18.01A Problem Set 1

(due Thurs., Sep. 13)

Part I (20 points)

TB = Simmons; SN = 18.01A Supplementary Notes (all have solutions) The problems marked 'other' are not to be handed in.

Class 1 (Wed., Sep. 5) Linear and quadratic approximations.
Read: SN: A.
Hand in: 2A/3, 7, 11, 12ae.
Others: 2A/1, 6, 12c.

Class 2 (Thurs., Sep. 6) Higher order approximations, Taylor series, Mean-value theorem.
Read: Orloff class notes on this topic, TB: 2.6 to middle p. 77, SN: MVT.
Hand in: 2G/1b, 2b, 6; 7C/1a,c; 7D/1ace, 2ef (skip radius of convergence)
Others: 2G/1a, 2a, 4.

Class 3 (Mon., Sep. 10) Indeterminate forms, L'Hospital's rule, growth rate of functions.
Read: TB: 12.2, 12.3 (examples 1-3, remark 1).
Hand in: 6A/1befgj, 5, 6c.
Others: 6A/1acd.

Class 4 (Tues., Sep. 11) Definite integral; summation notation, first fund. theorem, properties.
Read: TB: 6.3 through formula (4); skip proofs; 6.4, 6.5, 6.6.
Hand in: 3B/2ab, 3b, 4a, 5; 3C/2c, 3a, 5b, 6b; 4A/1a.
Others: 3C/1, 2a, 4; 4A/1b.

Continuation: (Wed., Sep. 13) Discussion, review and catch up.

Class 5 (Thurs., Sep. 14 **Problem Set 1 due**) Second fundamental theorem, $\ln x$ as an integral.
Read: SN: PI, FT.

Part II (27 points)

Directions: Try each problem alone for 20 minutes. If, after this, you collaborate, you must write up your solutions independently.

Problem 1 (Class 1, 6 pts: 2,1,1,1,1)
Supplementary Notes problem 2A-14.

Problem 2 (Class 2, 4 pts: 1,3)
A mass attached to a spring moves up and down with a certain frequency k_1, depending on its weight and the stiffness of the spring. If it is driven by an external force having a different frequency k_2, one possibility for the displacement of the mass from equilibrium (as a function of time) is given by

$$y = \frac{\sin k_1 t - \sin k_2 t}{k_1 - k_2} \tag{1}$$

a) Find the initial position y_0 and initial velocity v_0 of the mass (i.e. at time $t = 0$).

b) The problem is to find out what happens to y when $k_2 = k_1$, i.e. the spring-mass system is driven by an external force having the same frequency as its natural frequency. You can't put $k_2 = k_1$ in the equation for y. What we want is expressed instead by:

$$\lim_{k_2 \to k_1} \frac{\sin k_1 t - \sin k_2 t}{k_1 - k_2}$$

Determine this limit. (Hint: put $k_2 = k_1 + h$.)
Give a rough sketch of the resulting function.

Problem 3 (Class 2, 4 pts: 2,2)
Go to www-math.mit.edu/daimp and choose Taylor Polynomials.
Click the 'Terms' button to display the Taylor polynomial $p(x)$.
The cyan colored graph is the chosen function.
The yellow graph is the Taylor polynomial around a of degree n.
Play with the applet to get used to it. Practice setting different values of n and a. It is amusing to slide a and watch the approximating polynomial move. When setting n click a little to the right of the integer you want. At least in my browser there is a bug so that if you click a little to the left the degree of the Taylor polynomial is set to $n - 1$.

a) Choose $f(x) = e^x$ and set $a = 0$. What is the smallest degree Taylor polynomial needed to get a nearly perfect approximation for $2 > x > 0$? What about for $-4 < x < 0$?

b) Set $f(x) = \sin x$ and set $a = 0$. When going from $n = 1$ to $n = 2$ or from $n = 3$ to $n = 4$ the approximating polynomial does not change. Why is this?

Problem 4 (Class 4, 4 pts: 2,2)
a) Write down the Taylor series around $a = 0$ for e^x, $\sin x$, $\cos x$.

b) By differentiating the Taylor series in part (a) show that i) $\dfrac{de^x}{dx} = e^x$ and ii) $\dfrac{d\sin x}{dx} = \cos x$.

Problem 5 (Class 4, 5 pts: 1,1,2,1)
Consider the definite integral $\displaystyle\int_0^1 2^x dx$.

a) Use the First Fundamental Theorem to compute this integral.

b) Use a lower Riemann sum with n equal size intervals to approximate the integral.

c) Using the formula for the sum of a geometric progression (Simmons: formula (2) page 435) find a closed form expression for the sum in part (b).

d) Rederive the result in part (a) by finding the limit as $n \to \infty$ of the formula in part (c).

Problem 6 (Class 4, 4 pts: 1,2,1)
A snowball rolls down a hill for 10 seconds. It starts from rest and its angular speed increases linearly to 20 revolutions per second. While it rolls its diameter increases linearly from 1 to 3 feet.

a) Give formulas for the angular speed and diameter at time t, for $0 < t < 10$.

b) Give (with reasoning) an expression D for the distance rolled. (Divide up the period into equal intervals of length Δt and estimate how far the ball rolls during each small time period. Then let $\Delta t \to 0$ and write D as a definite integral.)

c) Evaluate D.