18.01A Problem Set 2b

(due Thurs., Sep. 27 (pset 2a due at same time))

Part I (15 points)

TB = Simmons; SN = 18.01A Supplementary Notes (all have solutions) The problems marked 'other' are not to be handed in.

Class 8 (Tues., Sep. 25) Integration: substitution, trigonometric integrals, completing the square.
Read: TB: 10.2, 10.3, 10.4.
Hand in: 5B/7, 9, 13, 16; 5C/6, 9, 11; 5D/1, 2, 7, 10.
Others: 5B/11; 5C/4, 5, 7.

Class 9 (Wed., Sep. 26) Integration: partial fractions.
Read: TB: 10.6, SN: F
Hand in: 5E/3, 5, 6, 10h (complete the square)
Others: 5E/2, 8b, 9b, 10ac.

Class 10 (Thurs., Sep. 27 **pset 2 due**) Integration by parts, numerical integration.
Read: TB: 10.7, 10.9.

Part II (20 points)

Directions: Try each problem alone for 20 minutes. If, after this, you collaborate, you must write up your solutions independently.

Problem 1 (Class 8, 3 pts: 2,1)
a) Textbook 10.3/26.

b) Use this to integrate $\tan^5 x$.

Problem 2 (Class 8, 3 pts)
Textbook 10.4/31. (Hint: substitute for $x - b$.)

Problem 3 (Class 9, 4 pts: 2,2)
a) Find a formula for $\int \sec x \, dx$ by writing $\sec x = \frac{\cos x}{1-\sin^2 x}$ and making a substitution for $\sin x$.

b) Connvert your answer in part (a) to the more familiar formula,

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C,$$

by multiplying the top and bottom of the fraction by $1+\sin x$. (Remember: $\ln \sqrt{u} = \frac{1}{2} \ln u$.)

(continued)

Problem 4 (Class 9, 5 pts: 3,2)
A simple model for the spread of an infectious disease is $\frac{dx}{dt} = kx(1-x)$, where x is the fraction of the population with the disease, $1-x$ is the healthy fraction of the population and $k > 0$ is a constant of proportionality. (The model says the rate of spread is proportional to the number of contacts between healthy and sick individuals.)
a) This is a differential equation which can be solved by 'separating variables', i.e.

$$\frac{dx}{x(1-x)} = k\,dt.$$

Integrate both sides of this equation and solve for x as a function of t.

b) What happens in the long run?

Problem 5 (Class 8, 5 pts: 2,3)
a) Textbook 10.3/29.

b) Use this same technique to find a reduction formula for $\int \sec^n x\,dx$.