18.01A Problem Set 2b
(due Thurs., Sep. 27 (pset 2a due at same time))

Part I  (15 points)

TB = Simmons; SN = 18.01A Supplementary Notes (all have solutions) The problems marked ‘other’ are not to be handed in.

Class 8 (Tues., Sep. 25) Integration: substitution, trigonometric integrals, completing the square.
   Read: TB: 10.2, 10.3, 10.4.
   Hand in: 5B/7, 9, 13, 16; 5C/6, 9, 11; 5D/1, 2, 7, 10.
   Others: 5B/11; 5C/4, 5, 7.

   Read: TB: 10.6, SN: F
   Hand in: 5E/3, 5, 6, 10h (complete the square)
   Others: 5E/2, 8b, 9b, 10ac.

Class 10 (Thurs., Sep. 27  pset 2 due) Integration by parts, numerical integration.
   Read: TB: 10.7, 10.9.

Part II  (20 points)

Directions: Try each problem alone for 20 minutes. If, after this, you collaborate, you must write up your solutions independently.

Problem 1  (Class 8, 3 pts: 2,1)
b) Use this to integrate \( \tan^5 x \).

Problem 2  (Class 8, 3 pts)
Textbook 10.4/31. (Hint: substitute for \( x - b \).)

Problem 3  (Class 9, 4 pts: 2,2)
a) Find a formula for \( \int \sec x \, dx \) by writing \( \sec x = \frac{\cos x}{1-\sin^2 x} \) and making a substitution for \( \sin x \).
b) Convert your answer in part (a) to the more familiar formula,

\[
\int \sec x \, dx = \ln | \sec x + \tan x | + C,
\]

by multiplying the top and bottom of the fraction by \( 1+\sin x \). (Remember: \( \ln \sqrt{u} = \frac{1}{2} \ln u \).)

(continued)
Problem 4  (Class 9, 5 pts: 3.2)
A simple model for the spread of an infectious disease is \( \frac{dx}{dt} = kx(1 - x) \), where \( x \) is the fraction of the population with the disease, \( 1 - x \) is the healthy fraction of the population and \( k > 0 \) is a constant of proportionality. (The model says the rate of spread is proportional to the number of contacts between healthy and sick individuals.)
a) This is a differential equation which can be solved by 'separating variables', i.e.
\[
\frac{dx}{x(1 - x)} = k \, dt.
\]
Integrate both sides of this equation and solve for \( x \) as a function of \( t \).
b) What happens in the long run?

Problem 5  (Class 8, 5 pts: 2.3)
a) Textbook 10.3/29.
b) Use this same technique to find a reduction formula for \( \int \sec^n x \, dx \).