AN APPLICATION OF OPTIMAL CONTROL TO STRATEGIC PLANNING

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INTRODUCTION TO WELCH'S

- Food manufacturer

- Make to stock
  Long production cycle time
  Short customer order cycle time

- Vertically integrated

- Several plant and warehouse locations

- Dynamic, uncertain demand

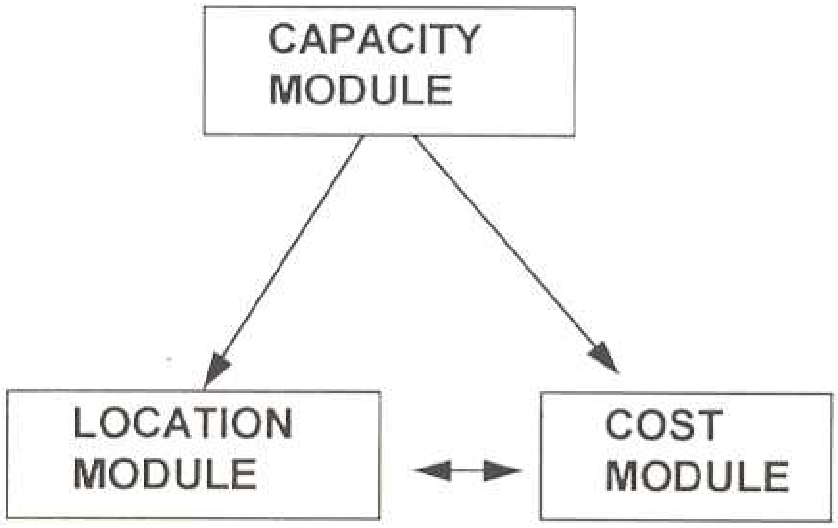
- Steeply escalating customer service demands

OBJECTIVE

Develop a mfg/warehousing strategic planning system which:

1. Signals the need for production and warehouse capacity expansion.
2. Provides a means of evaluating the total costs of candidate plant/warehouse locations.
3. Explicitly accounts for the costs associated with customer order cycle time.

THE PROPOSED SYSTEM



CAPACITY MODULE (Dynamic)

Plant Capacity
Warehouse Capacity } influence on order response time

LOCATION MODULE (Static)

Inbound/Outbound Transportation Cost
Customer Cycle Time
Warehouse Costs
Multi-Location Volume Assignments

COST MODULE (Static)

Volume Based Cost Assignments

CAPACITY MODULE

Requirements

1. Permits examination of dynamic demand scenarios.

2. Explicitly represents the affect on order response time of plant and warehouse utilization factors.

3. Model must be aggregated over all plants/ warehouses.

4. Permits exploration of trade-offs between costs of capacity expansion and deterioration in response time.

Approach: The above suggests that an optimal control model would be appropriate.

AN OPTIMAL CONTROL MODEL

Preliminary Model

Ignores Production and Shipping Rate Capacities

Over the planning horizon [O,T] minimizes costs of

1. Shipping backlog delay
2. Order filling delay due to inventory deficiencies
3. Changing shipping rate
4. Holding inventory

Subject to:

1. Inventory and shipping backlog dynamics
2. Non-negativity constraints on

    Inventory
    Customer order backlog
    Production rate
    Shipping rate

NOTATION

$I(t)$ = inventory level, units, u

$P(t)$ = production rate, u/t

$S(t)$ = shipping rate, u/t

$V(t)$ = shipping backlog, u

$U_1(t)$ = change in production rate, u/t^2 (control)

$U_2(t)$ = change in shipping rate, u/t^2 (control)

$D(t)$ = demand rate, u/t (given)

$B(t)$ = buffer inventory, u (given in terms of $D(t)$, demand std dev, customer service)

Costs

C_I = holding cost, \$/u-t
C_D = customer delay cost - \$/u-t
C_C = cost of changing shipping rate - $\$(t^2/u)^2$

SIMPLEST POSSIBLE MODEL

$$\text{Min } \hat{J} = \int_{0}^{T} \left\{ D(t) \left[\frac{V(t)}{S(t)} + \frac{aB(t)}{I(t)} \right] + \hat{C}_I I(t) + \hat{C}_C U_2^2(t) \right\} dt$$

$$\hat{J} = J/C_D \; ; \; \hat{C}_I = C_I/C_D \; ; \; \hat{C}_c = C_c/C_D$$

Subject to:

$$\mathring{I} = P - S$$

$$\mathring{P} = u_1$$

$$\mathring{S} = u_2$$

$$\mathring{V} = D - S$$

$I, P, S, V \geq \mathbf{0}$; u_i not restricted.

Note

a = delay (time) for inventory insufficiency.

$B(t) = k_s \cdot (cv)_D \, D(t)$

k_s = customer service level factor

$(cv)_D$ = coefficient of variation of demand.

SOLUTION PROCEDURE

1. Divide the planning horizon into N intervals.

2. Choose a feasible solution for the 2N variables.

   $u_1(t) \tilde{=} U(I)$, $I = 0, 1, \ldots, N-1$

   $u_2(t) \tilde{=} U(I)$, $I = N, N+1, \ldots, 2N-1$.

3. Solve the state equations using a finite difference scheme.

4. Obtain finite difference forms for $\hat{J}$, $\nabla \hat{J}$

5. Solve the 2N dimensional non-linear programming problem using the gradient projection method due to Goldfarb. This requires initial values for

   a. The matrix of active constraints: $\mathbf{A}_q$

   b. The variable metric matrix: $\mathbf{H}_q$

GETTING STARTED

1. **Artificially impose the condition:**

$$u(I) \leq VMAX$$

2. **As beginning values of $u(I)$ choose**

$$u^{\circ}(I) = VMAX$$

3. **Then the initial matrices are:**

$$A_q = A_N = -I_N$$

$$H_q = \phi$$

and the potentially troublesome matrix

$$(A_q^T A_q)^{-1} = I_N$$

with I_N the identity matrix of order N