18.02a Problem Set 8

(due Thurs., Dec. 6)

Grading

You will get a grade for 18.02A first half, and a separate grade for 18.02A second half. The two will be averaged together to give you a grade for 18.02A. If you finish during the spring semester, this grade (if A. B or C)) will appear on your transcript. If you finish during IAP, it will not; only P (pass) will be recorded. If you get a D or F, nothing will appear on the transcript regardless of when you finish, provided you are still a freshman.

Part I (15 points)

TB = textbook; SN = Supplementary Notes (all have solutions)

Class 30 (Tues., Nov 27) Non-independent variables, chain rule.
Read: TB: 19.6, SN: N.1-N.3
Hand in: 2J/1a, 2, 3a, 4a, 5a, 6, 7
Others: 2J/any others

Continuation: (Wed., Nov. 28) Discussion, review and catch up.

Class 31 (Thurs., Nov 29, **pset 7 due**) Double and iterated integrals.
Read: TB: 20.1, 20.2
Hand in: 3A/1ad, 2b, 3b, 4c. 5ac
Others: 3A/1bc, 2a, 3a

Class 32 (Mon., Dec 3) Polar coordinates, double integrals in polar coordinates.
Read: TB: 16.1, 20.4
Hand in: 3B/1ac, 2cd, 3bc
Others: 3B/1d, 2ab

The following are **not to be handed in**, but are on the *midterm*

Class 33 (Tues., Dec 4) Change of variable.
Read: SN: CV
Hand in: 3D/1, 2, 3, 4
Others: None

Class 34 (Wed., Dec 5) Applications of double integration.
Read: TB: 20.3
Hand in: 3C/1ac, 2, 4, 5
Others: 3C/1b

Review: Dec. 6-12

Midterm: **Tuesday Dec. 18 9-11 AM**

(continued)

Part II (22 points)

Problem 1 (Class 30, 7 pts:1,2,2,2)
Using the usual rectangular and polar coordinates, let w be the area of the right triangle in the first quadrant having its vertices at $(0,0)$, $(x,0)$ and (x,y). Using the equation expressing w in terms of x and y and the equations expressing y in terms of x and θ, calculate the two partial derivatives $\left(\frac{\partial w}{\partial x}\right)_\theta$ and $\left(\frac{\partial w}{\partial \theta}\right)_x$ in three different ways.

a) Directly, by first expressing w in terms of the independent variables x and θ.

b) By using the chain rule – for example $\left(\frac{\partial w}{\partial x}\right)_\theta = w_x\left(\frac{\partial x}{\partial x}\right)_\theta + w_y\left(\frac{\partial y}{\partial x}\right)_\theta$, where w_x and w_y are the formal partial derivatives.

c) By using differentials.

d) Using the triangle picture and geometric intuition, estimate $\left(\frac{\Delta w}{\Delta x}\right)_\theta$ and $\left(\frac{\Delta w}{\Delta \theta}\right)_x$ and show they agree with the two corresponding partial derivatives.

Problem 2 (Class 30, 4 pts)
Let $u = f(x,t)$ and $r = x - ct$, $\quad s = x + ct$. Show that $\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial r \partial s}$.

Problem 3 (Class 30, 3 pts)
In thermodynamics any two of the variables p, T, V, U, H, S can be taken as independent with the remaining variables being dependent. If p and T are independent we have the law

$$\left(\frac{\partial U}{\partial p}\right)_T + T\left(\frac{\partial V}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial p}\right)_T = 0.$$

Give the equation for this law when we take U and V to be the independent variables.

Problem 4 (Class 31, 3 pts)
A rectangular prism is made by taking a long piece of wood with a rectangular cross-section, sawing off one end perpendicularly to the four sides, and the other end at an arbitrary angle (so that the four long edges have four different lengths).

By using double integration, show that the volume of the prism is the product of its cross-sectional area and the average of the lengths of its four long edges.

Hint: place the prism so that one long edge lies along the z-axis and the perpendicular end lies in the first quadrant of the xy-plane. So that everyone will use the same notation, let

a = length of edge lying along the x-axis;
b = length of edge lying along the y-axis;
$z = Ax + By + C$ = the equation of the plane forming the slanted top of the prism.

Problem 5 (Class 31, 3 pts)
Evaluate by changing the order of integration: $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dy\, dx$.

Problem 6 (Class 31, 2 pts)
Explain why there must be a 'typo' in the following expression: $\int_1^2 \int_1^x \frac{1}{xy}\, dx\, dy$.