A study of ocean wave statistical properties using nonlinear, directional, phase-resolved ocean wavefield simulations

Legena Henry
PhD Candidate
Course 2 – OE
legena@mit.edu
abstract

We study the statistics of wavefields obtained from phase-resolved simulations that are non-linear (up to the third order in surface potential). We model wave-wave interactions based on the fully non-linear Zakharov equations. We vary the simulated wavefield's input spectral properties: peak shape parameter, Phillips parameter and Directional Spreading Function. We then investigate the relationships between these input spectral properties and output physical properties via statistical analysis. We investigate:

1. Surface elevation distribution in response to input spectral properties,
2. Wave definition methods in an irregular wavefield with a two-dimensional wave number,
3. Wave height/wavelength distributions in response to input spectral properties, including the occurrence and spacing of large wave events (based on definitions in 2).
$z$-direction. The fully non-linear equations integrated using pseudo-spectral methods in SNOW are as follows:

Conservation of mass for an incompressible fluid:

$$\nabla^2 \phi = 0 \quad \text{for } -\infty \leq z \leq \eta$$  \hspace{1cm} (2.2)

Dynamic boundary condition:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{P_a}{\rho} + gz = 0 \quad \text{on } z = \eta,$$  \hspace{1cm} (2.3)

Kinematic boundary conditions:

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial \phi}{\partial y} \frac{\partial \eta}{\partial y} - \frac{\partial \phi}{\partial z} = 0 \quad \text{on } z = \eta$$  \hspace{1cm} (2.4)

$$\nabla \phi \rightarrow 0 \quad \text{as } z \rightarrow -\infty$$  \hspace{1cm} (2.5)
\[ S(k, \theta) = \frac{2\omega}{g} \alpha \frac{g^2}{k^5} e^{-1.25(k_m/k)^4} \gamma \exp\left(\frac{(k-k_m)^2}{2(\sigma k_m)^2}\right) D(\theta), \tag{2.1} \]

where the following definitions apply:

\[ D(\theta) = |\cos \frac{1}{2} \theta|^2s, \text{ where } s = 1 \text{ (See Section 3.3.)} \]

\( \theta = \text{spreading angle i.e. angular input for } D(\theta) \text{ where } -\pi < \theta < \pi \)

\( \gamma = \text{peak shape parameter} \) (see Section 3.1)

\( \alpha = 5.061\left(\frac{H^2}{T^4}\right)(1 - 0.287\ln(\gamma)) \) (see Section 3.2)

\( \sigma = 0.07 \text{ for } f \leq f_m \text{ and } \sigma = 0.09 \text{ for } f > f_m \)

\( \lambda = \text{wavelength} \)

\( k = 2\pi/\lambda \)

\( k_m = \frac{2\pi f_m^2}{g} \)

\( f_m = 3.5(g/\bar{U}) \bar{x}^{-0.33} \) where \( \bar{U} \) is mean wind speed and \( \bar{x} \) is fetch length

\( g = \text{constant gravitational acceleration}. \)
1. Surface elevation distribution in response to input spectral properties:

   - Peak Shape Parameter
   - Phillips' Parameter
   - Directional Spreading Function
1. Surface elevation distribution in response to input spectral properties:

Peak shape parameter
Peak shape parameter produces higher impact on even moments of surface elevation, (i.e. surface elevation kurtosis and surface elevation variance) than on the observed odd moments (skewness) of surface elevation.

<table>
<thead>
<tr>
<th>Case</th>
<th>B</th>
<th>H</th>
<th>I</th>
</tr>
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<tbody>
<tr>
<td>peak shape parameter, $\gamma$</td>
<td>1.0</td>
<td>3.3</td>
<td>5.0</td>
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The highest elevations in a wavefield with mean peak shape parameter, 3.3 are more stable than those in wavefields with mean peak shape parameter, 1.0, and 5.0.

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We find surface elevation kurtosis in non-linear wavefields is much smaller than surface elevation slope kurtosis. We also see that higher values of peak shape parameter produce higher kurtosis of surface slope in the mean direction of propagation. We also see that higher values of peak shape parameter produce a higher distinction between surface elevation kurtosis and surface elevation slope kurtosis.

In linear equivalent wavefields the random rules in surface elevation kurtosis and surface slope kurtosis where kurtosis values are all very near the Gaussian value 3.0. We note that this is quite unlike what is seen in non-linear wavefields.
Phillips’ parameter, $\alpha = f \left( \frac{H}{\lambda} \right)$

1. Surface elevation distribution in response to input spectral properties:

Phillips' Parameter and Surface Elevation
In nonlinear simulations of surface elevation, a higher Phillips' parameter produces higher surface elevation kurtosis and higher surface elevation skewness but negligible influence on surface elevation variance.

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<td>Phillips' parameter, $\alpha$</td>
<td>0.0032</td>
<td>0.0160</td>
<td>0.0163</td>
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The stability of the maximum spectral energy in nonlinear wavefields is dependent on Phillips' Parameter. Higher Phillips' Parameter produces less stable maximum spectral energy.

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With all other spectral parameters equal, the influence of Phillips' parameter on the likelihood of large wave formation (measured in Benjamin-Feir Index) is unapparent.

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A lower Phillips' parameter produces smaller deviations between the three kurtosis evolutions: surface elevation kurtosis, kurtosis of surface elevation slope in the x-direction, and kurtosis of surface elevation slope in the y-direction. If Phillips' parameter is small enough, it produces kurtosis evolutions that are so close that they resemble linear simulations. In a case with a very low Phillips' parameter, the observed effect of the non-linearity order in the simulation is negligible. i.e. a low Phillips' parameter reduces/eliminates the effect of non-linearity in the wavefield.
1. Surface elevation distribution in response to input spectral properties:

Directional Spreading Function and Surface Elevation
- We observe nonlinear wavefields with constant Peak Shape Parameter and Phillips' Parameter and various input spreading angles between 1 and 180 degrees.

- Three regimes are seen to form:
  - Unidirectional (spreading angle between 1 and 10 degrees)
  - Mildly directional (spreading angle between 20 degrees and 40 degrees)
  - Strongly directional (spreading angle equal to or above 80 degrees)
We find the unidirectional wavefields have kurtosis, skewness and variance which compare unpredictably to those in strongly directional wavefields.

We see these regimes when we compare the order of magnitude of kurtosis and skewness.

Table 3.3: Simulated wavefield cases used in testing the effects of directional spreading with peak shape parameter, $\gamma = 3.3$ and Phillips’ parameter, $\alpha = 0.0160$

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<th>M</th>
<th>K</th>
<th>L</th>
<th>G</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>180</td>
<td>80</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>1</td>
</tr>
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</table>

![Graphs showing kurtosis, skewness, and variance for different cases.](image-url)
The highest spectral energy in unidirectional wavefields has much lower stability than equivalent directional cases. Larger directional spreading in a wavefield is associated with more stable but slightly smaller maximum spectral energy.
Directional spreading angle has regime-type effects on the spectral moments of surface elevation since we see one regime (cases with less directional effects) having growing spectral moments and the other regime (cases with strong directional effects) having falling spectral moments.
Directional spreading angle also has regime-type impact on surface slope kurtosis. Unidirectional wavefields - larger difference between the kurtosis : surface elevation and kurtosis : surface elevation slopes. Greater directional effects: surface elevation x-directional slope kurtosis less than surface elevation y-directional slope kurtosis.
Small detour: Future work

• PhD thesis
  – Analytically estimate the kurtosis or the skewness of the derivative of the surface elevation, both in x and y direction, both from free and bound modes.

  Justification:
  – Derivative of the surface elevation is relevant in ship impact and satellite measurements.
1. Wave definition methods in a nonlinear wavefield with a two-dimensional wave number

We present three solutions to the problem of defining waves on a non-linear, evolving free surface with a 2D wavenumber, \( k = (k_x; k_y) \). These solutions add useful insight to common field practices, since we present methods of defining waves suited to three specific field applications.
2. Wave definition methods in a nonlinear wavefield with a two-dimensional wave number zero-crossing method:
We study the uni-directional, narrow-band, zero-crossing method of wave-height definition defining wave heights along parallel rays in the wavefield in the mean direction of wavefield propagation. This approach is suitable in considering the waves interacting with arrays of fixed offshore columns.
Cross section of wave field, parallel to mean direction of wave travel.
2. Wave definition methods in a 3D, irregular wavefield

3D analogue of the 2D half-cycle excursion method: non-narrow-band, directional approach to wave heights in wavefields, considering waves interacting with floating wave-power devices.

(b) Half-cycle excursions, labeled $x_1$, $x_2$, ..., $x_{11}$, are non-narrow-band wave heights. We see examples of Type I excursions in $x_1$, $x_2$, $x_6$, $x_7$, $x_8$ and $x_{11}$ and examples of Type II excursions in $x_4$, $x_5$, $x_9$ and $x_{10}$.
2. Wave definition methods in a nonlinear wavefield with a two-dimensional wave number

We study a directional, narrow-band method of wave-height definition, finding the distribution of the largest waves in the wavefield. This approach is suitable in considering the waves interacting with ships and large offshore structures.
...and...
there's a lot more where that came from

www.legenahenry.com

- spectral parameters and wave height distributions
- spectral parameters and the spacing of large waves
- etc…

THANK YOU

legen@mit.edu