NSF GRANT # 0423484
NSF PROGRAM NAME: Design, Manufacturing, and Innovation

Analysis of Recycling Systems

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Abstract: This paper outlines past and future work on the topic of recycling systems. This project focuses on the performance of recycling systems from a range of perspectives. The recyclability of products, separation efficiencies for recycling processes and systems built from these processes, and economic factors contributing to the success of recycling enterprises are all considered. The goal of this project is to develop a basic understanding of the factors effecting the performance of recycling systems, with the eventual ambition of developing techniques for predictive analysis of these systems. These analysis techniques will allow us to evaluate the economic, ecological, and energy impact of recycling systems. This increased understanding will help guide the design of recycling systems.

1. Introduction: In the United States and other industrialized nations, the rate of material consumption is unsustainable. Recycling is an important factor in any strategy for reducing the rate of material consumption. In-depth analysis of the performance of recycling systems can provide guidance for the development of new recycling systems and the improvement of old systems.

This project attempts to analyze the performance of recycling systems on several different levels. Research into material mixing and its effect on product recyclability is presented. Research into the separation efficiency of compound separation processes with recycled streams is also presented. A model for comparing processes with different separation efficiencies is presented, as well as a model for estimating the market value of recycled materials.

2. A Measure of Material Mixing: Recyclable product choice is an important factor in the success of a recycling system. Two measures are necessary to assess the material recycling potential of a product, the value of the materials present in the product, and the material mixing in the product. In 1994, Allen and Behamensh proposed that an approach similar to Sherwood's characterization of the relationship between the price of a material and its concentration in its feed stream [1, 2]. The Sherwood plot shown in Figure 1 encompasses a variety of materials in dilute solution including metals, biological and biomedical materials, and pollutants.

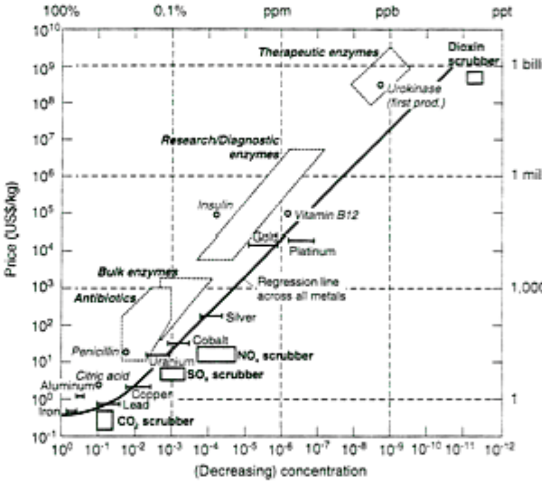

Figure 1: Sherwood plot of market value of a target material and its feed concentration. From Grübler [3].

Applying this relationship to the recyclability of products presents a problem because a typical product consists of several desirable materials in relatively high concentrations as opposed to a single desirable material in a dilute solution. Another relevant characteristic of these recycled product systems is the typical form of the separation processes. Most recycling systems can be modeled as a series of binary separation steps, with multiple collected material outputs.

An alternate cost scaling heuristic that takes these two factors into account is proposed here. The core features of this proposal are that: 1) the processing costs scale with the measure of complexity of the separation system, 2) all material outputs of the separation system, including the waste stream, are targeted, 3) all separation processes are equal in cost, and 4) only material recycling, as opposed to component recycling, is considered.

Several alternatives are possible for scaling the complexity of the separation system. One simple approach is scaling by the number of target materials, *M*, or minimum number of separation steps, *M-1*. Alternatively, an approach based on the thermodynamic work of mixing will also produce useful results. A third approach based on information theory is presented here. This information theory-based approach encompasses both the material counting approach and the thermodynamic approach.

The basis for this metric, first proposed by Dahmus and Gutowski, is information theory [4, 5]. A situation analogous to the complexity of recycling systems arises in the case of interpreting code words. Both systems can be modeled as decision trees, with each step discerning a bit, in the case of the code word, or separating materials, in the case of a recycling system. A useful measure of the effort required to process the word or material stream is to find the path length of the separation steps required to determine the code word or separate out the material, and create a weighted average for the path length based on the probability of the code phrase appearing or initial concentration of the material. Shannon developed a measure of uncertainty, *H*, which is a lower bound for the average word length, and thus a good bounding cost metric for the effort required to identify a code word [6, 7]. Here a parallel measure is presented for recycling systems.

The requirements placed on the measure of material mixing, *H*, described in terms parallel to Shannon's, are that:

1. H should be continuous in the material concentrations, c_i.
2. If all c_i are equal, that is, $c_i = 1/M$, where *M* is the number of materials, then *H* should be a monotonically increasing function of *M*.
3. *H* should be additive. That is, if a mixture can be broken down into a mixture of mixtures, then the final *H* should be the weighted sum of the individual values of *H*.

Shannon showed that the only *H* satisfying these three assumptions is of the form

$$H = -K \sum_{i=1}^{M} c_i \log c_i \qquad (1)$$

where K is a constant
M is the number of materials
c_i is the concentration of material *i*.

For simplicity, *K* can be assigned to be 1. The assumptions about the similar measure in information theory can be directly applied to this measure of mixing for recycling systems [4]. One important assumption is that each end result is a unique code word or material. The result is that the average number of separation steps for the process is greater than *H*, the measure of material mixing.

3. Material Mixing and Product Recyclability: With a measure of material mixing in hand, the profitability requirement for product recycling is

$$\sum_{i=1}^{M} m_i k_i > H k_b \qquad (2)$$

where m_i is the mass of material I (kg)
k_i is the value of material ($ per kg)
k_b is the processing cost per bit ($ per bit)
H is the measure of material mixing (bits).

The recycling rates for 20 different products with a variety of material compositions and values were compared to examine the relationship between complexity and recycling rate [5]. Material counting was based upon identifying all valuable recycled materials which could be separated from the product. The material values were obtained from market data on recycled materials, as reported by the RecycleNet Corporation [8], and estimated from market data on virgin materials, as priced on the New York spot market [9]. The amounts of the materials and their concentrations were obtained from published bills of materials.

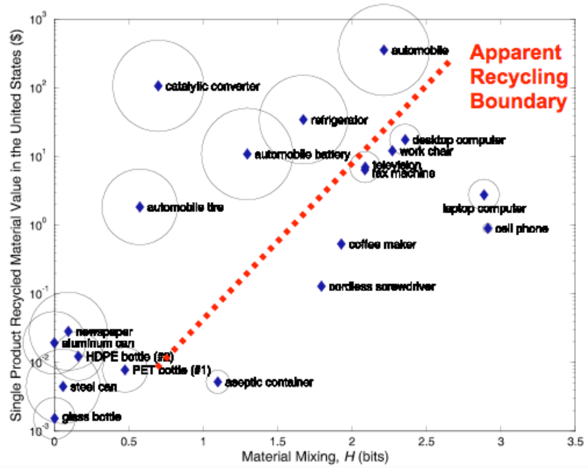

Figure 2: Plot of single product recycled material values, material mixing, and recycling rates for 20 products in the U.S. From Dahmus and Gutowski [5].

The graph in Figure 2 plots the material value contained in a product against the material mixing parameter for the product, H. The circles around each product point represent the recycling rate for the product. Products with no circle have a recycling rate of essentially zero.

The recycling trend is very apparent in the higher-value portion of the graph. Products in the upper left portion of the graph with low H and high material value are frequently recycled, while products with higher H and lower material value are not recycled. The divide between these products is represented by the *Apparent Recycling Boundary*. In the lower left corner of the graph, the division is less clear. The products shown in that corner the graph, with values of H below about 0.5, have a mix of recycling rates (20-70%). With the range of recycling rates for these products, extending the Apparent Recycling Boundary into the region may ignore other important factors.

While improvements in recycling technologies are altering the Apparent Recycling Boundary, shifting it toward the high H, low material value product region, current design trends are also shifting in this direction, making products less recyclable [10]. Figure 3 shows this trend for three different products: refrigerators, automobiles, and computers. These products are all becoming more complex over time. A notable exception to this trend is sport utility vehicles (SUVs). This is due in part to the increase in size and weight of materially simple systems such as the body panels and frame in SUVs, and in part to the overall increase in material content.

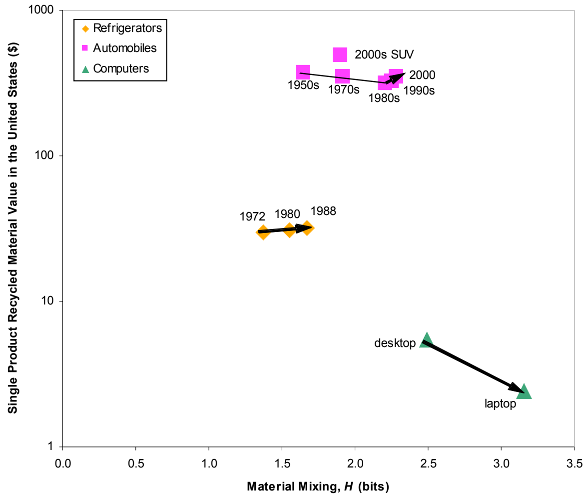

Figure 3: Product recyclability trends. From Dahmus and Gutowski [10].

4. Bayesian Material Separation Model: The Bayesian material separation model provides a simple characterization for the separation efficiencies of a recycling process. This model assumes a binary mixture of a target material, A, and a non-target material stream, A'. The separation efficiency of the target material is r, that is, the probability of correctly identifying and capturing the target material is r. Similarly, the separation efficiency for the non-target material, or probability of correctly rejecting the non-target material, is q. The final assumption of this model is that the separation efficiencies do not vary based on the concentration ratio of the two materials. The following figure represents a single Bayesian separation process.

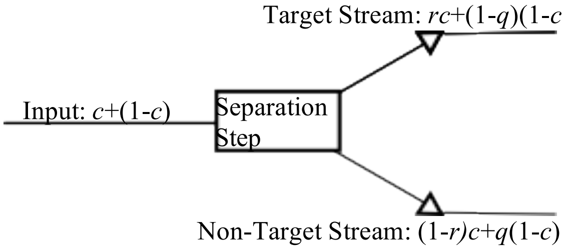

Figure 4: A single Bayesian separation.

The target output stream consists of the correctly identified target material, rc, and the incorrectly identified waste material, $(1-q)(1-c)$, while the non-target stream consists of the incorrectly identified target material, $(1-r)c$, and the correctly identified non-target material, $q(1-c)$.

One important consequence of this model in terms of recycling systems is that neither the target stream nor the non-target stream are pure materials. Some target material is lost into the non-target, or depleted, stream, while some of the non-target material continues in the refined product stream.

5. Multiple Step Bayesian Separation Systems with Recycling Streams: Larger separation systems can be built up from multiple Bayesian separation steps. Here we consider a purifying process, in which the output target stream of a separation step is fed into another separation step, for some number of steps, n. The target stream of the final step is taken as the target stream for the entire process, with the non-target stream for each step being collected as waste. A process of this nature leads to a highly concentrated output stream, but a large portion of the target material may be discarded along with the waste streams. The waste streams of the later separation steps may be relatively rich in target material. A detailed example of these two phenomena can be found in Gutowski et al. [11].

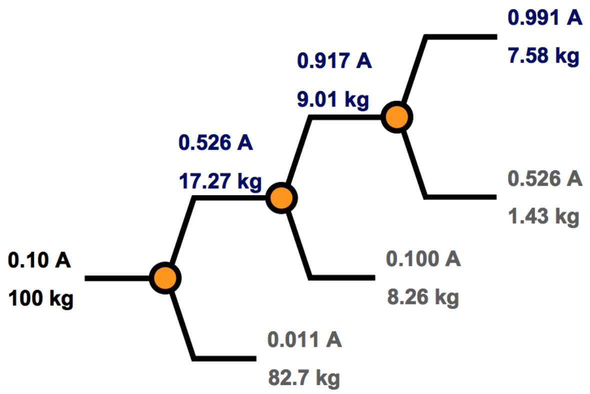

Figure 5: Separation process tree [11].

Figure 5 shows a separation process with *r*=*q*=0.909. At each branch, the concentration of material A is given along with the total mass in the system. Even with relatively high separation efficiencies for both the target and non-target material, only about 75% of the target material is captured. The waste stream of the last separation step is over 50%, five times that of the system input stream.

One way to mitigate the loss of the relatively rich later waste streams is to take depleted streams with concentrations equal to or greater than that of the original system input and feed them back into an appropriate earlier step, effectively recycling the later waste streams. As long as the concentration of the target material in the recycled stream is higher than the concentration of the input stream it is combined with, the output concentration of the target stream of the system will not be degraded. A simple model with a single recycled stream is shown in Figure 6.

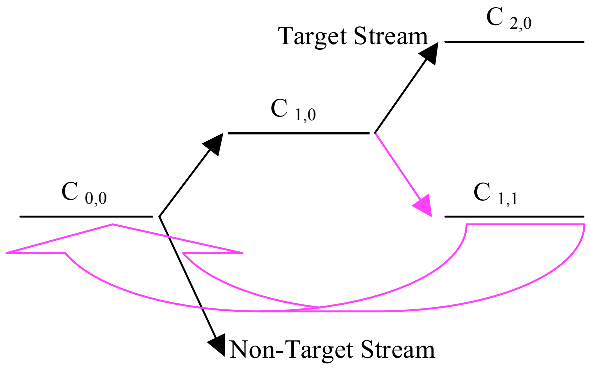

Figure 6: A simple system with a single recycling stream. From Albino [12].

The simple model shown above can be extended to systems with any number of re-entrant streams. The mathematical model of this system gives that for a multiple step system with a given number of recycling streams, *n*, that the target stream output mass for each step in the process will be

$$\begin{aligned} \gamma_n &= r^n \gamma_0 t(0)/t(n) \\ \gamma_{n-1} &= r^{n-1} \gamma_0 t(1)/t(n) \\ \gamma_{n-2} &= r^{n-2} \gamma_0 t(2)/t(n) \\ &\vdots \qquad \vdots \\ \gamma_1 &= r \gamma_0 t(n-1)/t(n) \end{aligned} \tag{3}$$

where γ_i is the mass of the target material present in the output step of step *i*.

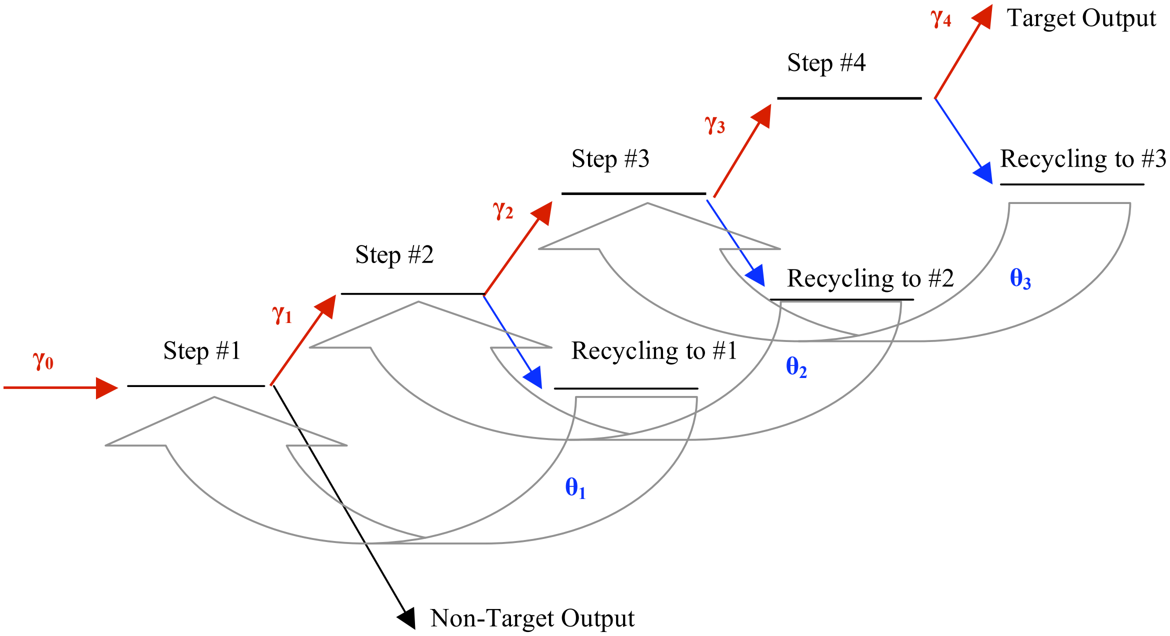

Figure 7: A separation system with three recycling streams [12].

r is the target separation efficiency and
$t(i)$ is the recursive function describing the denominator for the mass function for a process with i steps, as given in the following equation:

$$t(i) = t(1) - \frac{r^2(1-r)^2}{t(i-1)} - \frac{r^3(1-r)^3}{t(i-1)t(i-2)} - \ldots - \frac{r^i(1-r)^i}{t(i-1)\ldots t(1)} \quad (4)$$

where $t(0)=1$ and $t(1)=1-r(1-r)$. This solution was first presented by Albino [12].

As an example, consider the separation process shown in Figure 7. In this process, there are four separation steps with three recycled streams. Applying the formulas given above, we find that the mass output of the first separation process is

$$\gamma_1 = \frac{r\gamma_0[1-2r(1-r)]}{1-3r+4r^2-2r^3+r^4} \quad (5)$$

This gives the mass of the target material output by the process in terms of the target material input into the first step of the process. In terms of the percent of the material recovered, $R=\gamma/\gamma_0$, the portion of the total target material captured in the first separation step is

$$R_1 = \frac{r[1-2r(1-r)]}{1-3r+4r^2-2r^3+r^4} \quad (6)$$

For each step in the separation process, the portion of total target material captured in the target output of each step is

$$\begin{aligned} R_2 &= \frac{r^2[1-r(1-r)]}{1-3r+4r^2-2r^3+r^4} \\ R_3 &= \frac{r^3}{1-3r+4r^2-2r^3+r^4} \\ R_4 &= \frac{r^4}{1-3r+4r^2-2r^3+r^4} \end{aligned} \quad (7)$$

R_4 is then the final target material separation efficiency for the system.

A complete analysis of the performance of a multiple step separation process should also include tracking of the non-target material. The purity of the final output stream is effected by the flow of the non-target material, the portion of the non-target material in each recycled stream is important for assuring that the streams are being fed into the process at an appropriate point, and the final amount of material rejected is also an important indicator of the success of the process. Albino includes a similar analysis to that of the target material for the mass flow of the non-target material [12]. For an n re-entrant stream system, the non-target output material mass in the target stream is

$$\begin{aligned} \rho_n &= (1-q)^n \rho_0 u(0)/u(n) \\ \rho_{n-1} &= (1-q)^{n-1} \rho_0 u(1)/u(n) \\ \rho_{n-2} &= (1-q)^{n-2} \rho_0 u(2)/u(n) \\ \vdots &= \quad \vdots \\ \rho_1 &= (1-q)\rho_0 u(n-1)/u(n) \end{aligned} \quad (8)$$

where ρ_i is the mass of the non-target material present in the target output of step i.
q is the non-target separation efficiency and
$u(i)$ is the recursive function describing the denominator for the mass function for a process with i steps, as given in the following equation:

$$u(i) = u(1) - \frac{q^2(1-q)^2}{u(i-1)} - \frac{q^3(1-q)^3}{u(i-1)u(i-2)} - \ldots - \frac{q^i(1-q)^i}{u(i-1)\ldots u(1)} \quad (9)$$

where $u(0)=1$ and $u(1)=1-q(1-q)$. Continuing with the analysis of the system with three recycled streams as shown in Figure 5, the fraction of the non-target material in the final output target stream, Q_4, is given by

$$Q_4 = \frac{(1-q)^4}{1-3(1-q)+4(1-q)^2-2(1-q)^3+(1-q)^4} \quad (10)$$

The effectiveness of incorporating these recycling streams can be shown by calculating the improvement factor, F. The improvement factor is the ratio of the recycling streams system's target material output to the output of a corresponding system without recycling streams. For an n step process without recycling streams, the overall portion of the material recovered is

$$R_n = r^n \quad (11)$$

Then, the output improvement factor is then

$$F_n = \frac{1}{t(n)} \quad (12)$$

as given by Albino [12]. For our system with three recycling streams, the improvement factor is then

$$F_3 = \frac{1}{1-3r+4r^2-2r^3+r^4} \quad (13)$$

Figure 8 shows the improvement factor for the three recycling stream system as a function of target separation efficiency, r. From this graph, the effect of recycling streams in this system is apparent. Recycling streams are most important for material capture at lower separation efficiencies, but can still greatly improve the performance of systems with high recycling efficiencies.

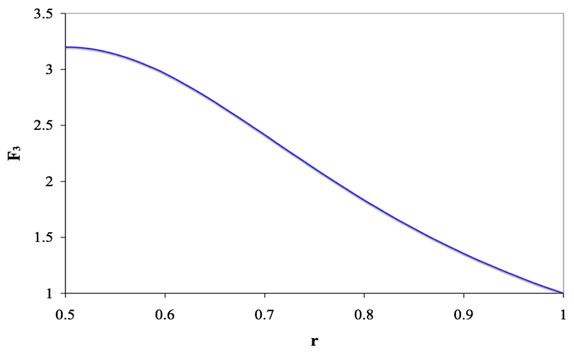

Figure 8: Improvement factor for a three recycling stream system.

7. Current Modeling Work: Evaluating the performance of a recycling system requires knowledge of many aspects of the system. Current work on this project focuses on developing models for different relationships for recycling systems.

One important economic aspect of recycling systems is the value of the material output. The main factor in the value of the output material is its purity. In order to compare results for different materials, output material values should be normalized. Here, the price of the virgin material from the New York spot price market is used to normalize the cost of several different materials. The graph in Figure 9 shows the normalized prices for several recycled materials. The recycled materials considered here include a variety of metals as well as PS, ABS, and PC-ABS plastics. Some groupings include a variety of recycled products or recycled material grades.

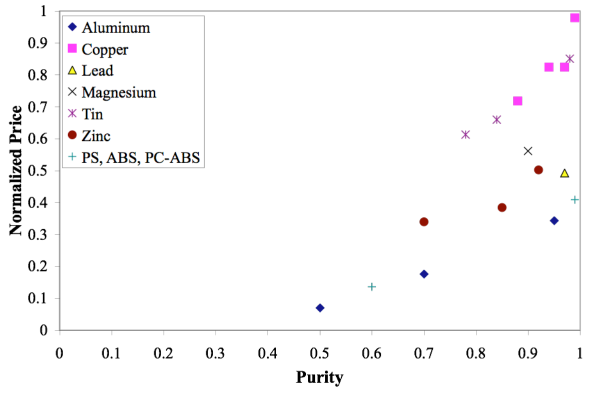

Figure 9: Normalized price-purity graph for recycled materials.

The data as plotted in the graph suggests that there is a relationship between the price of the recycled materials and its purity. While the value of the recycled material clearly increases with the material purity, the exact nature of the relationship is unclear. A linear model or an exponential model may be appropriate.

Several factors need to be considered in creating an appropriate model. One important factor is that different types of recycled products may not have the same price-purity relationship. The current data reflects mostly the recycling of metals. Plastics, paper, or other materials may have a different relationship that may need to be modeled separately from that of the metals. Another important factor is the discrete nature of the materials market. The market demand for some materials may be such that only certain price-purity sale points are available. Recycled materials with a purity between those of two sale points may have to be sold at the lower value. A price-purity may or may not attempt to incorporate this aspect of the market.

Another important aspect of recycling processes is the separation efficiencies of each process. Comparing the effectiveness of processes may not be as simple as just looking at the target material separation efficiency. A comparison may need to take the non-target separation into account. If two processes for the same material system have the same target material separation efficiency, but one process has a better non-target material separation efficiency, that process may be a better process. All cases may not be that clear cut; one process may have a high target separation efficiency, while another may have a high non-target separation efficiency.

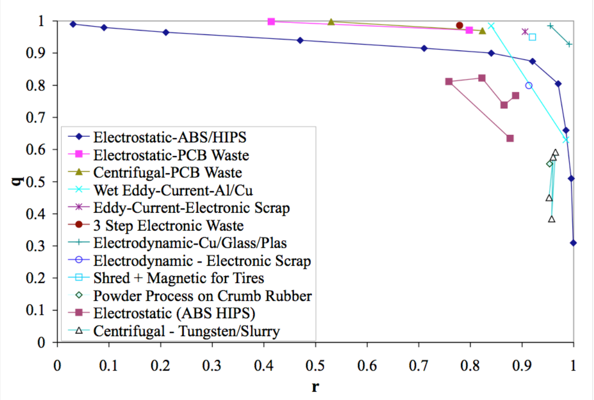

Figure 10: Separation efficiency data for several processes [13, 14, 15, 16, 17, 18, 19].

Figure 10 shows a variety of process separation data. The upper right corner of the graph represents the ideal process, where the target separation efficiency, r, and non-target separation efficiency, q, are both 1. For most processes with multiple separation points, a trade-off between r and q seems to exist. A good metric for the process separation efficiency should reflect this.

Another important factor that should be reflected in the model is that in the case of recycling systems, often both outputs of the process will be collected. Mathematically, the process should be symmetric for r and q.

A scoring model fulfilling the above criteria could be of the form

$$(r_i^{n_i} + q_i^{n_i}) = 1 \tag{14}$$

where n_i is the score assigned to process i with target and non-target separation efficiencies r_i and q_i. Other models that may fit the data include exponential models.

Creating a model with this type of data requires some caution. Variations in performance can be associated with many factors. Some processes have actively variable separation efficiencies; some have separation efficiencies that are dependent on the input material concentrations or feed rates. An understanding of the physical processes involved in a given separation process may provide additional insight. While it may neglect some of the complexities of the variation of separation efficiencies, a simple effectiveness score provides a quick way to compare dissimilar process.

8. Future Work: Continuing work on this project will follow several different paths. First, data collection for the price-purity correlation and separation efficiency scoring will continue. The current price-purity data focuses on metals recycling. More data on other materials, such as glass, plastics, and paper, would provide a more complete picture of the relationship between normalized price and purity. Different correlations may be necessary to provide a complete model for predicting the price of a recycled material based on its purity.

More separation efficiency data is necessary to evaluate potential separation efficiency scoring metrics. Once a model is formulated, it may be possible to group different types of processes based on their efficiency scores, or to sort similar separation processes based on their effectiveness as evaluated by their efficiency score.

Another important near-term goal is to verify several of the important assumptions of the information theory-based material mixing model and the Bayesian material separation model. One of the most important assumptions for the Bayesian material separation model is that the separation efficiencies for the target and non-target materials do not vary with input concentration. We are partnering with Axion Recycling, Ltd. of Manchester, England to create a series of tests to verify this critical assumption. With Axion Recycling, Ltd.'s cooperation, we will be able to investigate several recycling processes, including emerging technologies such as spectral sorting for plastics. Another important model assumption for the Bayesian separation is that all processes are roughly equal in cost. This assumption can be verified by gathering data from different manufacturers and recycling companies.

The ultimate goal of this project is to integrate the separate models describing the performance of recycling systems into a larger predictive model. A computer model incorporating many of the models presented in this paper could be used to simulate the performance of typical recycling systems. The model would be able to provide estimates of the economic, ecological, and energy-use performance, as well as characterize the relationship between these three important parameters for a given recycling system. With an understanding of the interplay of economics, energy use, and ecological concerns, the performance of successful recycling systems can be optimized and the failure of other recycling systems can be understood.

9. Acknowledgements: We would like to acknowledge the support of the National Science Foundation Award DMI #0423484.

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