ (Exact answer is 10.668.)

(continued)

Problem 13. Normal = gradient = $\langle 2x + y^2,\ 2xy + z^2,\ 2yz \rangle$.
At $(1, 2, 3)$: grad = $\langle 6, 13, 12 \rangle$
Tangent plane: $\boxed{6(x-1) + 13(y-2) + 12(z-3) = 0 \quad \text{or} \quad 6x + 13y + 12z = 68.}$

Problem 14. a) You draw the picture.
V = volume = $xyz = 4$, S = surface area = $2xy + 2xz + yz$.
Substituting $z = 4/xy \Rightarrow \boxed{S = 2xy + 8/y + 4/x.}$

b) Find critical points:
Solve $\frac{\partial S}{\partial x} = 2y - 4/x^2 = 0$, $\frac{\partial S}{\partial y} = 2x - 8/y^2 = 0$.
$\Rightarrow\ y = 2/x^2$, $x = 4/y^2 \ \Rightarrow\ x = x^4 \ \Rightarrow\ x = 0, 1$.
$x = 0$ gives undefined $y \ \Rightarrow$ only critical point is $(1, 2)$

Answer: $x = 1$, $y = 2$, $z = 2$.

c) $D = \begin{vmatrix} A = w_{xx} & B = w_{xy} \\ B = w_{xy} & C = w_{yy} \end{vmatrix} = \begin{vmatrix} 8/x^3 & 2 \\ 2 & 16/y^3 \end{vmatrix} = \begin{vmatrix} 8 & 2 \\ 2 & 2 \end{vmatrix} = 12.$
$D > 0$ and $A > 0 \ \Rightarrow$ minimum (second derivative test).

d) $\boxed{R = \text{first quadrant. Boundary} = \text{positive axes.}}$

e) When $x = 0$ or $y = 0$ the formula in part (a) gives $S = \infty$.

f) Lagrange:

$$\nabla w = \lambda \nabla v,\ V = 4 \ \Rightarrow \ \begin{aligned} 2y + 2z &= \lambda yz \\ 2x + z &= \lambda xz \\ 2x + y &= \lambda xy \\ xyz &= 4 \end{aligned}$$

Solving symetrically: $\frac{2}{z} + \frac{2}{y} = \lambda$; $\frac{2}{z} + \frac{1}{x} = \lambda$; $\frac{2}{y} + \frac{1}{x} = \lambda$.
The first 2 equations give: $x = \frac{y}{2}$. The second and third give: $z = y$.
Substituting in the formula for V gives $y^3/2 = 4 \ \Rightarrow\ y = 2$.
Answer: $x = 1$, $y = 2$, $z = 2$. (Same as before.)

Problem 15. Call the function w.
Derivatives: $w_x = 2x - 2y^2$, $w_y = -4xy + 4y$,
$A = w_{xx} = 2$, $B = w_{xy} = -4y$, $C = w_{yy} = -4x + 4 \ \Rightarrow\ D = AC - B^2 = -8x + 8 - 16y^2$.
Critical points: $w_x = 2x - 2y^2 = 0$, $w_y = -4xy + 4y = 0$
Second equation $\Rightarrow\ y = 0$ or $x = 1 \ \Rightarrow$ critical points are $(0, 0)$, $(1, 1)$, $(1, -1)$.

$(0, 0) : D = 8 > 0$, $A > 0 \ \Rightarrow$ minimum.
$(1, 1) : D = -16 \ \Rightarrow$ saddle.
$(1, -1) : D = -16 \ \Rightarrow$ saddle.

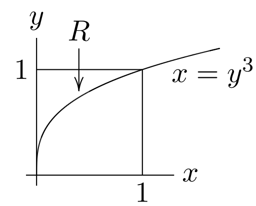
(continued)

Problem 16.

The region of integration is R.

Reverse the limits:

Outer var. y: from 0 to 1; inner var. x: from 0 to y^3.



$\Rightarrow$ Integral $= \displaystyle\int_0^1 \int_0^{y^3} e^{y^4}\, dx\, dy$.

Inner: $x e^{y^4}\Big|_0^{y^3} = y^3 e^{y^4}$.

Outer: $\displaystyle\int_0^1 y^3 e^{y^4}\, dy = \frac{1}{4} e^{y^4}\Big|_0^1 = \boxed{\frac{1}{4}(e^1 - 1).}$

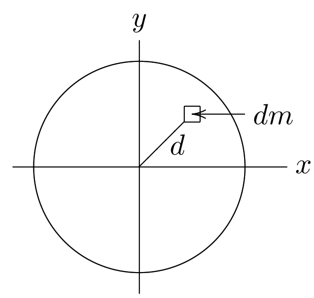
Problem 17.

a) $I = \delta \displaystyle\int\int_R d^2\, dm$, where $dm = \delta dA$,

and $d =$ moment arm $= r$, $\delta =$ (constant) density.

For this part, put point at origin

Limits for R: outer var. $\theta : 0$ to 2π; inner var. r: 0 to a.



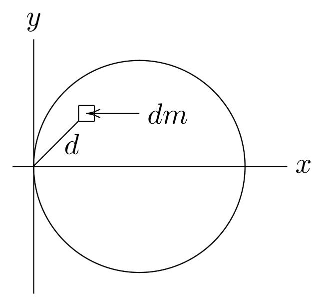
$$\boxed{I = \delta \int_0^{2\pi} \int_0^a r^2\, r\, dr\, d\theta.}$$

b) Put point at origin.

Limits for R:

outer var. $\theta : -\pi/2$ to $\pi/2$; inner var. r: 0 to $2a\cos\theta$.

Moment arm $d = r$.

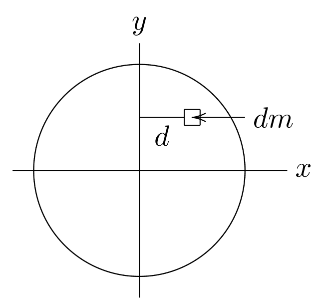


$$I = \delta \int_{-\pi/2}^{\pi/2} \int_0^{2a\cos\theta} r^2\, r\, dr\, d\theta = \boxed{2\delta \int_0^{\pi/2} \int_0^{2a\cos\theta} r^2\, r\, dr\, d\theta.}$$

c) Line $= y$-axis

Limits for R: same as part (a).

Moment arm $d = x = r\cos\theta$.



$$\boxed{I = \delta \int_0^{2\pi} \int_0^a r^2 \cos^2\theta\, r\, dr\, d\theta.}$$

(continued)

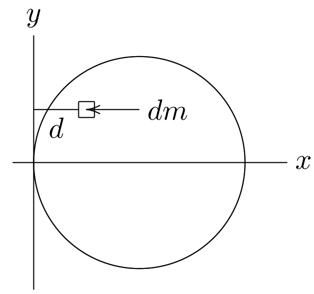
d) Line = y-axis

Limits for R: same as part (b).

Moment arm $d = x = r\cos\theta$.

$$I = \delta \int_{-\pi/2}^{\pi/2} \int_0^{2a\cos\theta} r^2 \cos^2\theta \, r\, dr\, d\theta$$

$$= \boxed{2\delta \int_0^{\pi/2} \int_0^{2a\cos\theta} r^2 \cos^2\theta \, r\, dr\, d\theta.}$$

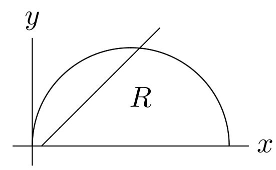


e)

Limits for S: outer var. θ: 0 to $\pi/4$; inner var. r: 0 to $2\cos\theta$

Moment arm $d = r$.

$$\boxed{I = \delta \int_0^{\pi/4} \int_0^{2\cos\theta} r^2 \, r\, dr\, d\theta.}$$



Problem 18.

Mass $= \int\int_R \delta\, dA$.

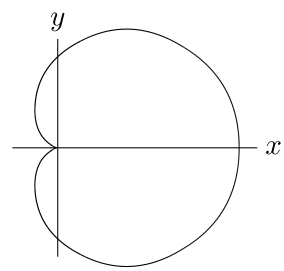
Limits of integration:

outer variable θ: 0 to 2π; inner variable r: 0 to $a(1+\cos\theta)$.

$$\text{Mass} = \int_0^{2\pi} \int_0^{a(1+\cos\theta)} \frac{1}{r}\, r\, dr\, d\theta = \int_0^{2\pi} \int_0^{1+\cos\theta} dr\, d\theta.$$

Inner integral: $r\big|_0^{a(1+\cos\theta)} = a(1+\cos\theta)$.

Outer integral: $\displaystyle\int_0^{2\pi} a(1+\cos\theta)\, d\theta = \boxed{2\pi.}$



Cardiod: $r = a(1+\cos\theta)$

Problem 19. Convert integrand: $f(x, y) = v^2u^2$.

Compute dA: $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -5$

$\Rightarrow\ dA = \frac{1}{5}\, du\, dv$.

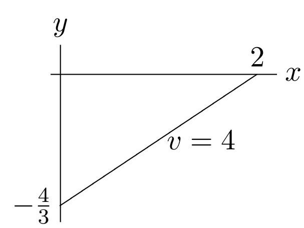
Limits of integration:

$x = 0\ \Rightarrow\ u = y,\ v = -3y\ \Rightarrow\ u = -v/3$.

$y = 0\ \Rightarrow\ u = x,\ v = 2x\ \Rightarrow\ u = v/2$.

Outer var. v: 0 to 4; inner var. u: $-v/3$ to $v/2$.

(Note: in the other order you need to break the region into 2 pieces.)



Integral $= \dfrac{1}{5}\displaystyle\int_0^4 \int_{-v/3}^{v/2}$.

Inner: $\displaystyle \frac{1}{15} v^2 u^3 \Big|_{-v/3}^{v/2} = \frac{1}{15} v^5 (\frac{1}{8} + \frac{1}{27})$.

Outer: $\displaystyle \frac{1}{15}\frac{v^6}{6}(\frac{1}{8} + \frac{1}{27})\Big|_0^4 = \boxed{\frac{4^6}{90}(\frac{1}{8} + \frac{1}{27}).}$

If you've read to here, thank you. It's been fun