Photoelectric determination of Planck’s constant

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Planck’s constant “h”, the ratio of a photon’s energy to its frequency, is determined by irradiation of a potassium photocell with the principal lines of mercury. The retarding voltage required to negate the photocurrent is the principal mechanism for the calculation of “h”.

1. INTRODUCTION

The photoelectric effect, first discovered and investigated in 1895 by Heinrich Hertz, was one of the physical phenomena that inspired Einstein to publish his 1905 paper claiming that the energy delivered by a photon to an atom with excitable electrons was proportional to the frequency of the incident light. In 1912, Richardson and Compton detailed the photoelectric effect to a degree sufficient to support Einstein’s quantum picture of light.

Nowadays, the supremacy of the quantum model over the classical one is easy to experimentally verify in the laboratory through the same method used by Compton and Richardson - by opposing the photocurrent with an opposing voltage and plotting the data in such a way as to yield h.

2. THEORY

It is well-known that radiation striking a metal surface causes current to flow. In the quantum model, a photon with incoming energy \( h\nu \) striking a surface with work function \( \phi \) liberates an electron with energy \( h\nu - \phi \) to flow in a circuit.

By providing a retarding voltage \( V_r \) in the direction opposite to the current, \( eV_r \) of the electron’s energy is negated. The resultant current is proportional to the energy of a liberated electron and to the intensity of the incident radiation. That is, a current

\[
I \sim h\nu - \phi - eV_r
\]  

is observed. By increasing the voltage, the current can be made to decrease to the point when the potential entirely offsets the energy of each electron. We wish to measure this point at which the retarding voltage entirely counteracts the current caused by the photoelectric effect: \( eV_r = h\nu \).

As the voltage increases, the current does not decrease to zero - it becomes a constant negative, due to the different potential between the anode and the cathode. This is called the reverse current \( I_{V=\infty} \), and is proportional to the work function.

\[
I_{V=\infty} \sim -\phi
\]  

There are qualitative and quantitative consequences of the quantum model we wish to demonstrate. Of the former, that the relationship of incident photon energy to frequency is linear, and that there is some frequency below which no photocurrent flows. Of the latter, we wish to plot the retarding voltage \( V_r \) necessary to overcome the current caused by the photoelectric effect across different frequencies \( \nu \), and can measure the ratio \( \frac{V_r}{\nu} \), and can measure the ratio \( \frac{V_r}{\nu} = \frac{h}{e} \).

Furthermore, the intercept of this line with the \( V_r = 0 \) is the work function \( \phi \).

3. EXPERIMENT

3.1. Setup

The experimental setup consisted of a mercury lamp, a photocell which exhibits the photoelectric effect, several narrow-band filters, a retarding power supply wired to the photocell, and an electrometer to measure the induced current, as shown in Figure 1.

A Thermo-Oriel Model # 65130 high-powered spectral calibration mercury lamp, which was the source of radiation for the experiment, had been positioned to point at a Leybold photocell with a potassium photocathode. A filter wheel with five band-pass filters had been positioned in between the radiation source and the photocell. The

FIG. 1: Experimental setup. Figure modified from [1].
filters corresponded to principal lines of mercury: 3650 Å, 4047 Å, 4358 Å, 5461 Å, 5770 Å all with tolerance ±20 Å.

An Agilent variable DC Power Supply was connected to the photocell to provide a retarding voltage in opposition to the photocurrent, and a Keithley electrometer to measure the resulting current. After the lamp had been turned on, the orientation and height of the photodiode was adjusted until the current reached a stable maximum. We attempted to ensure that the positive output of the power supply, as well as the rest of the apparatus, was properly grounded.

3.2. Procedure

For each filtered wavelength, we slowly increased the retarding voltage while recording the measured current in the photocell. We repeated this procedure four times, making additional measurements around the voltage that makes the current falls to zero. The power supply is accurate within an interval of ±0.01 V, and we were careful to record the current to the number of significant digits, as appropriate.

4. DATA AND ANALYSIS

A plot of current versus retarding voltage for each wavelength is shown in Figure 2. Two features are salient. First, when the amperage is low, the plots become curved. We posit that this is a consequence of background radiation, since the principal lines of mercury are too sharp and intensive for this to be a consequence of their spread through the filter.

Second, the observed current asymptotically reaches a value lower than zero. As expressed earlier, this is a consequence of the fact that the anode is at a higher potential than the cathode. We call this the reverse current.

Furthermore, the λ = 5461 Å, 5770 Å plots look significantly smaller than those of the other wavelengths. Using established values of Planck’s constant, we calculate the energy of the incident λ = 5770 Å radiation

\[
E_{5770} = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^8)}{(1.60 \times 10^{-19})(5770 \times 10^{-10})} = 2.15 \text{eV} \quad (3)
\]

which is not enough to liberate an electron from the φ ≃ 2.3 potassium cathode [2]. This, along with the dramatically decreased intensity of the observed current, leads us to conclude that the data collected for the 5770 Å wavelength is the result of ambient light, and we discount it.

A similar calculation with the 5461 Å line leads to \( E_{5461} = 2.27 \text{eV} \), which is close enough to the work function of the metal that a 1% error in the stated work function (if φ = 2.27) would cause a photocurrent. In fact, a significant current is observed which is larger than the current caused just by ambient light. Despite its problematic nature, we decide to include this data set in our analysis and calculation of \( h \).

We use two methods to determine the retarding voltage, and use the retarding voltage to calculate \( h \) and, later, φ.
4.1. Naive determination of the retarding voltage

A naive method is to use the value of the voltage that produces a current closest to zero. For the wavelengths 3650 Å, 4047 Å, 4358 Å, and 5461 Å, the voltages are 1.15 V, 0.93 V, 0.79 V, and 0.37 V, respectively. We plot voltage against frequency in Figure 3, and find that a least-squares linear fit has a slope of \( \frac{h}{e} = 2.79 \times 10^{-15} \).

We now make a rigorous analysis of the error inherent in this calculation, using formulae from Bevington and Robinson [3].

The maximum error in the measurement of the voltage is given by

\[
\frac{\sigma_V}{V} = \frac{0.01}{0.37} = 0.027
\]

The maximum error in the frequency is with the 3650 Å line, which is given by

\[
\frac{\sigma_\nu}{\nu} = -\frac{\sigma_\lambda}{\lambda} = -\frac{20\text{ Å}}{3650\text{ Å}} = 5.48 \times 10^{-3}
\]

We use a conservative propagation of error and say that the error in the slope of the calculated line is directly proportional to the largest error in the \( V \), and inversely proportional to the largest error in the \( \nu \).

We assume that the errors in \( \nu \) and \( V \) are not correlated, and throw out the covariance terms in the error expression, which simplifies to:

\[
\frac{\sigma_h}{h} = \sqrt{\left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_\nu}{\nu}\right)^2} = 0.0276
\]

This gives a value of

\[
h = 4.47 \pm 0.12 \times 10^{-34} \frac{m^2\text{kg}}{s}
\]

4.2. Alternate determination of the retarding voltage

We use an alternate method as described in Melissinos [4]. For each wavelength, we draw a straight line tangent to the data before deviations are low that models the effect of the retarding voltage on the observed photocurrent, and a horizontal line that models the current in the circuit after the retarding voltage has entirely cancelled out the photocurrent. The intersection of these two lines is another way to calculate \( V_r \) - the voltage at which the photocurrent is just negated.

Our methodology for deriving these lines is as follows. For the tangent lines, we fit a line to points in the top half of the data set, where there seems to be little deviation. For the horizontal lines, we simply average the observed photocurrent in the three right-most data points. The intersection of these lines is another way to calculate \( V_r \) - the voltage at which the photocurrent is just negated.

The intersection of these lines is shown in Figure 4, and from these we calculate the retarding voltages of .86 V, .71 V, .58 V, and .27 V, for the wavelengths 3650 Å, 4047 Å, 4358 Å, and 5461 Å, respectively. Plotting these in Figure 3, and fitting these with a least-squares line, we get a slope of \( \frac{h}{e} = 2.2 \times 10^{-15} \).

We again make an analysis of the error, beginning with...
the relative error in the calculation of the retarding voltage. This is significantly more involved than the same calculation under the naive method.

The retarding voltage is given by the intersection of a fitted line with an averaged value of $I_{V=\infty}$. The error in the voltage-value of the intersection has error contributions from the $V$ and $I$ of the datapoints used to fit the tangent line, and random relative error from $I_{V=\infty}$.

We again throw out the covariances, and simplify the error expression to:

$$\frac{\sigma V_r}{V_r} = \sqrt{\left(\frac{\sigma V}{\Delta V}\right)^2 + \left(\frac{\sigma I}{\Delta I}\right)^2 + \left(\frac{\sigma I_{V=\infty}}{I_{V=\infty}}\right)^2}$$ \hspace{1cm} (8)

Conservatively, the relative errors $\frac{\sigma V}{\Delta V}$ and $\frac{\sigma I}{\Delta I}$ are given by the largest relative errors in $V$ and $I$, respectively, for each wavelength. The relative error $\frac{\sigma I_{V=\infty}}{I_{V=\infty}}$ can be calculated by looking at the largest relative error in the points used to fit $I_{V=\infty}$ for each wavelength.

Using this method, we calculate values for $\frac{\sigma V_r}{V_r}$ of .059, .069, .068, and .13 for the 3650 Å, 4047 Å, 4358 Å, and 5461 Å lines. Continuing with our conservative style of error analysis, we choose the largest of these as the uncertainty in the $V_r$ data.

The uncertainty in the frequency measurement is given by (5). Following the formula in (6), the relative error in $h$ is

$$\frac{\sigma h}{h} = \sqrt{\left(\frac{\sigma V_r}{V_r}\right)^2 + \left(\frac{\sigma \nu}{\nu}\right)^2} \approx .13$$ \hspace{1cm} (9)

This gives a value of

$$h = 3.5 \pm .5 \times 10^{-34} \frac{m^2 kg}{s}$$ \hspace{1cm} (10)

4.3. Determination of the Work Function

The $y$-intercept in the plots of $h$ are the calculated values of the work function $\phi$ because it is this deficit in energy that must be overcome before there is a positive energy associated with a given frequency. The relative error in $\phi$ is the relative error in the slope ($h/e$) used to find the intercept.

From the naive method,

$$\phi = 1.14 \pm .03 \text{ eV}$$ \hspace{1cm} (11)

From the alternate method,

$$\phi = 0.93 \pm 0.12 \text{ eV}$$ \hspace{1cm} (12)

5. CONCLUSIONS

The calculated values of Planck’s constant are

$$h_1 = 4.47 \pm 0.12 \times 10^{-34} \frac{m^2 kg}{s}, \quad h_2 = 3.5 \pm 0.5 \times 10^{-34} \frac{m^2 kg}{s}$$ \hspace{1cm} (13)

And the values of the work function are

$$\phi_1 = 1.14 \pm 0.03 \text{ eV}, \quad \phi_2 = 0.93 \pm 0.12 \text{ eV}$$ \hspace{1cm} (14)

We have successfully shown, albeit qualitatively, that the ratio of the energy of incident radiation to its frequency is a constant, and that there are low frequencies which do not produce a photocurrent. However, we have failed on the quantitative front.

The values of “$h$” reported here clearly differ from the established value; the first calculation by eighteen standard deviations, and the second calculation by six. The values of $\phi$ are not consistent with each other or the given value in [1].

Throughout the course of the experiment, we have tried to ensure that no systemic sources of error were present. We made sure our equipment was properly grounded, checked the readings on the power supply and electrometer with a hand-held voltmeter, and ensured that background noise was not significantly affecting the observed data.

We can provide no conclusive explanation for our results.

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