18.02A Problem Set 6

(due Thurs., Nov. 15)

This is due after exam 4, but the material for classes 21-23 is on the exam.

Part I (20 points)

TB = Simmons; SN = 18.02A Supplementary Notes (all have solutions)
The problems marked 'other' are not to be handed in.

Class 21 (Tues., Oct. 31) Parametric equations.
Read: TB: 17.1, 17.2 to middle of page 598, 18.4
Hand in: 1E/3bc, 4; 1I/2b, 3ad, 5
(For 1E/4 simply accept that $x + 2y - z = 2$ is the equation of a plane)
Others: 1E/1, 3a, 6; 1I/1, 3bc

Continuation: (Wed., Nov. 1) Discussion, review and catch up.

Class 22 (Thurs., Nov. 2 **pset 5 due**) Vector derivatives: velocity, curvature (2 hours).
Read: TB: 17.4
Hand in: 1J/1ac, 3, 4ab, 5, 6, 9abc, find the curvature of the helix in 1J/6.
Others: 1J/2, 7

Continuation: (Mon., Nov. 6) Continuation.

Class 23 (Tues., Nov. 7) Continuation, Kepler's second law.
Read: SN: K
Hand in: None
Others: None

Continuation: (Wed., Nov. 8) Discussion, review and catch up.

Exam: (Thurs., Nov. 9) **Exam 4** (covers 17-23)

Class 24 (Mon., Nov. 13) Functions of several variables, partial derivatives.
Read: TB: 19.1
Hand in: 2A/1abe
Others: 2A/1cd

Class 25 (Tues., Nov. 14) Tangent plane, level curves, contour surfaces.
Read: TB: 19.2 SN: TA
Hand in: 2A/2ae, 3b; 2B/1b, 4, 6
Others: 2A/1acd, 2bc, 2B/3, 7

(continued)

Part II (29 points)

Problem 1 (Class 21, 5 pts: 1,1,2,1)
a) Find the position vector of the trajectory of circular motion in the plane around the origin starting at $(-1, 0)$ going clockwise at unit speed.

b) Find the position vector of the trajectory of circular motion in the plane around the origin starting at $(10, 0)$ going counterclockwise at speed 60.

c) Repeat part (b) if the speed is now 60 rpm (with t measured in minutes).

d) Find the position and velocity vectors of the trajectory with initial position (at time $t = 0$) $\mathbf{r_0} = \mathbf{j}$, initial velocity $\mathbf{v_0} = -\mathbf{i}$, and acceleration $\mathbf{a}(t) = \cos t\,\mathbf{i} - \sin t\,\mathbf{j} + \mathbf{k}$.

Problem 2 (Class 21, 8 pts: 2,3,2,1 + 2 E.C.)
a) A jet takes off from (1,1,0) at time $t = 0$ and moves with constant velocity $\mathbf{v} = \langle -5, 0, 1\rangle$. In a flight simulator, the trajectory is displayed in the yz-plane as it would appear to an observer at the point (1,0,0). Find the formula (in the form $y = y(t)$, $z = z(t)$) for the trajectory on the screen. (*Worked example (1) may help with this.*)

b) Repeat part (a) if the jet takes off from (a, b, c) and has constant velocity $\langle \alpha, \beta, \gamma\rangle$.

c) Your answers in parts (a) and (b) should be along a straight line (this may not be obvious in part (b)). What happens as $t \to \infty$?

d) Draw several trajectories as they would appear on the screen.

Extra credit) Give a geometric explanation for your answer in parts (c) and (d)

Problem 3 (Class 21, 4 pts: 2,2)
a) Let $P = (1, 1, 1)$. Find the distance from P to the plane $x + y = 7$.

b) Let P be as in part (a). Find the distance from the point P to the line $t(1, 2, 2)$.

Problem 4 (Class 22: 5 pts: 3,1,1)
a) Find the radius of curvature, center of curvature and the unit tangent and normal vectors to the parabola $(x, y) = (at^2, 2at)$, where a is a constant.

b) Find the radius of curvature of $y = 2x + 3$ for general x.

c) Find the point of maximum curvature on the parabola $y = x^2$.

Problem 5 (Class 20: 3 pts)
Find the center of the unique circle through the three points $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 1)$.

Problem 6 (Classes 22, 4 pts: 2,2)
a) Define the cycloid and derive parametric equations for it.

b) Compute the arclength of one arch of the cycloid.