PRACTICAL PRODUCTION SCHEDULING WITH CAPACITY CONSTRAINTS AND DYNAMIC DEMAND: FAMILY PLANNING AND DISAGGREGATION

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Scheduling manufacturing operations with finite capacity under conditions of uncertain, dynamic demand is difficult. However, it is not an art form. Important parts of the scheduling process can be carried out using rationally constructed models that will greatly reduce the burden on the plant production scheduler.

Welch's is a make-to-stock food manufacturer facing an environment of steeply escalating customer service demands which imposes the need to maintain safety stocks which reflect dynamic demand, forecast error and bias, manufacturing lead time, product quality "hold" time, and ABC classification. The present system of scheduling successfully incorporates all of these factors, basing production decisions on a dynamic safety stock. However, as capacity utilization increased, it became clear that the production planning system must be refined to include:

- A means of smoothing demand peaks to recognize production capacity limits
- A method of lot sizing which attempts to balance major setup costs and holding costs
- Variability in production output.

We will describe the new scheduling system and illustrate its use with real data from one of the manufacturing lines. Our objectives will be twofold: to demonstrate successful application of some simple management science tools in an area prone to trial-and-error decision making; to provide guidance to others who wish to implement more rational scheduling systems.

Before we begin, we wish to make very explicit the environment for which the system was designed: (1) dynamic, uncertain demand; (2) make to stock, i.e., a forecast-driven planning system with dynamic buffer stock requirements; (3) dedicated production lines; (4) for each line a set of two or more families of products with similar setup times and costs, and similar product values and production rates within each family; (5) family setup times and costs dominate those of individual products.

We make no claim that our system is optimal. It is rational, meets our stated objectives, and can be implemented on a spreadsheet for less than two minutes per production line. All references to computer time are based on a 33 mhz, 486, EISA architecture with no math coprocessor.

SYSTEM OVERVIEW

Inventory Planning Model (IPM)

This is the existing scheduling system; one of its functions is to determine dynamic buffer stocks for all products over an 11-week horizon based on demand forecasts. Reorder points are computed as demand during manufacturing lead time plus safety stock. Safety stocks account for forecast error and bias and desired customer service level. This system also has a lot-sizing capability based on the anticipated reorder point after lead time. However, it does not recognize production capacity limits and lot sizing does not incorporate setup and holding cost considerations. For a description of this system, see [3].

The existing system will continue to provide dynamic buffers to the new scheduling system which is essentially a modification of the hierarchical system proposed by Hax [2].

Family Planning Model (FPM)

In this portion of the new scheduling system, products on each dedicated production line are aggregated into two or more families having similar costs, production rates, and setup times. Family setups are expensive in both time and money, while product setups within a family are relatively inexpensive and require little time.

There are two major objectives for this part of the system: to smooth peak demands through time so that production capacity is not exceeded; to allocate near-term production requirements among the families in a way that balances family setup costs and family holding cost.

These objectives are realized by using a modified version of the mixed integer programming model of Chung and Krajewski [1], summarized by Vollmann, *et al.*, [4]. A six-month time horizon was chosen to capture demand spikes, and this horizon is divided into two portions: the next four weeks and five subsequent months.

During the next four weeks, we need to know what each family's regular and overtime production allocation should be, by week, to minimize holding costs, setup costs, and overtime costs while meeting demand and maintaining buffers. We also need the family production requirements by month over the following five months to meet demand and maintain buffers, but based only on holding and overtime costs, ignoring setups. This strategy limits the number of binary variables to a reasonable value and saves computer run time. The family planning model, FPM, is summarized in the Appendix. Note that in our model, we do not permit setups on overtime, consistent with current practice at Welch's. Computer run times for the FPM with four families are typically less than one minute.

Disaggregation of the Family Plans (DPM)

The results of the FPM are total family production requirements by period. We know that these requirements are feasible because FPM imposed feasibility through the constraint equations. We also know that in the disaggregation of the families, we must try to meet the total family production requirement as closely as possible. If the disaggregated schedule for a given family falls short of the FPM allocation, we face the potential danger of future stockouts. If we exceed the FPM allocation, we may exceed production capacity limits or incur unnecessary holding costs.

For individual products we must ensure that demand is met and that buffers are maintained. The output of our disaggregation planning model provides individual production requirements by time period. It may often be the case that these requirements are directly interpretable as lot sizes or they may require further modification to meet specific circumstances.

Our disaggregation model considers only the next four weeks, and to begin with, we will assume continuous (i.e., non-integer) production requirements are appropriate. A general model which balances individual product setup and holding costs is given in the Appendix as DPM.

Constraint (10) attempts to meet the FPM production requirement as closely as possible by penalizing excess production, E(t), and production shortfalls, S(t), in the objective function with very high costs. These penalties can be discarded for continuous lot sizes. It is not necessary to directly impose capacity constraints at this level since they are implicit in the family production requirements and these are enforced with the E(t) and S(t) penalties.

In most applications of this disaggregation model it is probably not necessary to account for setup and holding at the individual product level. In general, we find very little difference in the solution costs if holding and setup costs are dropped from the objective function (9) and constraints (7) are eliminated. This shouldn't be surprising since we have already accounted for major setup and holding costs over this four-week period, and at this level the problem has become so constrained that all feasible solutions look pretty much alike. Considerable computer time savings can be achieved for large families, say eight or more products, if the binary setup variable, N(i, t), can be eliminated from the model.

Further support for ignoring holding and setup costs at this level is provided by Hax [2], who was concerned only with filling the production requirements imposed by the next higher level in the hierarchy.

In practice, continuous lot sizes may not be the most natural form for individual product runs. For example, at Welch's, product run lengths are tied to the existing eight-hour shift structure. Furthermore, run lengths for the most part can be expressed as multiples of a one-quarter shift (i.e., 2 hours). These kinds of issues tend to be specific to given firms and industries. However, in the next section we will examine some alternative disaggregation models in which lot sizes can be put into correspondence with the set of integers as is the case at Welch's.

DISAGGREGATION TO INTEGER LOT SIZES

Suppose we define the integer variables: Z(i, t) = number of ¼ shifts allocated to production of end item i during period t, and the productivity per ¼ shift for item i is denoted by p(i). The DPM can be converted very simply to an integer lot-size model. The only changes required are to constraints (2) and (10) where we make the replacements

$$P(i, t) \rightarrow p(i)Z(i, t),$$

and we require Z(i, t) to be integer valued. Unfortunately, this disaggregation model requires a prohibitive amount of computer time, even for small families. If setup costs (and constraints) are discarded, disaggregation of only three end items still takes approximately 3.0 minutes.

There is a cure for this problem and it requires some simple preconditioning of the DPM. These are simple spreadsheet algebra computations requiring negligible computer time. First, replace the end-item productivity factors, p(i), by the average for the family, $\bar{p}$, everywhere in the formulation. Second, solve the material balance constraint for I(i, t):

$$I(i, t) = \sum_{k=1}^{t} \bar{p}Z(i, k) + I(i, 0) - \sum_{k=1}^{t} D(i, k)$$

where I(i, 0) are the end-item beginning inventories. Now substitute for I(i, t) in the objective function (9) and in the buffer constraints (6). Rearrange the buffer constraints and round the right-hand sides to integer values with an eye toward beginning inventory values. We are, in effect, overriding the buffers suggested by IPM. This will be self-correcting over time, since the schedule is updated weekly.

We now have a disaggregation model which accounts for setup and holding costs, but contains binary and integer variables. This model is shown in the Appendix as DPMIHS. We next examine the behavior of this formulation with and without setup costs.

DPM With Integer Lot Sizes Holding and Setup Costs (DPMIHS)

We have tested the simplified DPMIHS formulation to disaggregate three- and seven-product families. The three-product family required about one-half minute, and the seven-product family about five minutes. However, if we drop the binary and integer requirements (15), (16) and obtain a straight LP solution, the production lot sizes, Z(i, t), are all integer valued while the setup binary remains non-integer. The LP solution is virtually indistinguishable from the "exact" solution and is only 0.5% higher in cost.

If computer time is a critical factor, an LP relaxation will provide lot-size results that do not need to be rounded and costs close to "optimal."

DPM With Integer Lot Sizes Ignoring Holding and Setup Costs (DPMI)

As noted earlier, we can often neglect holding and setup costs in the disaggregation process without fear of undue cost penalties. When we do this, the disaggregation formulation takes on a very special structure as can be seen in the Appendix (DPMI). The decision variables in the constraint equations all have coefficients of +1. The over and under production penalty coefficients are −1 and +1, respectively. In addition, we now round the right-hand sides of (13) to integer values. This structure permits us to drop the integer restriction on the lot-size variables and obtain an LP

TABLE 1: Family Data

	Demand/Units				Buffers/Units				Time Available–Hours	
									Regular Time	Overtime
Period	D(1,t)	D(2,t)	D(3,t)	D(4,t)	B(1,t)	B(2,t)	B(3,t)	B(4,t)	AR(t)	AO(t)
(Weeks)										
1	52.50	4.50	9.20	8.30	65.03	13.12	21.94	11.53	80	36
2	32.73	6.48	10.14	13.63	68.61	12.98	23.17	13.27	80	36
3	32.73	6.48	10.14	13.63	70.50	14.17	26.00	18.57	80	36
4	32.73	6.48	10.14	13.63	70.88	14.17	26.56	19.80	80	36
(Months)										
5	48.63	42.30	38.94	44.65	29.16	15.39	38.16	14.76	320	144
6	44.26	72.74	31.22	44.78	29.16	15.39	38.16	14.76	320	144
I(i,0)	60.00	6.00	20.00	25.00	units, beginning inventories					
CH(i)	150.00	150.00	150.00	150.00	$/unit/week, holding costs					
CO(i)	172.00	124.00	212.00	121.00	$/units, overtime costs					
r(i)	.92	.67	1.14	.65	hours/unit, time required per unit of production					
CS(i)	400.00	400.00	400.00	400.00	$/setup, setup costs					
S(i)	8.00	8.00	8.00	8.00	hours, setup time					

TABLE 2: Results of the Family Planning Model (4 Families)

	Family Production Requirement				*Family Ending Inventories/Units*				*Family Overtime Production/Units*			
Period	*P(1,t)*	*P(2,t)*	*P(3,t)*	*P(4,t)*	*I(1,t)*	*I(2,t)*	*I(3,t)*	*I(4,t)*	*O(1,t)*	*O(2,t)*	*O(3,t)*	*O(4,t)*
(Weeks)												
1	47.43	8.98	5.57	0	65.03	19.46	21.94	16.70	10.10	8.98	5.57	0
2	36.31	0	14.00	10.20	68.61	12.98	25.80	13.27	0	0	0	0
3	33.93	7.08	5.17	9.47	70.50	20.65	26.00	18.57	0.69	7.07	5.17	9.46
4	33.11	0	10.70	14.86	70.88	14.17	26.56	19.80	0	0	0	0
(Months)												
5	6.91	43.52	50.54	39.61	29.16	15.39	38.16	14.76	0	0	0	0
6	44.26	72.74	31.22	44.78	29.16	15.39	38.16	14.76	0	0	0	0

solution which is naturally integer valued. Disaggregation of seven-product families is then virtually instantaneous.

It is interesting to note that constraints (12) of the DPMI formulation are nothing more than a generalized form of the feasibility conditions that arise in every production planning problem: cumulative production in any period must be at least as large as cumulative demand in that period. The modifications in (12) simply net out beginning inventories and add buffer stocks.

APPLICATION OF THE SCHEDULING MODELS

We will demonstrate the use of these models with data drawn from one of Welch's production lines. We illustrate the scheduling process with a batch/packaging line with 14 dedicated products. These products form four families of unit sizes 2, 2, 3, 7.

Family Planning

Aggregated data are given in Table 1. We will use only two months beyond the initial four weeks to limit the problem size for this demonstration.

These data were entered into FPM of the Appendix, with Q(i) = 500 for each family. The solution was obtained in less than one minute and the results are shown in Table 2.

Should a family planning run result in an infeasible condition, the overtime constraint can be relaxed. The resulting solution will then show if we are out of production capacity or if selective relaxation of one or more tight buffers might permit a feasible solution with the overtime constraint reinstated.

Disaggregation of a Family Plan

We demonstrate the disaggregation of family three which contains three end items. The data required are shown in Table 3.

Only integer lot sizes are of interest to us since they represent the most difficult computational problems in the disaggregation process. Furthermore, since ¼ shift lots (two hours) are most natural to Welch's, the integer lot size variable, Z(i, t), will represent two-hour time periods.

Table 4 shows the results of disaggregation with and without holding and setup costs accounted for. The results for the ending inventories, I(i, t), and production in units, P(i, t), must be computed "outside" the model since they have been eliminated in favor of the integer lot-size variables, Z(i, t). Actual costs are also computed outside the model.

TABLE 3: Product Data for Family 3 (3 Products)

	Demand/Units			*Buffers/Units*		
Period	*D(1,t)*	*D(2,t)*	*D(3,t)*	*B(1,t)*	*B(2,t)*	*B(3,t)*
(Weeks)						
1	6.70	0.50	2.00	17.99	0.93	3.02
2	6.78	1.48	1.88	19.03	0.98	3.16
3	6.78	1.48	1.88	17.12	3.16	5.72
4	6.78	1.48	1.88	16.73	3.60	6.23
I(i,0)	12.89	2.66	4.45	units, beginning inventories		
CH(i)	150.00	150.00	150.00	$/unit/week, holding costs		
CS(i)	100.00	100.00	100.00	$/setup, setup costs		
S(i)	0.50	0.50	0.50	hours, setup time		
p(i)	1.81	1.65	1.78	units/¼ shift, productivities		

TABLE 4: Disaggregation of Family 3 into Integer Lot Sizes

Holding and Setup Costs Included: Cost = $18,057.00

	No. of ¼ Shift Lots			*Ending Inventories*			*Production Requirements*				
Period	*Z(1,t)*	*Z(2,t)*	*Z(3,t)*	*I(1,t)*	*I(2,t)*	*I(3,t)*	*P(1,t)*	*P(2,t)*	*P(3,t)*	*Planned Production*	*Aggregate Requirements**
(Weeks)											
1	7	0	1	18.86	2.16	4.23	12.67	0	1.78	14.45	11.14
2	4	1	3	19.32	2.33	7.69	7.24	1.65	5.34	14.23	14.00
3	3	3	0	17.97	5.80	5.81	5.43	4.95	0	10.38	10.36
4	3	0	3	16.62	4.32	9.27	5.43	0	5.34	10.70	10.70

Holding and Setup Costs Excluded: Cost = $18,122.50

	No. of ¼ Shift Lots			*Ending Inventories*			*Production Requirements*				
Period	*Z(1,t)*	*Z(2,t)*	*Z(3,t)*	*I(1,t)*	*I(2,t)*	*I(3,t)*	*P(1,t)*	*P(2,t)*	*P(3,t)*	*Planned Production*	*Aggregate Requirements**
(Weeks)											
1	7	0	1	18.86	2.16	4.23	12.67	0	1.78	14.45	11.14
2	4	3	1	19.32	5.63	4.13	7.24	4.95	1.78	13.97	14.00
3	4	0	2	19.78	4.15	5.81	7.24	0	3.56	10.80	10.36
4	2	2	2	16.62	5.97	7.49	3.62	3.30	3.56	10.48	10.70

* Aggregate production requirements for family 3 from FPM (see Table 2, P(3,t)).

The solution which included holding and setup costs has a total cost only $66 (or 0.4%) less than the solution which ignores holding and setup costs. Note that rounding, which was done on the right-hand sides of the constraints upon removing I(i, t), has resulted in some "overproduction" in the first week. This will produce some excess inventory, which will be remedied the next time the schedule is run.

When family four, containing seven end items, is disaggregated with setups and holding costs included and the binary constraint on N(i, t) imposed, excessive computer run times result (40 minutes). However, the LP relaxation provided almost identical natural integer values for the lot-size variables Z(i, t) at a cost premium of $80 out of $14,256, or some 0.6%. The family-four solution ignoring holding and setup costs produces a natural integer LP solution at a cost of $14,260.50, or a premium of $4.50 above "optimum."

CONCLUSIONS

We have developed a practical method for weekly scheduling of lot sizes under conditions of finite capacity and dynamic demand using a two-stage hierarchical system. This system is compatible with a decentralized, desktop computer environment and the use of spreadsheets. The system will greatly reduce the headaches and costs associated with sending infeasible production plans to the production scheduler. The system explicitly accounts for major family setup costs and lessens the problem of consuming production capability with excessive setups.

This planning system, while it operates in a spreadsheet, requires compatible software which can handle binary variables (e.g., WHAT'S BEST). Currently, spreadsheets possess only limited LP capabilities and do not permit such variables.

We are currently in the early stages of implementation of this system at Welch's. We expect the complete system will add no more than approximately two hours of computer time per week to the company-wide production planning process.

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APPENDIX

Family Planning Model (FPM)

Define

CH(i) = holding cost for family i, $/unit/period
CO(i) = overtime production cost for family i, $/unit
CS(i) = setup cost for family i, $/setup.
I(i, t) = inventory level of family i at the end of period t, units
P(i, t) = regular time production of family i during period t, units
O(i, t) = overtime production of family i during period t, units
D(i, t) = demand (forecast) for family i during period t, units
B(i, t) = buffer stock requirement for family i at the end of period t, units — supplied by IPM.
r(i) = time required to produce one unit of family i on regular time or overtime, hours/unit.
S(i) = setup time for family i, hours
AR(t) = total regular time hours available in period t
AO(t) = total overtime hours available in period t
N(i, t) = 1 if family i is produced in period t, 0 otherwise
Q(i) = fictitious demand, at least as large as the total demand over the appropriate time horizon

$$\text{Min:} \quad \sum_i \sum_{t=1}^{4} [CH(i)I(i,t) + CO(i)O(i,t) + CS(i)N(i,t)] + \sum_{t=5}^{9} [CH(i)I(i,t) + CO(i)O(i,t)] \quad (1)$$

Subject to:

$$I(i, t-1) - I(i,t) + P(i,t) + O(i,t) = D(i,t); \quad \text{for all } i, t. \quad (2)$$

$$\sum_i [r(i)P(i,t) + S(i)N(i,t)] \le AR(t); \quad t = 1 \text{ to } 4. \quad (3)$$

$$\sum_i [r(i)P(i,t)] \le AR(t); \quad t = 5, 6, \ldots, 9. \quad (4)$$

$$\sum_i [r(i)O(i,t)] \le AO(t); \quad \text{for all } t. \quad (5)$$

$$I(i,t) \ge B(i,t); \quad \text{for all } i, t. \quad (6)$$

$$P(i,t) - Q(i)N(i,t) \le 0; \quad \text{for all } \ i, t = 1 \text{ to } 4. \quad (7)$$

$$O(i,t) - P(i,t) \le 0; \quad \text{for all } \ i, t = 1 \text{ to } 4. \quad (8)$$

For Q(i), we require $Q(i) \ge \sum_{t=1}^{4} D(i,t)$, and N(i, t) binary.

Note that the first four periods are weeks and the next five periods are months. Care must be taken with any time-based quantities.

Disaggregation Planning Model (DPM)

We will adopt the FPM notation with i now representing the individual product number rather than the family. Denote the total family requirement (regular plus overtime) in period t as determined by FPM for a specified family as $P(\cdot, t)$. For each family, solve the mixed LP problem:

$$\text{Min:} \quad \sum_i \{\sum_t [CH(i)I(i,t) + CS(i)N(i,t)]\} + \sum_t M[E(t) + S(t)] \quad (9)$$

Subject to: Constraints (2), (6), (7), and

$$\sum_i P(i,t) - E(t) + S(t) = P(\cdot, t); \quad \text{for all } t \quad (10)$$

where M is very large compared to other objective function coefficients.

Disaggregation to Integer Lot Sizes, Including Holding and Setup Costs (DPMIHS)

$$\text{Min:} \quad \sum_i \{\sum_{t=1}^{4} \bar{p}CH(i) \sum_{k=1}^{t} Z(i,k) + CS(i)N(i,t)\} + \sum_{t=1}^{4} M[E(t) + S(t)] \quad (11)$$

Subject to:

$$\text{Buffers:} \quad \sum_{k=1}^{t} Z(i,k) \ge \{\sum_{k=1}^{t} D(i,k) + B(i,t) - I(i,0)\}/\bar{p} \quad \text{for all } i, t. \quad (12)$$

Prod Reqmts: $\sum_{i} Z(i, t) - E(t) + S(t)$

$$= P(\cdot, t)/\bar{p} \quad \text{for all } t \quad (13)$$

Binary: $Z(i, t) - Q(i)N(i, t) \leq 0 \quad \text{for all } i, t \quad (14)$

$N(i, t)$ binary. (15)

$Z(i, t)$ integer. (16)

Disaggregation to Integer Lot Sizes, Ignoring Holding and Setup Costs (DPMI)

$$\text{Min:} \quad \sum_{t} M[E(t) + S(t)] \qquad (17)$$

Subject to: (12), (13).

Constraint (16) is not needed. The LP solution is integer valued.

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