Two-Period Planning for Hybrid Seed Corn: An Alternative Solution Procedure

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Reference:

Jones, Philip C., Timothy J. Lowe, Rodney D. Traub, Greg Kegler. 2001. Matching Supply and Demand: The Value of a Second Chance in Producing Hybrid Seed Corn. *Manufacturing & Service Operations Management*. Vol. 3, No. 2, pp. 122-137.

Introduction

This is an attempt to fathom the approach of Jones, et al to the problem stated in the title. We assume that the reader has this reference at hand. We also wish to avoid the need for a solver such as What's Best! as used in that study.

Throughout we rely on the results from Jones, et al that the marginal profit functions are well behaved and that local optima are global. For the most part, we adopt the notation used in the Jones study. However, we will assume that the probability distributions for crop yield and demand are statistically independent, **normal** distributions with the same means and standard deviations used by Jones in his Managerial Insights section. In that section, he examined 9 combinations of variability for yield and demand: high, medium and zero, using symmetrical, discrete distributions. We will confine our attention to only 2 of these combinations: yield and demand at high variability ; yield and demand at medium variability.

Some Notation

$i = 1,2$ index for planning periods

$K_i = \$900$, cost per acre

$c_i = \$10$, variable cost per bushel

$\pi = \$27.50$, shortage cost per bushel

$v = \$23.50$, salvage value per bushel

$p = \$60$, price per bushel

$y_i =$ yield in bushels per acre for period i, a random variable

$D =$ demand in bushels, a random variable

$\mu_{yi} = 40$ bushels per acre, mean yield

$\mu_D = 210$ (1000s of) bushels, mean demand

$\sigma_{yi} = 7.4833$, 4.4721 high and medium values of standard deviation for yield

$\sigma_D = 7.4833$, 4.4721 high and medium values of standard deviation for demand

k_{yi} = coefficient of variation for yields

k_D = coefficient of variation for demand

Q_i = decision variables for acreage planted in each period

w_i = bushels on-hand beginning period i, w_1 is specified and $w_2 = Q_1y_1$

V_i = marginal profit function for period i

Additional notation will be introduced as needed. For convenience, we assume as did Jones that $w_1 = 0$ and that yields and various costs remain the same from period 1 to period 2.

A General Description of the Method

The objective of the analysis is to find the optimal acreages Q_1 and Q_2 for two sequential planning periods with unknown crop carry-over w_2 at the start of period 2. The process, in general terms follows.

1. Equation (1) of Jones is the marginal profit function $V_2(Q_2,w_2)$ for the two-period planning problem. If we set the derivative of this function equal to zero, we can then search for the optimal Q_2 for each w_2 over a range of specified values. The result permits construction of the function $Q^*_2(w_2)$, so that the marginal profit function becomes $V^*_2(w_2)$. The search process involves numerical integration of the normal density function and its cumulative and the use of the Goal Seek utility in Excel.
2. Equation (2) of Jones is the marginal profit function $V_1(Q_1)$ for the first period, assuming an optimal choice of acreage for period 2, e.g. $V_1(Q_1)$ depends on $V^*_2(w_2)$. We next differentiate equation (2) with respect to Q_1, bearing in mind that $w_2=Q_1y_1$, so that we must employ the chain rule: $dV^*_2(w_2)/dQ_1=(dV^*_2(w_2)/dw_2)x(dw_2/dQ_1)$.
3. In order to determine $dV^*_2(w_2)/dw_2$, we must return to the modified form of equation (1) as determined in step 1. above. After differentiation and carrying out numerical integration for a specified range of w_2, we can construct an explicit function for the derivative of V^*_2.
4. We now return to the differentiated form of equation (2) and carry out a search for the optimal value of Q_1. Once again, this will require numerical integration and the use of Goal Seek.

In the next section, we will examine the details of this alternative approach and present results for the optimal choices for acreage in each period. As we shall see, they compare quite closely to those obtained by Jones.

Analysis and Results

Step 1.

Setting the derivative of $V_2(Q_2,w_2)$ with respect to Q_2 equal to zero gives:

$$\int_{-\infty}^{\infty}\left[F(k) - k_{y2}f_z(k)\right]f(z_D)dz_D = \frac{(K_2/\mu_{y2} + c_2 - v)}{(p + \pi - v)} \qquad (1)$$

where

$f_z(.)$ is the standard normal density function

$F(.)$ is the normal cumulative

z_D is the standard normal variate for demand (subscript for clarity only)

and

$$k = \left[\frac{(1 + k_D z_D - w_2/\mu_D)}{Q_2\mu_{y2}/\mu_D} - 1\right]/k_{y2} \qquad (2)$$

In equation (1) w_2/μ_D represents the proportion of mean demand met by first period ending crop inventory while $Q_2\mu_{y2}/\mu_D$ is the proportion of mean demand met by second period output.

We will require numerical integration to search equation (1) for Q_2 and for subsequent analysis. In all cases we used Simpson's Rule with the standard normal variate in the range [-5.0 , +5.0] in steps of size 0.2. The value of $w2/\mu_D$ was varied in the range [0 , 1] in steps of 0.1 and an additional point was evaluated at 0.95. We found that Goal Seek was unable to find a solution for values of $w_2/\mu_D > 1.02$.

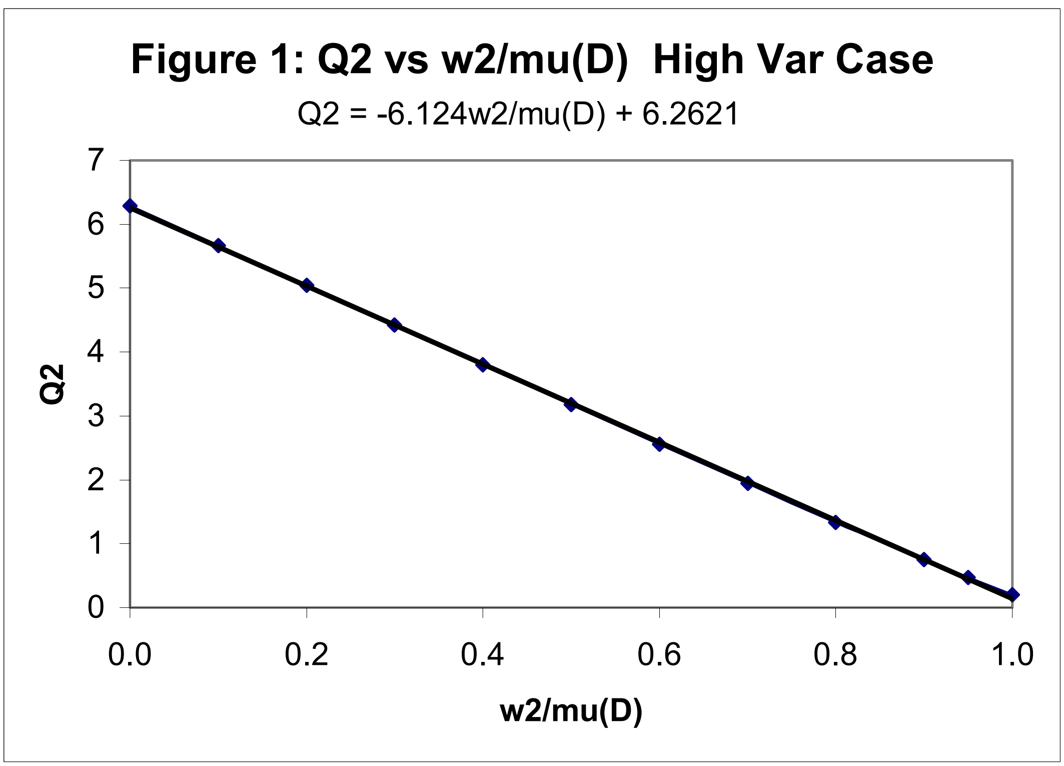
Figure 1 below,shows the results of the search for optimal Q_2 for the case of high variability. The linear nature of the relationship is surprising, but was observed at many different combinations of the coefficients of variation for yield and demand. Then we can write

$$Q^*_2 = a_0 + a_1 (w_2/\mu_D) \qquad (3)$$

Table 1 below lists the coefficients of equation (3) for the high and medium variability cases.

	a_0	a_1
High Var.	6.2621	-6.1240
Med Var.	5.8742	-5.7915

Table 1: Coefficients of equation (3).



Note that the coefficient a_0 in equation (3) is exactly the optimal value for acreage for a single period planning situation, since $w_2 = 0$. We are ready to take on step 2 of the procedure outlined earlier.

Step 2.

Our next task is to consider Jones' equation (2) for the marginal profit function in period 1, differentiating that expression with respect to Q_1 and equating the result to zero. That result, after rearranging, follows.

$$\int_{-\infty}^{\infty}(1+k_{y1}z_{y1})\left[\frac{dV_2^*(w_2)}{dw_2}\right]f(z_{y1})dz_{y1} = K_1/\mu_{y1} + c_1 \qquad (4)$$

After we establish an algebraic form for the term in brackets in equation (4) as a function of w_2 (in the next step), we then make use of the fact that $w_2=Q_1y_1$. This will permit a Goal Seek search for the Q^*_1 that satisfies equation (4).

Step 3.

First define two new constants:

$$\hat{a}_i = a_i(\mu_{y2}/\mu_D) \quad i=0,1$$

Differentiation of equation (1) with respect to w_2 after substitution of equation (3) gives

$$\frac{dV_2^*(w_2)}{dw_2} = b_0 + b_1\int_{-\infty}^{\infty}\left[(1+\hat{a}_1)F(k) - \hat{a}_1 k_{y2} f_z(k)\right]f(z_D)dz_D \qquad (5)$$

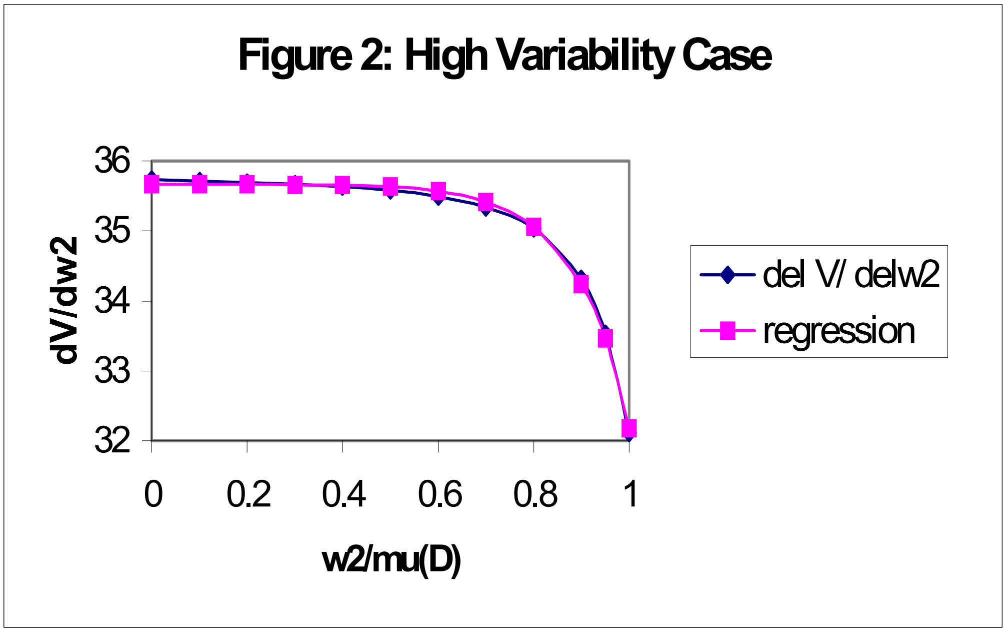
where

$$b_0 = -K_2\hat{a}_1/\mu_{y2} - c_2\hat{a}_1 + v(1+\hat{a}_1) \qquad , \qquad b_1 = p+\pi-v$$

and

$$k = \left[\frac{1+k_D z_D - w_2/\mu_D}{\hat{a}_0 + \hat{a}_1 w_2/\mu_D} - 1\right]/k_{y2} \qquad (6)$$

The procedure here is to evaluate equation (5) for a range of values of w_2/μ_D, plot the results and fit a function of w_2/μ_D to the derivative. Figure 2 illustrates such a plot for the high variability case. For the medium variability case, the curve is very similar. However, at very small values for the coefficients of variation (i.e.<.01), the plot becomes linear.



A function of the form

$$\frac{dV_2^*(w_2)}{dw_2} = d_0 + d_1 \exp(w_2/\mu_D)^6 \qquad (7)$$

provides an excellent, yet simple, fit for both high and medium variability cases with $R^2 > .996$. Table 2 below lists the coefficients for both cases.

	d_0	d_1
High Var.	37.691	-2.0287
Med Var.	35.462	-1.2214

Table 2: Coefficients for equation (7)

Now we turn our attention to determination of the optimal value of the first period acreage, Q_1.

Step 4.

Return to equation (4) and substitute equation (7). Next substitute Q_1y_1 for w_2 and $(\mu_{y1}+\sigma_{y1}z_{y1})$ for y_1.

Then Goal Seek can be used to find the optimal Q_1 . This result can then be used to compute $w_2 = Q_1\mu_{y1}$. Equation (3) will provide the optimal Q_2. Total supply is found from $w_2 + Q_2\mu_{y2}$. These results are shown in Table 3 below along with a comparison to the Jones results.

	High Variability		Medium Variability	
	Present Study	Jones Study	Present Study	Jones Study
Single Period Plan				
Q_1	6.26	6.0	5.89	6.0
Total Supply	250.40	246.0	234.80	240.0
Two Period Plan				
Q1	4.15	4.5	4.77	4.8
Q2	1.42	1.1	0.61	0.6
Total Supply	222.84	224.0	215.29	217.0

Table 3: Comparison of some key results

Differences in the results of the two approaches as revealed in the table above may be due to any of several factors:

1. The present study used normal distributions for both yield and demand random variables while the Jones analysis used discrete, symmetrical distributions.
2. Numerical integration does not provide exact answers.
3. The Goal Seek utility in Excel does not provide exact answers.
4. The values for high and medium coefficients of variation of demand stated in the Jones study (0.05 and 0.03) do not correspond to the values we computed using the discrete probability distribution of demand given in his Table 4. We obtained values of 0.0356 and 0.0213 respectively and used these throughout our analysis.