Spinning Black Hole Energetics

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We discuss black hole energy processes that could serve as the mechanism for relativistic jets. The energy-extraction process of Roger Penrose is presented. The existence of the negative energy region is derived from the extreme Kerr metric. Theoretical limits on the efficiency of the process are discussed. An analogous process for a Kerr-Newman hole is presented. The Blandford-Znajek process is presented with a review of recent literature in this field.

1. INTRODUCTION

Astrophysicists are faced with the difficult task of finding explanations for certain high-energy cosmic events, such as the many light-years-long relativistic jets present around active galactic nuclei and black hole systems, gamma ray bursts and quasars. When evidence that a spinning black hole could reside at the center of these locations came to light, astrophysicists struggled to find an explanation. An obvious candidate to power these events is the energy locked inside the spin of the black hole. Over the course of the past few decades, various schemes to utilize this energy in a natural process have been proposed.

2. KERR METRIC

It is known that a black hole can be entirely described through the use of only three parameters - the mass M, angular momentum J (often expressed in terms of units of mass $a = J/M$) and residual charge Q. Though the mass of observed black holes ranges over several orders of magnitude, it is believed that a tends to be nonzero and significant, and that the residual charge Q is nearly identically zero. Thus, we seek a metric for a spinning black hole with no charge. This is known as the *Kerr metric*, and within the equitorial plane in Boyer-Lindquist coordinates, it has the following form.

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 + \frac{4Ma}{r} dt d\phi - \frac{dr^2}{1 - \frac{2M}{r} + \frac{a^2}{r^2}} - \left(1 + \frac{a^2}{r^2} + \frac{2Ma^2}{r^3}\right) r^2 d\phi^2 \quad (1)$$

In some of the analysis in this paper, we will consider only maximally spinning black holes ($a = 1$), and introduce a *reduced circumference* R for convenience. The metric in this case is

$$d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 + \frac{4M^2}{r} dt d\phi - \frac{dr^2}{\left(1 - \frac{M}{r}\right)^2} - R^2 d\phi^2 \quad (2)$$

$$R^2 = r^2 + M^2 + \frac{2M}{r^3} \quad (3)$$

A spinning black hole has several noteworthy properties. First, part of the measured mass M is actually stored as rotational energy, and is in theory available for extraction, since it is not mass that has passed the event horizon. Second, there exist several critical values of r in the metric. Before the normal *event horizon*, there exists a *static limit* - a surface past which not even light can resist being dragged along with the black hole.

3. PENROSE PROCESS

In 1969, Roger Penrose, a post-doctoral student at Cambridge, published a paper on the role of general relativity in gravitational collapse[2]. In it, he presented a mechanism by which the rotational energy of a black hole might be extracted. Figure 1 is an illustration found in Penrose's paper of such a possible process.

Penrose noted that while the process was not "practical", there might yet be some "indirect relevance to astrophysical situations". Crucial to his argument is the existence of a region of negative total energy in the vicinity of a spinning black hole. Below, we investigate this region, present the process, and investigate its efficiency.

3.1. Negative Energy Region

The derivation below is based on the one given in Taylor et. al. [1]. The Euler-Lagrange equations of motion (derived from Equation 1) for a particle in the vicinity of an extreme Kerr hole indicate the conserved quantity which we denote *total energy* E/m.

$$\frac{E}{m} = \left(1 - \frac{2M}{r}\right) \frac{dt}{d\tau} + \frac{2M^2}{r} \frac{d\phi}{d\tau} \quad (4)$$

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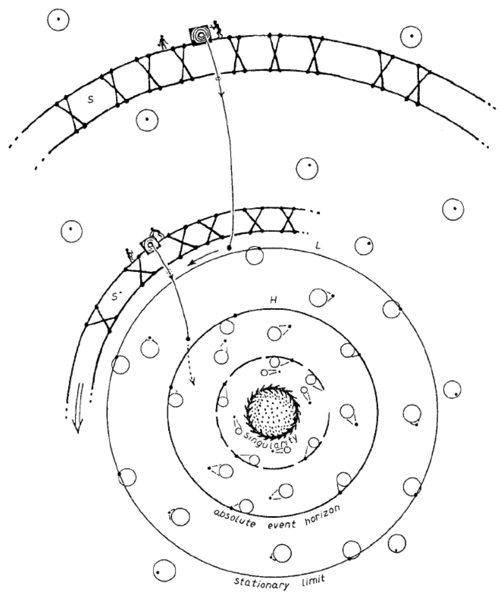

FIG. 1: Penrose's original illustration, showing energy extraction. The energy of the box when it crosses the inner horizon is negative as measured from S. Penrose later comments that a ballistic method is also possible.

We are interested in regions of parameter space over which the energy is negative. Rearranging Equation 4, and multiplying through by R to recover the expression for tangential velocity, we can derive the formal condition for negative energy.

$$R\frac{d\phi}{dt} < R\frac{2M-r}{2M^2} \tag{5}$$

How does this restriction relate to the tangential velocity of light, which sets the speed limit for any matter moving with or against the rotation of the hole, at different values of r? Setting $d\tau = dr = 0$ for light, Equation 3 becomes a quadratic in $d\phi/dt$. There are two roots, which after algebraic manipulation are

$$R\frac{d\phi}{dt} = \frac{2M^2}{rR} \pm \frac{r-M}{R} \tag{6}$$

Note that for $r < M$, both values of tangential velocity are *positive*: not even light can escape moving in the direction of rotation when in the ergoregion. Plotting Equation 5 and 6 in Figure 2, we see that negative energy configurations do in fact exist, but only past the static limit. This also shows explains why no analogue of this process exists in the Schwarzschild hole: configurations with $r < 2M$ are already inside the horizon.

3.2. A "Practical Process"

We now show that regions of negative energy can be used to extract energy from a black hole. Boiled down

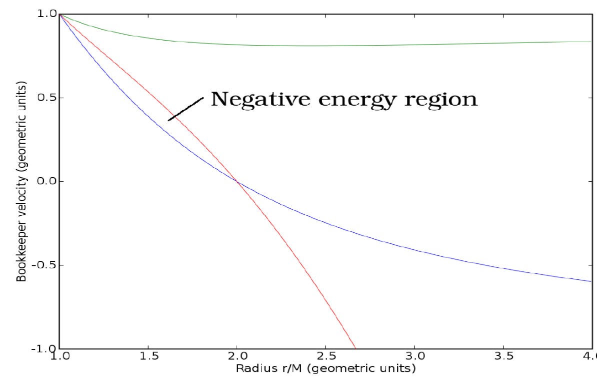

FIG. 2: Plot of limits on the tangential velocity of a particle. The green and blue lines are the (upper and lower) speed of light for a co-rotating and contra-rotating trajectory, respectively. The red line is the upper limit required for a negative energy configuration. Only at $r < 2M$ are the conditions simultaneously realizable. Taken from [2].

to its essentials, the Penrose Process is simple:

1. Two particles of equal mass are at rest at infinity.

2. Both are moved into a region of negative energy in vicinity of the black hole.

3. One particle is captured, decreasing the mass of the hole.

4. The other particle escapes to infinity, with more energy than the initial energy of both particles.

Evidently, the black hole has imparted some of its rotational energy to the particle.

At this point, the problem becomes optimizing the parameters of a generic orbit to find the maximum energy-out to energy-in ratio. In 1983, Chandrasekhar [3] showed that this efficiency was about 20.7%. We make a similar calculation, using the more idealized process presented in [1].

Assume that the two particles of mass m are made of matter and antimatter, are at rest at infinity ($E = 2m$) and are sent by some agent into the vicinity of a black hole. In the region of negative energy, let them annihilate each other and convert all of their mass into radiation. Half of the radiation is sent into the black hole, and the other half is sent so that it escapes out towards infinity. The energy gained by the agent at infinity is equal to the negative of the energy of the radiation captured by the black hole.

To calculate this energy, we turn to the expression for the energy of a particle moving along a rotating ring in [1].

$$E = \left(\frac{r-M}{R} + \frac{2M^2}{rR}v_{ring}\right)E_{ring} \tag{7}$$

Since the "particle" in question is light, we set $|v_{ring}| = 1$ and $E_{ring} = m$. To obtain the minimal energy, we choose the negative sign for v_{ring} to represent light moving against the rotation of the hole.

Equation 7 is maximal just outside the horizon at $r = M$. Here, the energy of the inward falling radiation is $-m$, and the energy-at-infinity of the escaping radiation is $3m$. This idealized process thus has a theoretical efficiency limit of 50%. We will return to this.

4. KERR-NEWMAN PENROSE PROCESS

In our analysis above, we have freely used two of the parameters available to us to describe the black hole - spin J and mass M. In 1985, Bhat, et. al., a group of researchers at the University of Pune in India, studied how the negative energy region was affected by the presence of a residual net charge Q[4].

Justification of the relevance of this analysis is not trivial, since astrophysical objects are not in general known to carry significant residual charge. However, because of the scale differences at which electromagnetic and gravitational effects become important, even a relatively small residual charge could have a dramatic effect. Specifically, Bhat et. al. set $Q/M << 1$, assumed it could be left out of the metric of the black hole, but that added the Lorentz force to the equations of motion.

In this way, the title of this section is misleading: the Kerr metric is the one actually used, and an electrostatic term is artificially added to the expression for energy. In particular, this can be realized by adding the electromagnetic energy potential $V(r) = -qQ/r$ to the energy quantity for a particle with charge q. Now we examine the effect this term has region of negative energy.

4.1. Effect on Negative Energy Region

Consider an incoming particle at radial coordinate r with reduced angular momentum $l = L/m$ and product of the charge $\lambda = qQ$. Figure 3 shows how the energy curve E/m as a function of distance varies with these parameters. There are several salient features in these plots.

Consider the left plot. Varying the angular momentum certainly affects the shape of the curve, but it takes extreme values of l to move the r-value at which the energy is negative to any appreciable extent. The right plot tells a different story. A modestly charged particle can change the curve dramatically, in two ways: the extremal value of negative energy becomes very large, and the negative energy region occurs at higher values of r - in some cases, easily outside the static limit.

Despite these differences, the mechanism here is still fundamentally the ballistic one of Penrose. Surprisingly, Bhat et. al. show that theoretical trajectories for two particles exist that give *any* desired value for the efficiency of the process. Although in practice, the efficiency would be limited by such considerations as the typical l and λ values of incoming particles and the residual charge Q, this lack of a limitation is an important improvement over the standard Penrose Process.

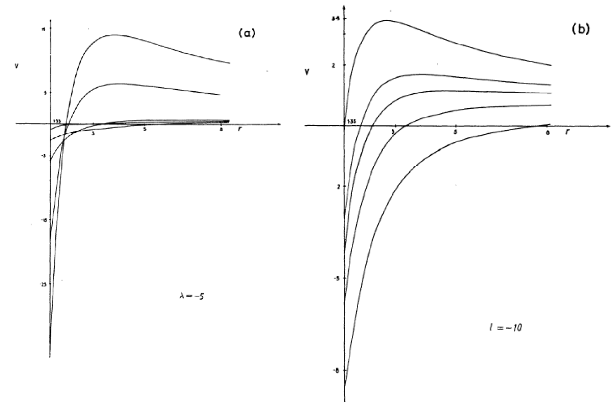

FIG. 3: Sample effective potentials for a charged Kerr hole. In both cases $J = 0.8M$ and $Q = 0.5M$. The vertical axis is drawn through the r-value of the horizon. On the left, $l = -100, -50, -10, 0, 5$ while $\lambda = -5$. On the right, $\lambda = -10, -5, -2, 0, 5$ while $l = -10$. Taken from [4].

5. BLANDFORD-ZNAJEK PROCESS

Clearly the addition of electromagnetic phenomena has a powerful effect. In 1977, Blandford and Znajek, a post-doc and graduate student at Cambridge, devised another practical energy extraction process [5]. In their description, the black hole is electrically neutral, but electromagnetic forces still play an important role. Observational evidence shows that many spinning black holes exhibit a rotating *accretion disk* in their equatorial plane. The density of matter in the disk rises sharply close to the horizon, and matter that has magnetized over time will create strong magnetic field lines in the vicinity.

Thorne [6] argues that while the black hole slowly swallows up the accretion disk, the field lines of the magnetized matter remain, threaded through the horizon. Price's theorem that this asymmetry be radiated away does not apply here, as the hole is not in isolation: it is surrounded by a massive and complex accretion disk. The frame of the field lines is dragged along with the rotation of the black hole. These rotating field lines induce an electromagnetic force that accelerates charged plasma at relativistic speeds along the axis of rotation. Due to the radial component of the field, the particle spirals as it leaves.

Blandford and Znajek showed that this process was analogous to a simple circuit in which the hole functioned as the voltage supply, the field lines as the wires and the plasma as the load. This is vividly illustrated in Fig-

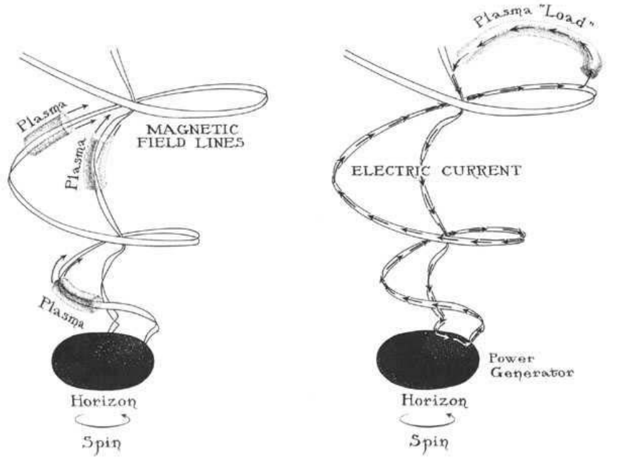

FIG. 4: Illustration of the Blandford-Znajek Process and circuit analogue. Taken from [6].

ure 4. In fact, relativistic jets believed to be caused by some electromagnetic process are sometimes known as "Poynting-dominated" jets, to indicate that the axis of rotation is a major direction of the flow of electromagnetic flux.

5.1. Discussion

We have glossed over some of the finer details. There is some debate about the theoretical values of such quantities as the strength of the field from the accretion disk, the ratio of the height to the length of the disk, its mass, density and viscosity, the Lorentz factor of emitted plasma, which types of black holes or neutron stars could support such a process, and many others. Experimental investigation is still extremely difficult. Thus, these topics are important to both experimentalists attempting to confirm the Blandford-Znajek mechanism and to theoreticians attempting to calculate its ability to cause relativistic jets.

Recently, much literature has been published on whether or not the Blandford-Znajek mechanism can be responsible for the formation of relativistic jets and the physical details required for such a process. We list a few references here. Komissarov [7] demonstrated that rotating magnetic field lines cause jet-like phenomena in plasma for a wide range of a-values. Koide et. al. [8] and Armitage et. al. [9] conduct numerical investigations of the formation of relativistic jets using different models, both with positive results. Lee et. al. [10] argue (and in their paper published a year before this one, as well) that the Blandford-Znajek mechanism is appropriate for powering gamma ray bursts, as well.

There are, of course, detractors and not all of the evidence is yet available. The full scope and suitability of this mechanism is still being vigorously investigated.

6. CONCLUSION

We have presented three energy extraction mechanisms of increasing complexity. All fundamentally rely on the spin of the black hole to power the process, though in different ways. The Penrose Process has fallen out of favor as a possible mechanism, for several reasons. It requires the breakup (spontaneous fission) of particles at relativistic speeds in opposite directions, which is not known to happen. Additionally, it operates at a maximum efficiency of about 20%. This is not enough to accelerate a particle at rest to speeds past $.4c$ or so. The Kerr-Newman Penrose Process, though more efficient, is on shakier theoretical ground.

Only the Blandford-Znajek Process seems to be both realistic and efficient. It provides a believable mechanism for the generation of relativistic jets, the early observational evidence is amicable, and there is great hope that the BZ Process will go on to explain one of the great mysteries of the cosmos.

This paper was written for a collaborative class project on spinning black holes. For an analysis of possible orbits around a Kerr hole, consult the work of William Throwe. For an in-depth discussion of the static limit and inner/outer horizons, consult the work of Matias Lavista.

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