Supply Chain Risk

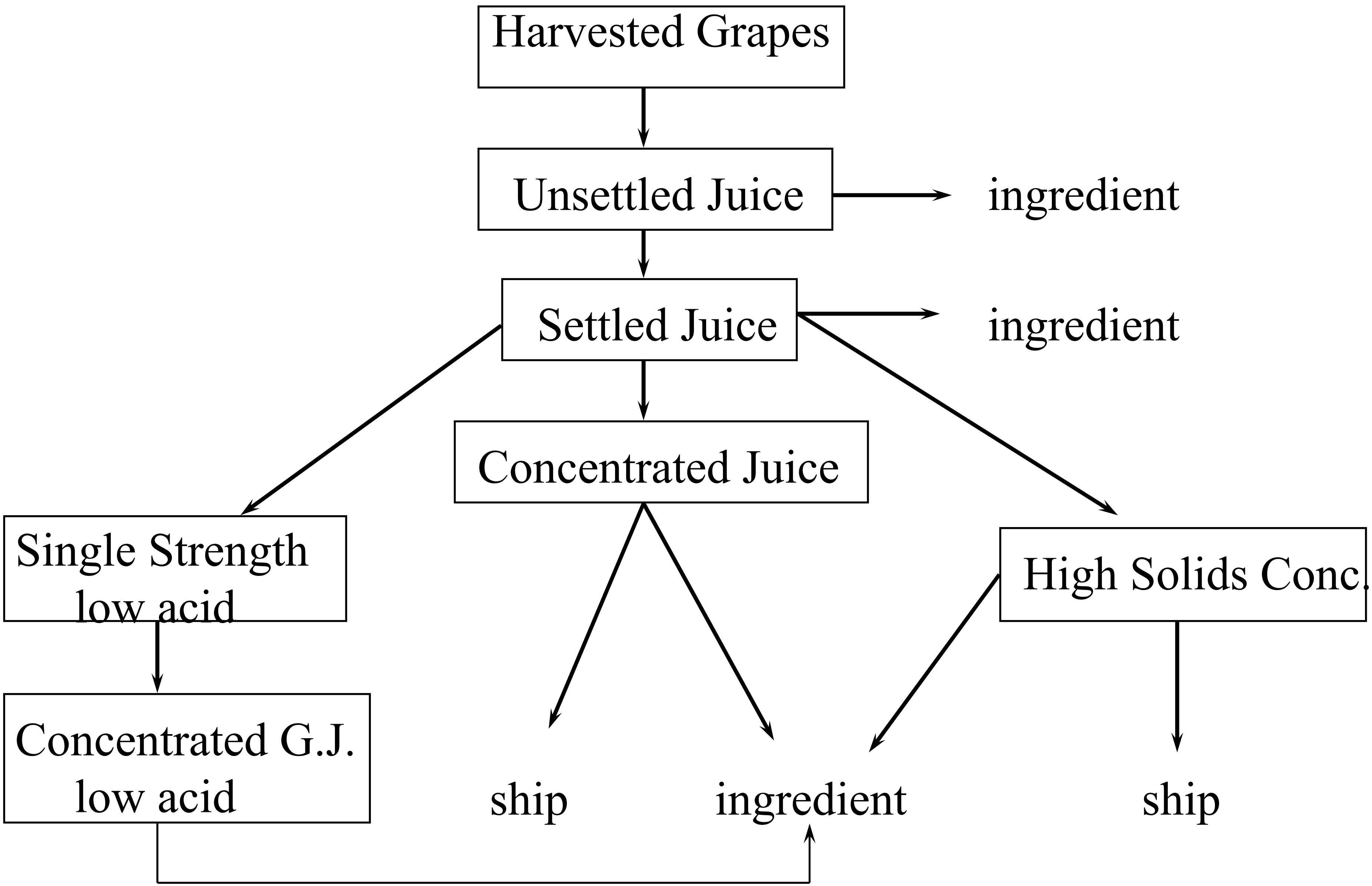
Modeling the Harvest

1996 - 2002

Stuart J.Allen, Professor Emeritus Penn State

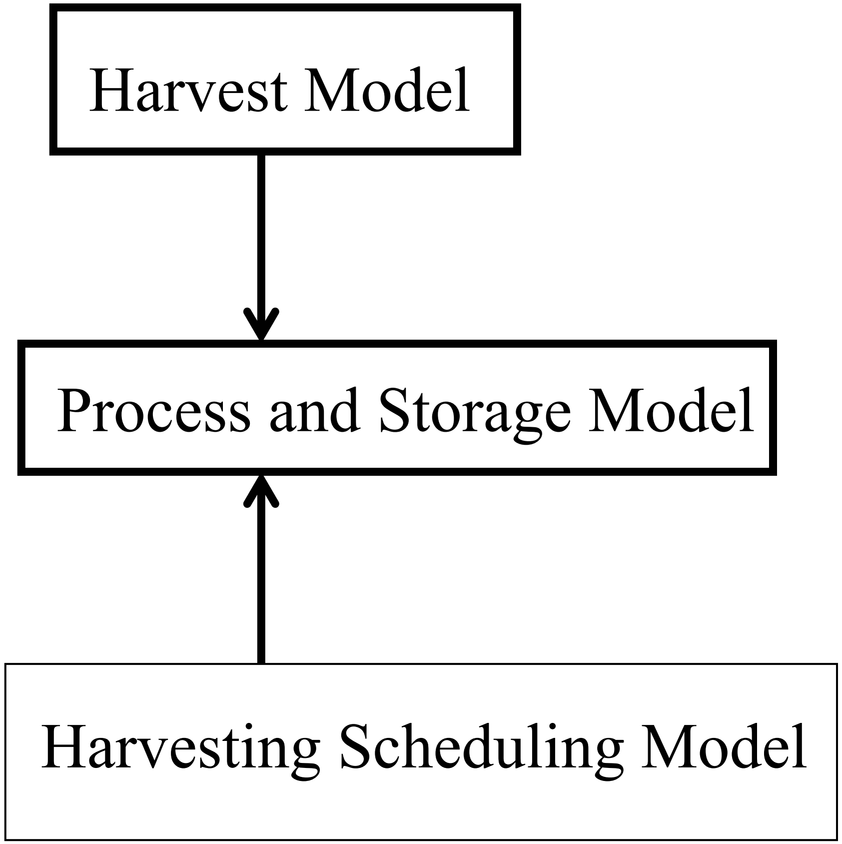
Edmund W. Schuster, MIT

Steps in Grape Juice Processing



Harvest Model Introduction

- The harvest involves one of the oldest decisions involving risk
- The “harvest model” determines the optimal rate to harvest the crop based on risk
- The model is categorized as probabilistic, with an analytical solution.
- The model gives reasonable solutions.
- Savings of $1.5 MM in practice



Output

Rate of processing per day
to meet "policy."

Amount of investment
to meet "policy"

Scheduling of the harvester
operators, and communication

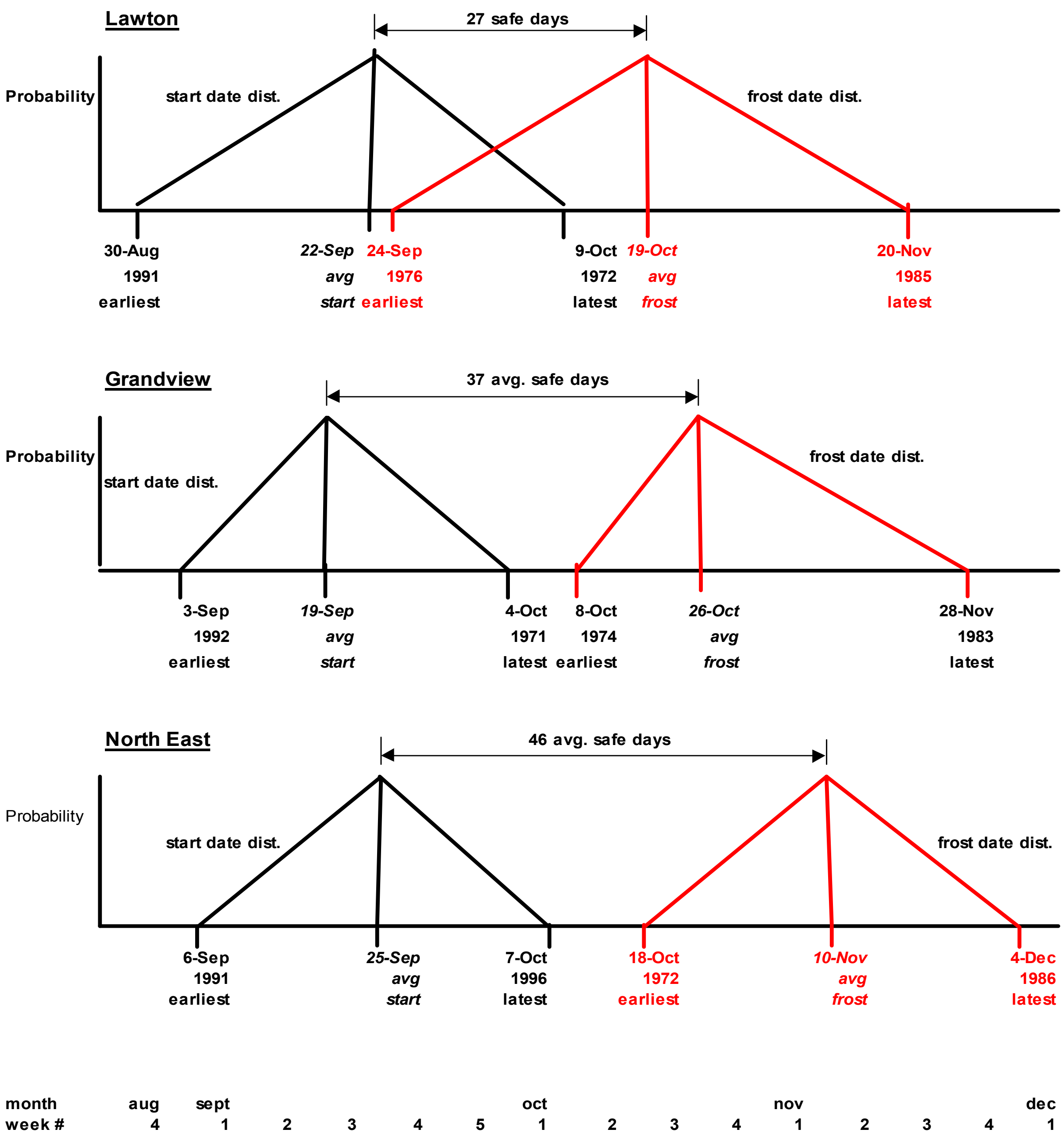
Purpose of the "Harvest Model"

- Balance the growers desire to harvest all grapes before a hard frost verses capital expenditures required for maximum through-put rate.
- Historically, cooperatives used <u>fixed-length of harvest</u> to plan the though-put rate.
- The fixed-length of harvest method ignored the risk of a hard freeze

Definition of “Policy”

- Take 100% of the crop, 85% of the time
- Implies a harvest rate (R) required to meet the policy
- By defining a “statistical” policy for receiving grapes we can make trade-offs between harvest capacity and investment in equipment
- We calculated a “loss function” and found the 85% policy to be optimal

Qualitative Comparison of Start Dates and First 28 Degree Day
With Estimated Triangular Distributions



Comments (avg. safe days = 27):

Lawton has a significant risk of a frost occurring shortly after the start of harves

Six times during the past 27 years (22%) Lawton had less than 10 safe days.

Comments (avg. safe days = 37):

The West has little, if any, chance of a frost occurring shortly after harvest begins.

Over a 50% chance of frost occurring in October

Comments (avg. safe days = 46):

North East has little chance of frost in October.

Largest amount of average safe days.

Data Required for the Harvest Model

- Harvest Size - we use the average of the LRP for Concord, for each growing area
- Historical analysis shows the harvest size to be normally distributed with $\mu(H)$ representing the average (forecast) $\sigma(H)$ and representing the standard deviation of the crop

Data (continued)

- We use the “start date” and “end date” provided by National to calculate the length of season, L.
- We assume the distribution of the season length to be normal (based on observations of histograms), again $\mu(L), \sigma(L)$.
- L is not correlated with harvest size, H.
  - .14 correlation with significance of 53%.

Mathematical Development of the Harvest Model

For a growing area:

R = Harvest rate in tons per day, **a decision variable**
T = Time required to harvest the entire crop in days

The time to harvest the crop is $T = H/R$

and we define the "slack" time as the difference

between the length of harvest season, L, and the time

required to harvest 100% of the crop;

$S = L - T$

Now the slack time S is normally distributed and if we know the **mean** and **standard deviation**, we can enforce a policy requirement that the entire crop be harvested with stated probability, p:

P(S>=0) = p, where p might range from 85% to 98%

The estimated expected value of the slack is:

$$\mu(S) = \mu(L) - \mu(H) / R$$

We concluded statistical independence between length of harvest season and harvest size, then we have:

$$\sigma^2(S) = \sigma^2(L) + \sigma^2(H) / R^2$$

Assuming the slack time, S, is normally distributed, we

define Z(p) as the standard normal value of Z

associated with an upper-tailed policy level of p

$$\mu(S) \geq -Z(p)\sigma(S)$$

For realistic policy requirements, $p>.50$ and $Z(p)<0$.

Using the above equations and doing some substitutions:

$$Z^2[\sigma^2(L) + \sigma^2(H) / R^2] \leq \mu(L) - \mu(H) / R]^2$$

With R being the decision variable, on both sides of the equation, we can solve through trial an error until both sides of the equation are equal, or we can use goal seek in Excel to find the solution.

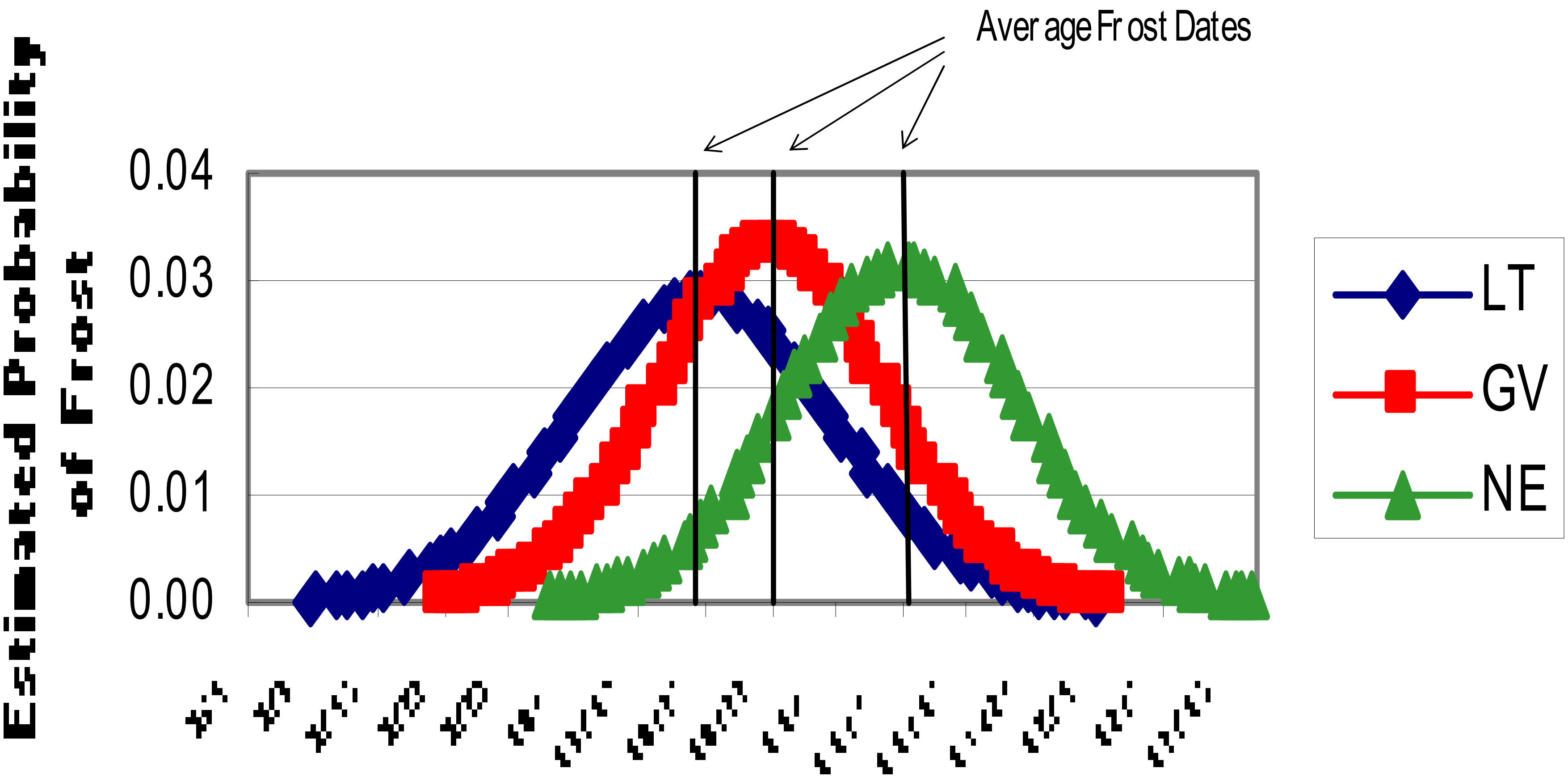
Harvest Model Output: Average Concord Receiving Rate per Day (all values in tons)

Policy*	Mi	Wa	NE/Wfd
70%	1,850	2,740	2,670
80%	2,290	3,220	3,000
85%	<u>2,610</u>	<u>3,520</u>	<u>3,200</u>
90%	3,450	4,250	3,675
95%	5,700	5,680	4,450
Current	**1,900**	**4,000**	**3,650**

*Standard policy is to “receive 100% of the crop, 85% of the time

NOTE: data from 1997

Climate Difference: Distribution of Freeze Dates



Harvest Risk With Costs

“The Food Chain: Managing Harvest Risk”

C(H) = Cost of lost harvest (run too slow)

C(R) = Cost of harvesting the crop too early (run too fast)

The result is a flexible model that gives the optimal harvest
Rate and risk based on the cost information.

Applications to the short product life-cycle lot sizing problem.

An Example (Sweaters)

Average number of sweaters sold per season	= 800
Standard Deviation	= 150
Average length of selling season	= 84 days
Standard Deviation	= 10 days
Cost of unsold inventory	= $60
Cost of a lost sale	= $40

The Optimal Solution

Order 834 sweaters

Expected % Recovery = 96%