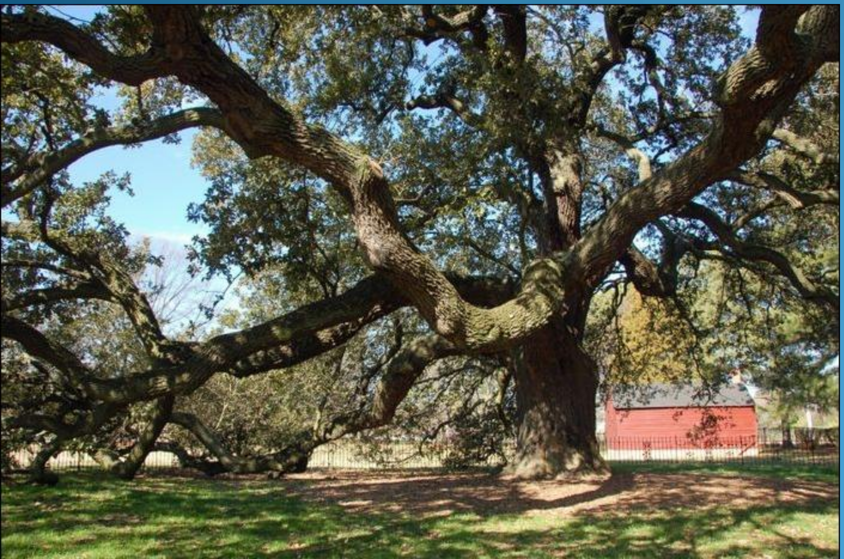
Electron-Ion Collider Collaboration Meeting
Hampton University, 19-23 May, 2008

Gluon Saturation and Black Holes

1. Introduction
2. BFKL evolution & saturation in DIS
3. Critical gravitational collapse
4. Saturation/black hole holography?
5. Open questions & future work



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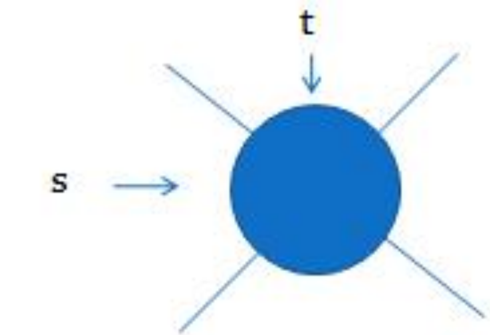
Miguel Angel Vázquez-Mozo

1. Introduction

High energy limit of scattering amplitudes in QCD:



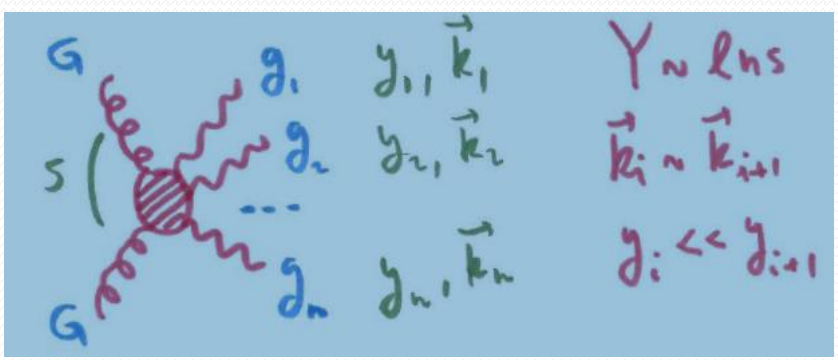
$s \gg |t|, \; Q^2 \gg \Lambda^2_{QCD}$

$\alpha_s(Q^2) \ll 1$

Large logarithms in s compensate small coupling and a full resummation is needed:

$$\mathrm{BFKL} \sim \sum_{n=1}^{\infty} (\alpha_s \ln s)^n$$

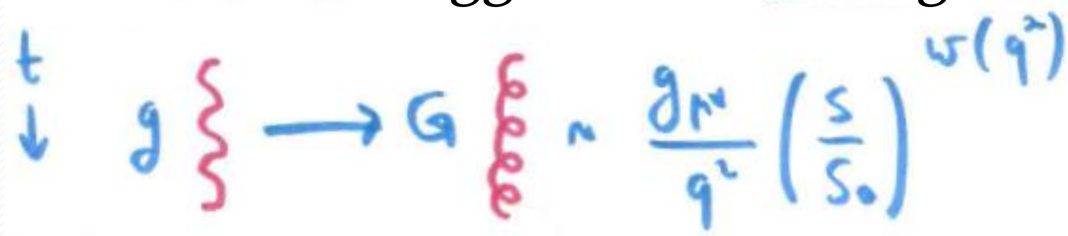
In multi-Regge kinematics:



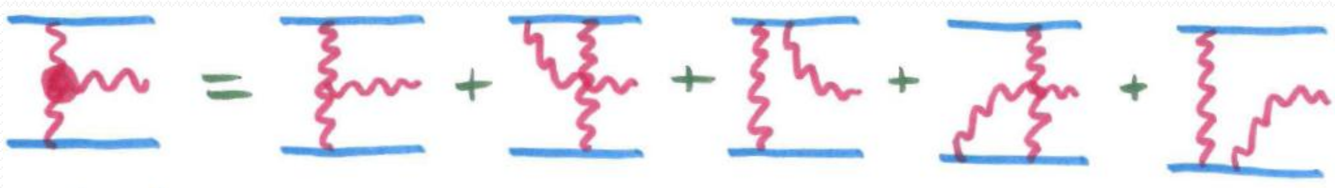
$$\bar{\alpha}_s^n \int_0^Y dy_1 \int_0^{y_1} dy_2 \cdots \int_0^{y_{n-1}} dy_n \sim \frac{(\alpha_s Y)^n}{n!}$$

In this limit new effective degrees of freedom appear

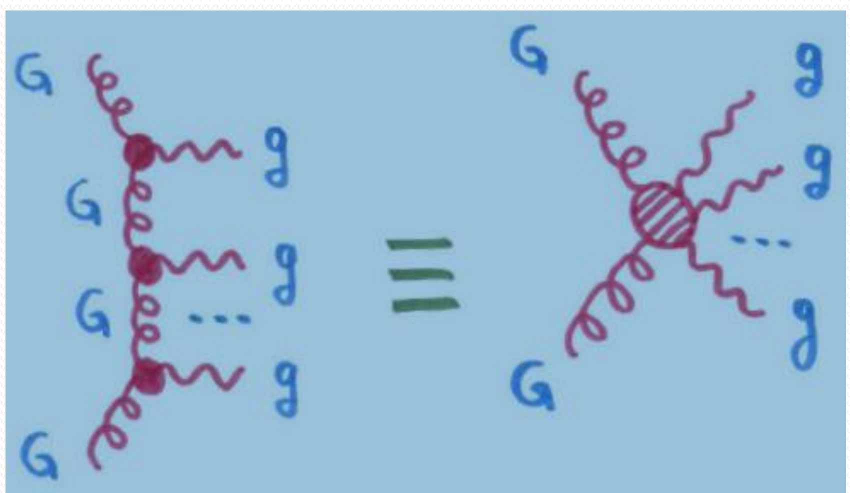
VIRTUAL contributions Reggeize t-channel gluons



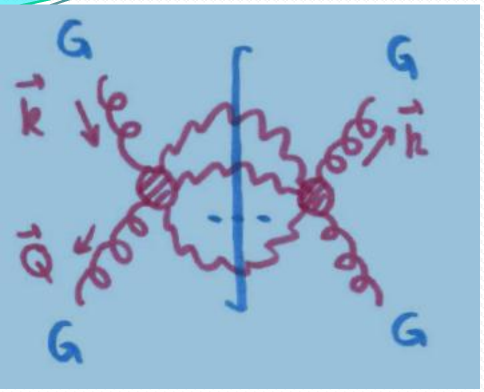
REAL emissions create a gauge invariant effective vertex



2 to 2+n soft gluon amplitudes have ladder structure



Multijet cross sections:



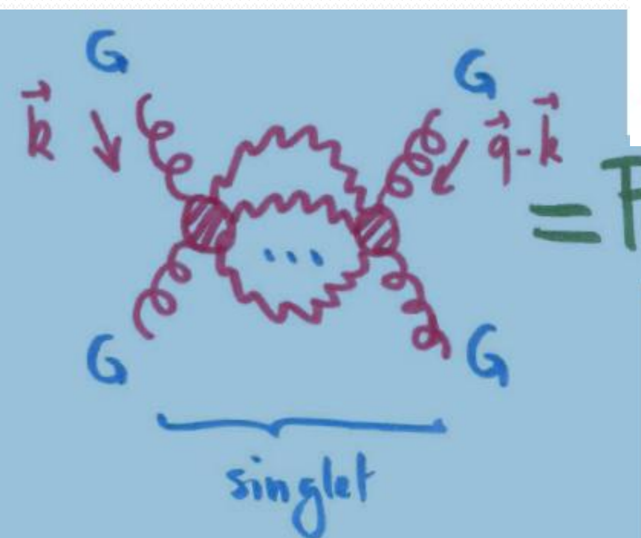
$$f(\vec{k},\vec{Q},Y) \sim \sum_{n=-\infty}^{\infty} \int \frac{d\omega}{2\pi i} e^{\omega Y} \int \frac{d\gamma}{2\pi i} \left(\frac{\vec{k}^2}{\vec{Q}^2}\right)^{\gamma} \frac{e^{in\theta}}{\omega - \bar{\alpha}_s \chi_n(\gamma)}$$

$$\chi_n(\gamma) = 2\psi(1) - \psi\left(\gamma + \frac{|n|}{2}\right) - \psi\left(1-\gamma+\frac{|n|}{2}\right)$$

$n \rightarrow$ conformal spins

Diffractive events:

Hard Pomeron = bound state of 2 Reggeized gluons.

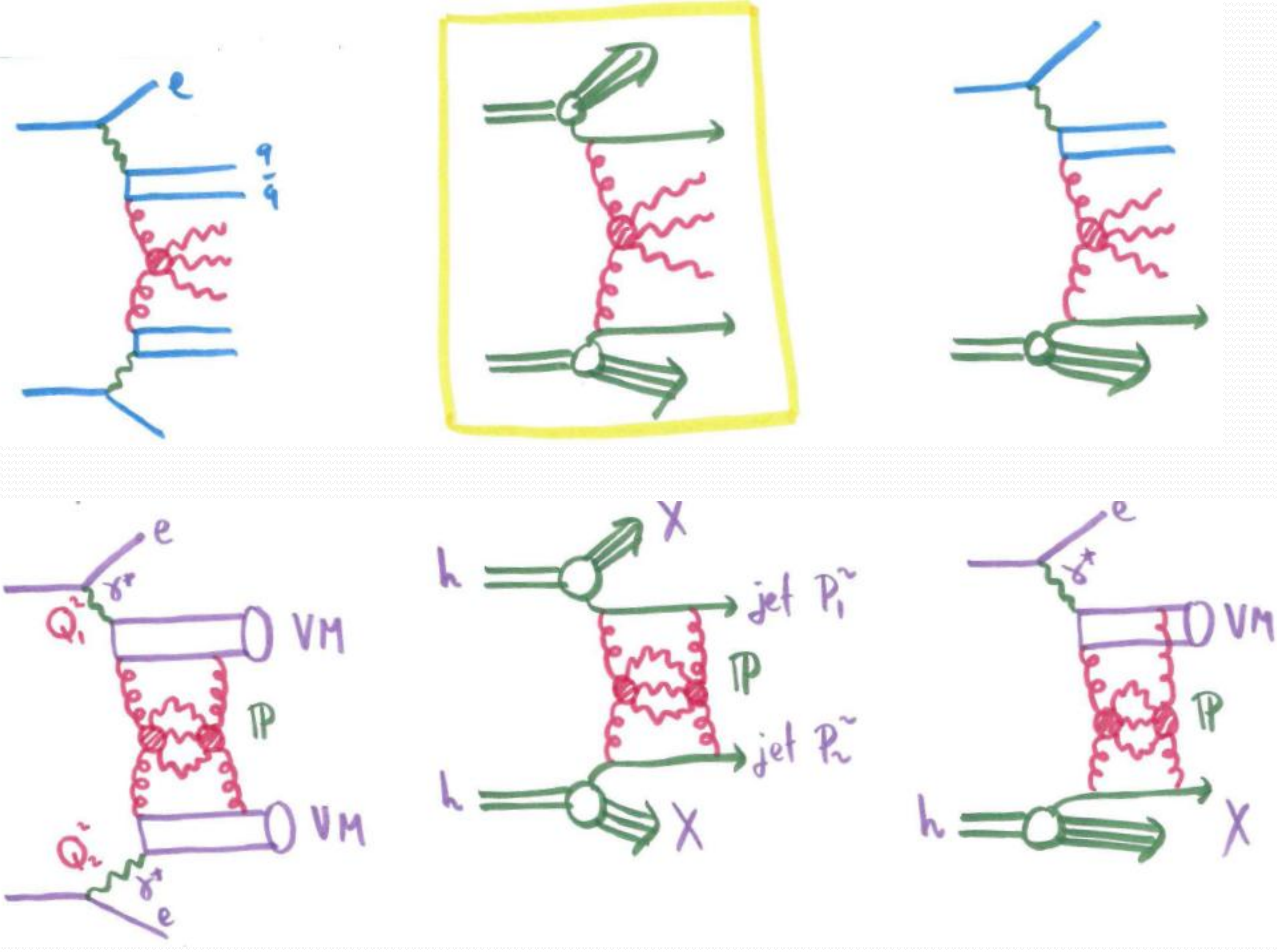


= Pomeron

$$\frac{\partial}{\partial(\alpha_s Y)} f(\vec{k},\vec{q},Y) = \int d\vec{k}' \, K(\vec{k},\vec{k}',\vec{q}) \, f(\vec{k}',\vec{q},Y)$$

$$q = q_x + i\, q_y \qquad q^* = q_x - i\, q_y$$

$$K(\vec{k},\vec{k}',\vec{q}) \qquad z_i \rightarrow z_i' = \frac{a z_i + b}{c z_i + d}$$



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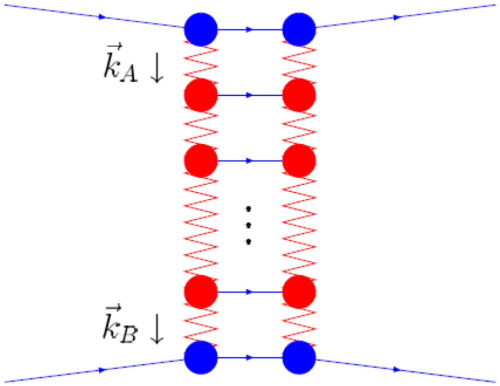
2. BFKL evolution & Saturation in DIS



$$\bar{\varphi}\left(k_A, k_B, \mathrm{Y}\right) = \frac{1}{\pi k_A k_B} \int \frac{d\gamma}{2\pi i} \left(\frac{k_A^2}{k_B^2}\right)^{\gamma - \frac{1}{2}} e^{\chi(\gamma)\bar{\alpha}_s \mathrm{Y}}$$

At large energies the saddle point $\gamma = 1/2$ dominates

$$\chi(\gamma) \simeq 4 \log 2 + 14\,\zeta_3 \left(\gamma - \frac{1}{2}\right)^2 + \cdots$$

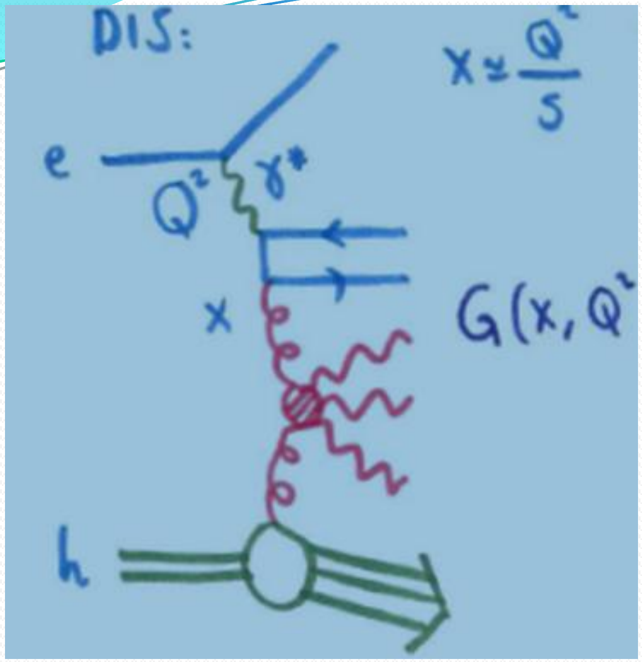
$$\bar{\varphi}\left(k_A, k_B, \mathrm{Y}\right) \simeq \frac{1}{2\pi k_A k_B} e^{\Delta \mathrm{Y}} \frac{1}{\sqrt{14\pi\zeta_3\bar{\alpha}_s \mathrm{Y}}} e^{\frac{-t^2}{56\zeta_3\bar{\alpha}_s \mathrm{Y}}}$$ with $t \equiv \log\left(k_A^2/k_B^2\right)$

IR/UV symmetric diffusion in transverse momenta for

$$\Phi\left(k_A, k_B, \mathrm{Y}\right) \equiv k_A\, k_B\, \bar{\varphi}\left(k_A, k_B, \mathrm{Y}\right) \qquad \frac{\partial \Phi}{\partial(\bar{\alpha}_s \mathrm{Y})} = 4 \log 2\, \Phi + 14\,\zeta_3 \frac{\partial^2 \Phi}{\partial t^2}$$

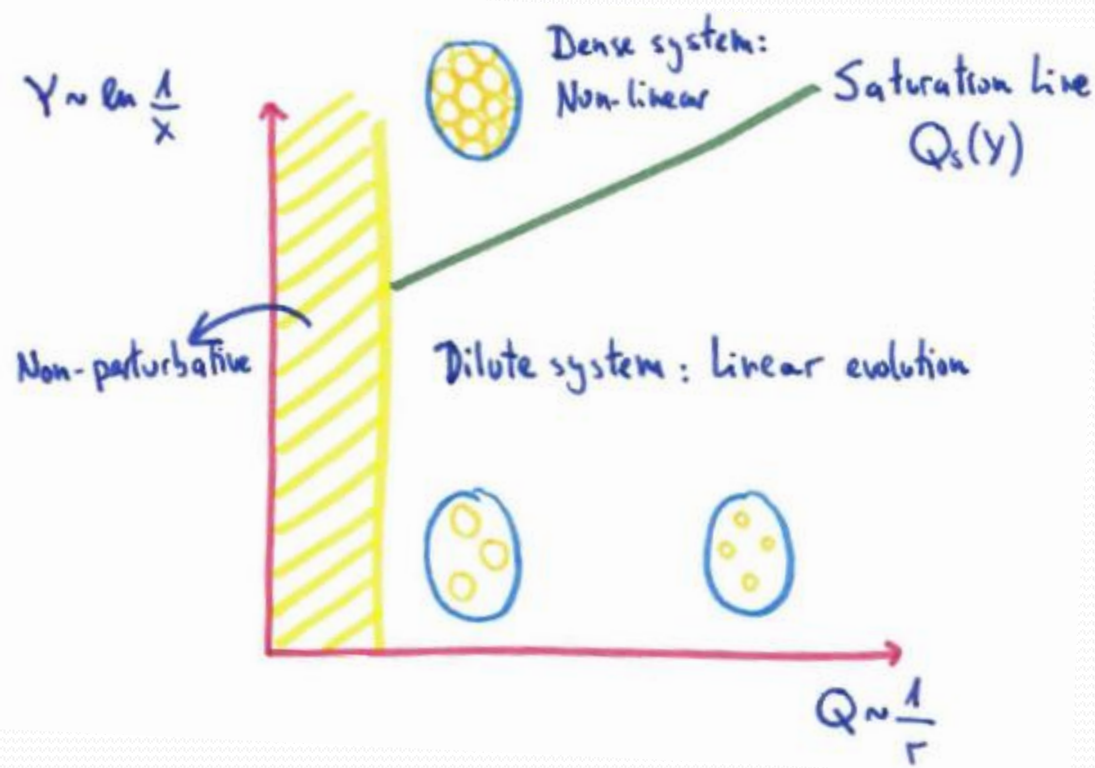
$$\chi(\gamma) = 2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)$$

$\gamma \to 1 - \gamma$ invariant



$$f(x, \mathbf{k}^2) \sim \left(\frac{x}{x_0}\right)^{-\lambda}$$ violates unitarity bounds

BFKL increases number of gluons of a fixed transverse size 1/Q



Perturbative degrees of freedom at high density dominated by nonlinearities

Non-linearities needed to damp this growth

For large targets BK equation is a good candidate:

$$\frac{\partial \Phi\left(k_A, k_B, \mathrm{Y}\right)}{\partial(\bar{\alpha}_s \mathrm{Y})} = -\Phi\left(k_A, k_B, \mathrm{Y}\right)^2$$
$$+\int_0^1 \frac{dx}{1-x}\left[\Phi\left(\sqrt{x} k_A, k_B, \mathrm{Y}\right)+\frac{1}{x} \Phi\left(\frac{k_A}{\sqrt{x}}, k_B, \mathrm{Y}\right)-2 \Phi\left(k_A, k_B, \mathrm{Y}\right)\right]$$

Non-linearities can be introduced with weighted diffusion in linear evolution:

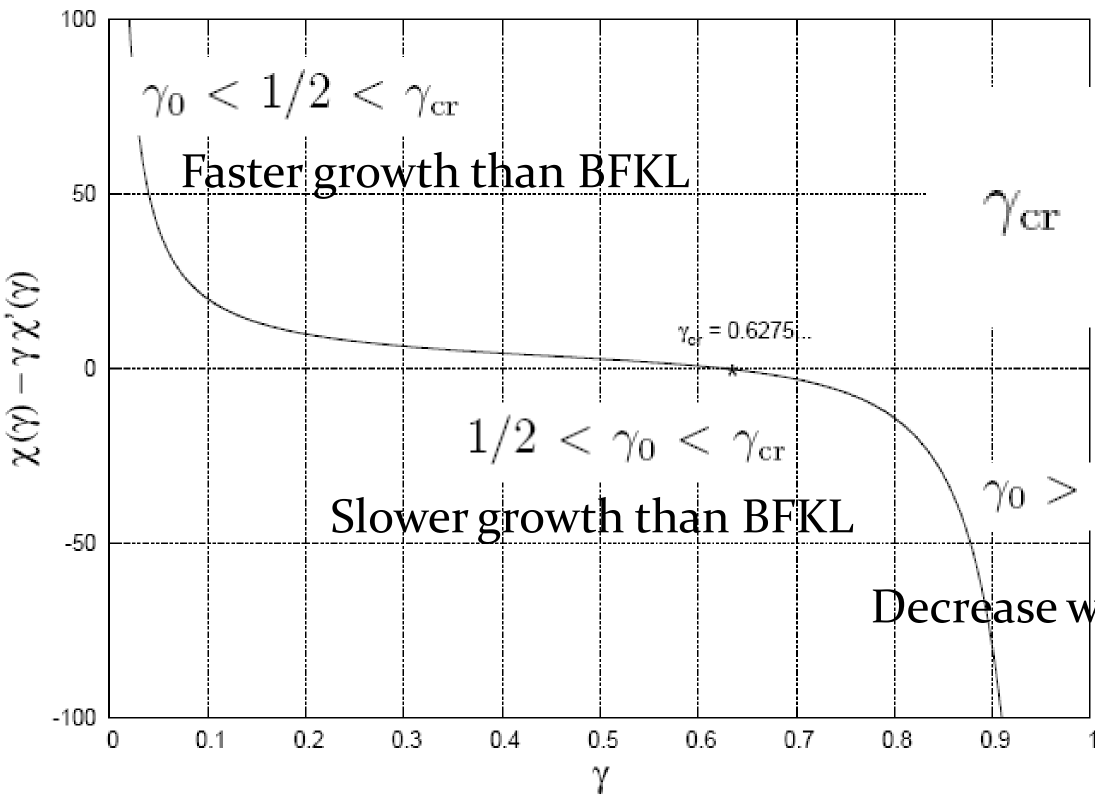
$$\bar{\varphi}\left(Q_{\text{targ}}, Q_{\text{proj}}, \mathrm{Y}\right) = \frac{1}{\pi Q_{\text{targ}}^2} \int \frac{d\gamma}{2\pi i}\left(\frac{Q_{\text{targ}}^2}{Q_{\text{proj}}^2}\right)^{\gamma} e^{\chi(\gamma) \bar{\alpha}_s \mathrm{Y}}$$

forced to have a
different saddle point

$$\chi'(\gamma_0) \bar{\alpha}_s \mathrm{Y}+\log\left(\frac{Q_{\text{targ}}^2}{Q_0^2}\right) = 0$$

$$\chi(\gamma) \simeq \chi(\gamma_0)+\chi'(\gamma_0)(\gamma-\gamma_0)+\frac{1}{2} \chi''(\gamma_0)(\gamma-\gamma_0)^2+\cdots$$

$$\bar{\varphi}\left(Q_{\rm targ}, Q_{\rm proj}, {\rm Y}\right) \simeq e^{\gamma_0 t_0 + \bar{\alpha}_s {\rm Y}(\chi(\gamma_0) - \gamma_0 \chi'(\gamma_0))} \frac{e^{\frac{-t_0^2}{2\chi''(\gamma_0)\bar{\alpha}_s {\rm Y}}}}{\pi Q_{\rm targ}^2 \sqrt{\chi''(\gamma_0) 2\pi \bar{\alpha}_s {\rm Y}}}$$



$\gamma_0 < 1/2 < \gamma_{\rm cr}$

Faster growth than BFKL

$1/2 < \gamma_0 < \gamma_{\rm cr}$

Slower growth than BFKL

$\gamma_0 > \gamma_{\rm cr}$

Decrease with energy

$$\gamma_{\rm cr} = \frac{\chi(\gamma_{\rm cr})}{\chi'(\gamma_{\rm cr})} \simeq 0.6275...$$

For $\gamma_0 = \gamma_{\rm cr}$ there is no growth with energy

$$\gamma \to 1 - \gamma$$

symmetry broken

$$\gamma_0 = \gamma_{cr}$$

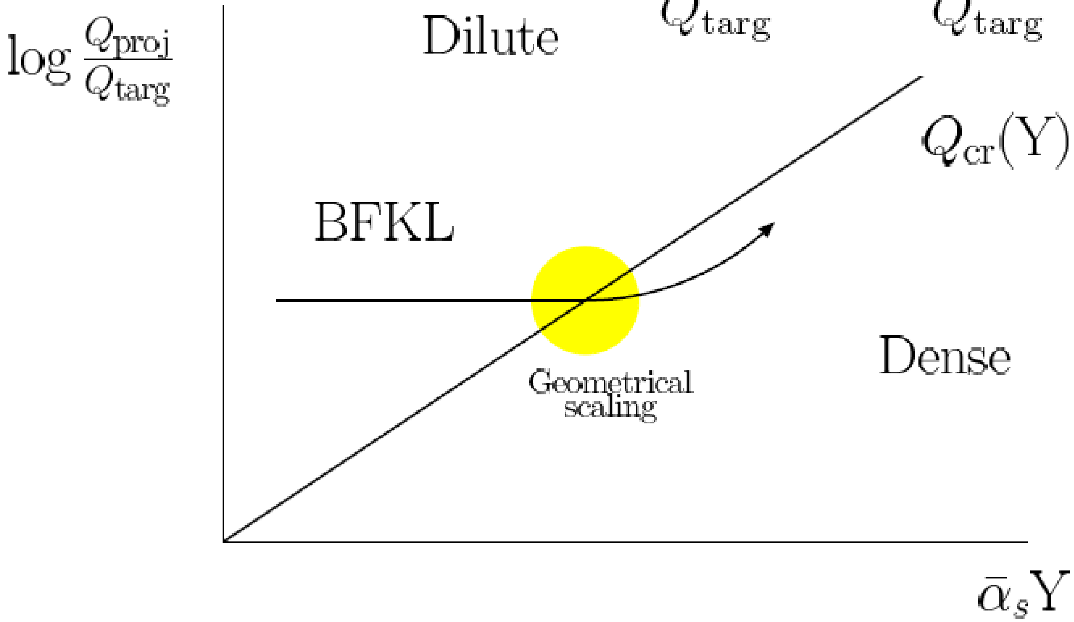
IR suppression

$$\bar{\varphi}\left(Q_{\rm targ}, Q_{\rm proj}, Y\right) \simeq \left(\frac{Q_{cr}(Y)}{Q_{proj}}\right)^{2\gamma_{cr}} \frac{e^{\frac{-t_{cr}^2}{2\chi''(\gamma_{cr})\bar{\alpha}_s Y}}}{\pi Q_{\rm targ}^2 \sqrt{\chi''(\gamma_{cr}) 2\pi \bar{\alpha}_s Y}}$$

Critical line: $Q_{cr}(Y) \; = \; Q_{\rm targ} \exp\left[\frac{\chi'(\gamma_{cr})}{2}\bar{\alpha}_s Y\right]$

Solution invariant under geometrical scaling:

$$\bar{\alpha}_s Y \;\rightarrow\; \bar{\alpha}_s Y + \log\lambda,$$
$$\frac{Q_{\rm proj}}{Q_{\rm targ}} \;\rightarrow\; \frac{Q_{\rm proj}}{Q_{\rm targ}}\lambda^{\frac{\chi'(\gamma_{cr})}{2}}.$$



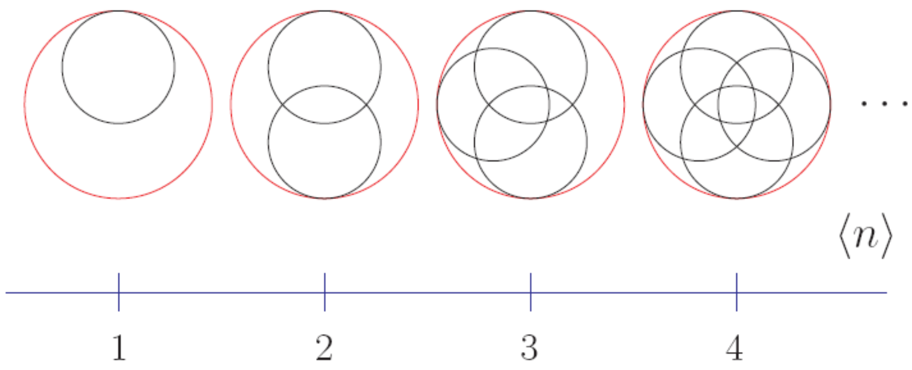
with critical exponent

$$\boxed{\chi'(\gamma_{cr})/2 \simeq 2.4417...}$$

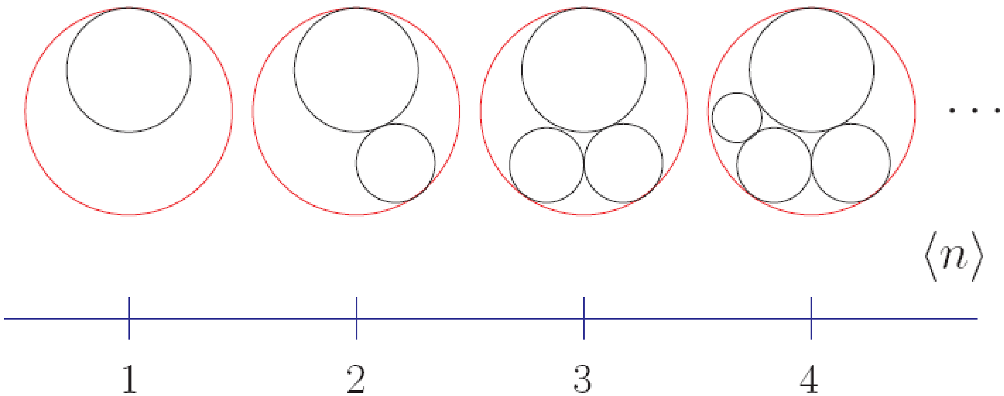
for the crossover dilute-dense transition

Main features of saturation:

1. Dilute/dense transition
2. Scaling symmetry
3. Critical exponent 2.44
4. IR/UV competition



At asymptotic energies linear evolution has no memory on transverse sizes



When memory is introduced infrared modes are suppressed

$$\mathcal{T}_{\rm cr} \;=\; \mathcal{T}_{\rm targ} \exp\left[-\frac{\chi'(\gamma_{\rm cr})}{2}\bar{\alpha}_s \mathrm{Y}\right]$$

3. Critical gravitational collapse

References:

Álvarez-Gaumé, Gómez, Vázquez-Mozo, PLB (2007)

Álvarez-Gaumé, Gómez, SV, Tavanfar, Vázquez-Mozo, arXiv:0710.2517 [hep-th], arXiv:0804.1464 [hep-th]

LO BFKL:

- The coupling is fixed and carries colour factor

$$\bar{\alpha}_s \equiv \frac{\alpha_s(\mu) N_c}{\pi}, \ \mu \text{ is the } \overline{\text{MS}} \text{ scale.}$$

- No fermions
- The same kernel in all SUSY theories [Lipatov]
- Holomorphically separable and SL(2,C) invariant [Lipatov]
- Iterated in s-channel with periodic BC corresponds to an integrable Heisenberg ferromagnet. [Faddeev, Korchemsky]

Holographic interpretation at large coupling? [Brower-Polchinsky-Strassler-Tan] [Cornalba-Costa-Penedones]

Gravity dual of the saturation line? [Hatta-Iancu-Mueller]

$$Q_s(Y) = Q_0 e^{\bar{\alpha}_s Y \, 2.44}$$

Important: in the gauge theory side we are at small coupling

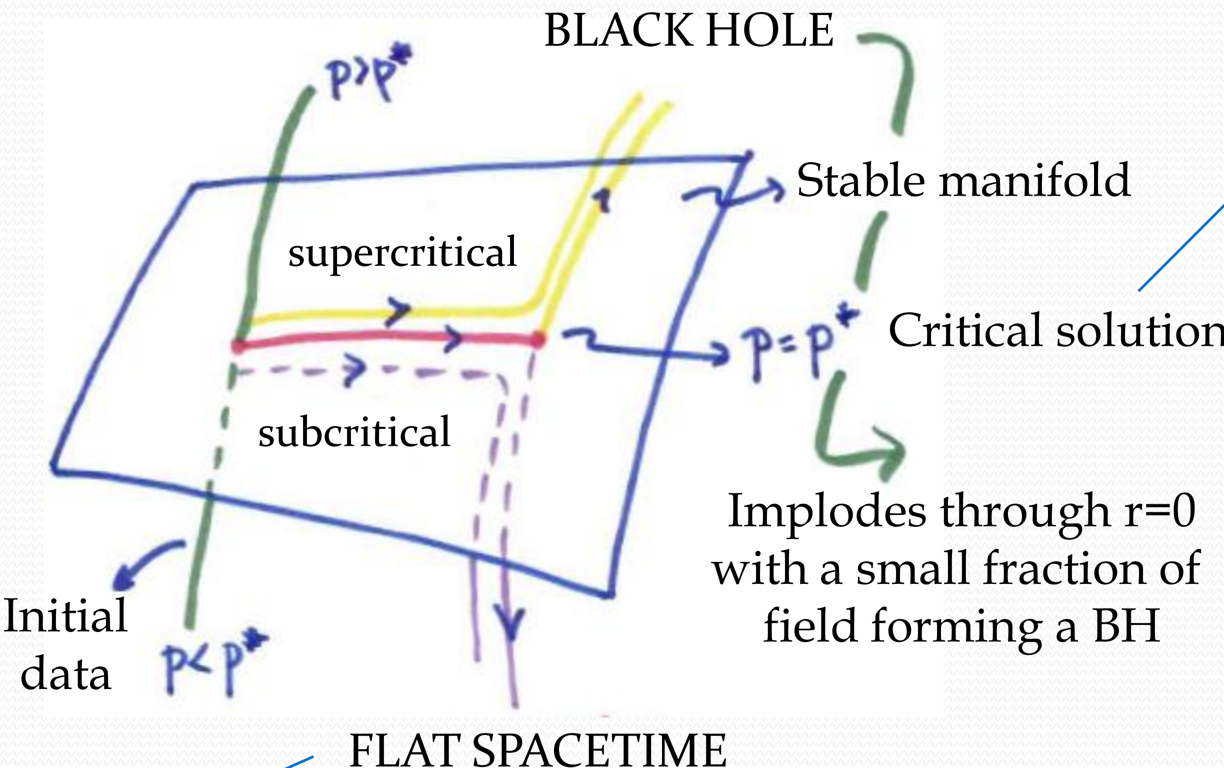
Dual of perturbative parton saturation in gravity?

$Q_s(Y) = Q_0\, e^{\bar{\alpha}_s Y\, 2.44}$

First hint: Black hole formation in gravitational collapse of matter

Choptuik´s numerical study of spherical symmetric collapse of a massless field

p: a parameter for the gravitational self-interaction of initial data of imploding scalar waves with different radial density



Scalar wave packet implodes and then disperses

This is a line of universal critical dynamics

In all dimensions the radius of the BH scales as

$$r_{\rm BH} \simeq |p - p^*|^{\frac{1}{\lambda_c}}$$

In dimension five:

$$\lambda_c \simeq 2.44$$

[Alvarez Gaume-Gomez-Vazquez Mozo]

$p = p^*$ critical solution is discrete self-similar (DSS)

$Z_*(t,r) = Z_*(e^{\Delta} t, e^{\Delta} r)$

with "echoing" $\Delta \simeq 3.44$

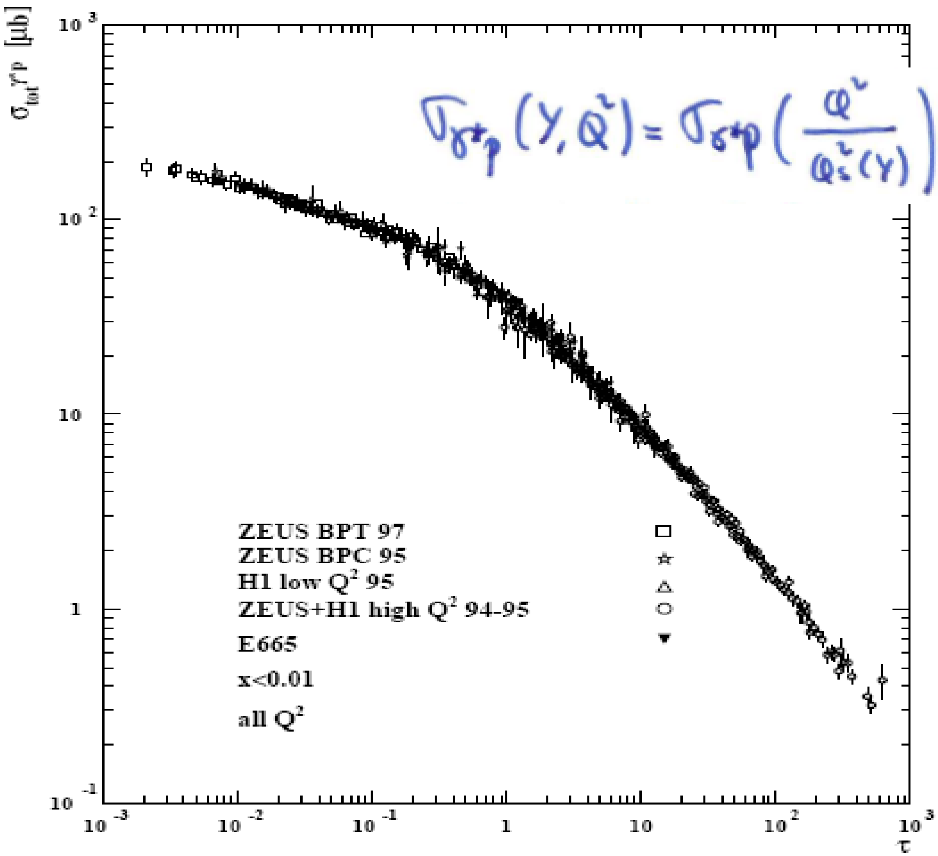
Metric/field components reproduce themselves after an echoing period

This echoing is not present in QCD

QCD has a continuous self-similarity (CSS):

Geometric scaling in DIS data at small x

$\sigma_{\gamma^* p}(Y, Q^2) = \sigma_{\gamma^* p}\left(\frac{Q^2}{Q_s^2(Y)}\right)$



[Stasto-Golec Biernat-Kwiecinski]

CSS in any gravitational collapse?

Spherical collapse of perfect fluid with equation of state

$P = k\rho$

We have studied the gravitational collapse of a perfect fluid in any dimension

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p\, g_{\mu\nu}$$

With barotropic equation of state: $p = k\,\rho, \qquad 0 \le k \le 1.$

At initial time a density of matter is distributed in the radial coordinate r

There is spherical symmetry to avoid gravitational waves:

$$ds^2 = -\alpha(t,r)^2 dt^2 + a(t,r)^2 dr^2 + R(t,r)^2 d\Omega^2_{d-2}$$

This type of collapse was studied exactly by Choptuik in a classical work in numerical relativity.

For a generic initial density, parametrized by p, there is no collapse

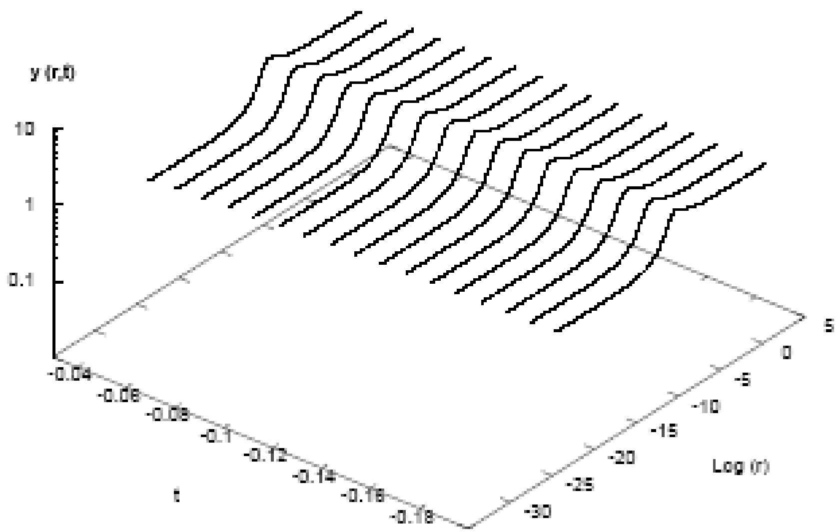
For critical initial density, p*, a small fraction of matter goes through a region dominated by a continuous self-similar scaling law and forms a tiny black hole

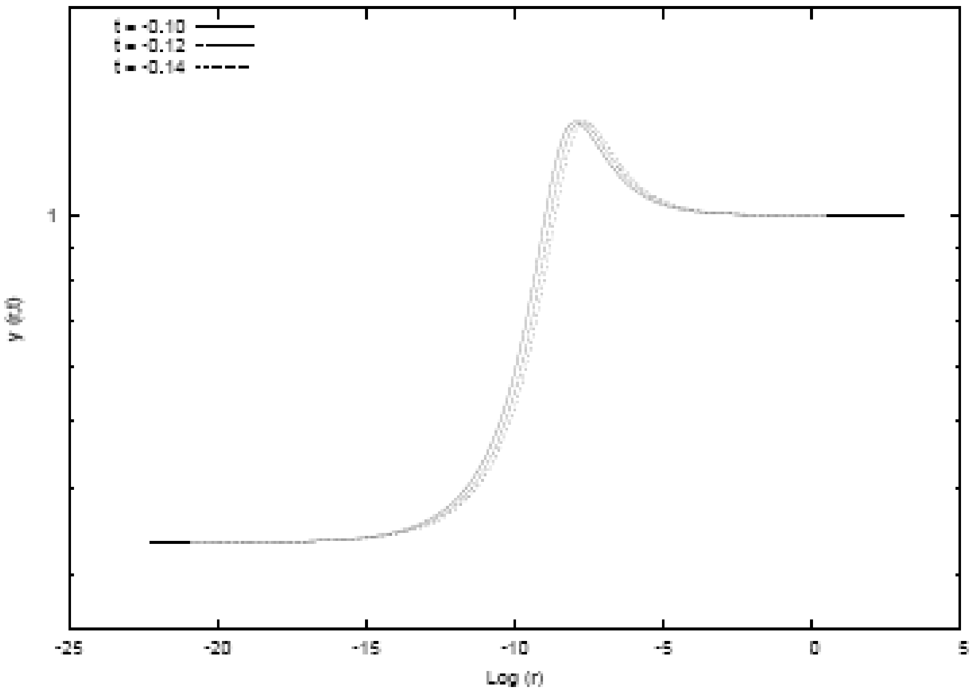
The size of this black hole scales with the formula $\boxed{r_{\rm BH} \quad \sim \quad \ell_0 \left|p - p^*\right|^{1/\lambda_{\rm BH}}}$

Our approach for any dimension is more modest

We impose CSS in Einstein´s equations: critical solution: z=r/t: Z(r,t)=Z(z)

Ratio of the mean density inside the sphere of radius r to the local density at r:

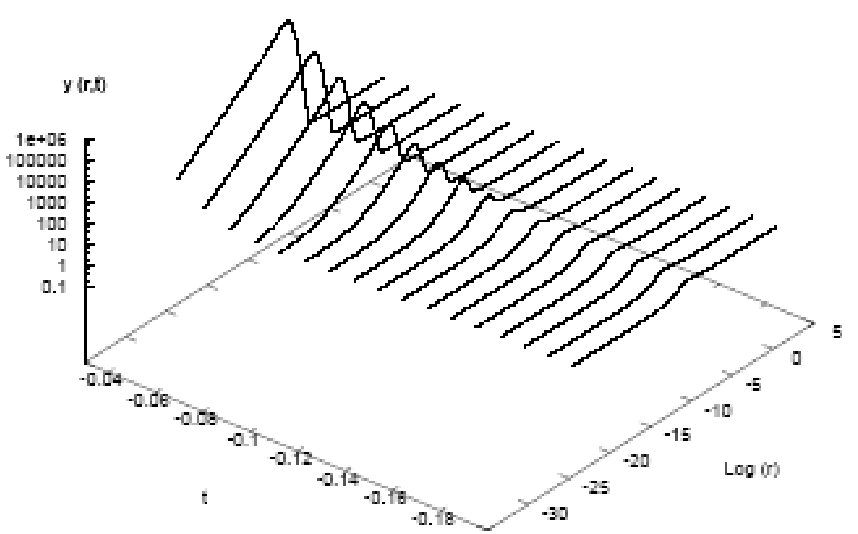


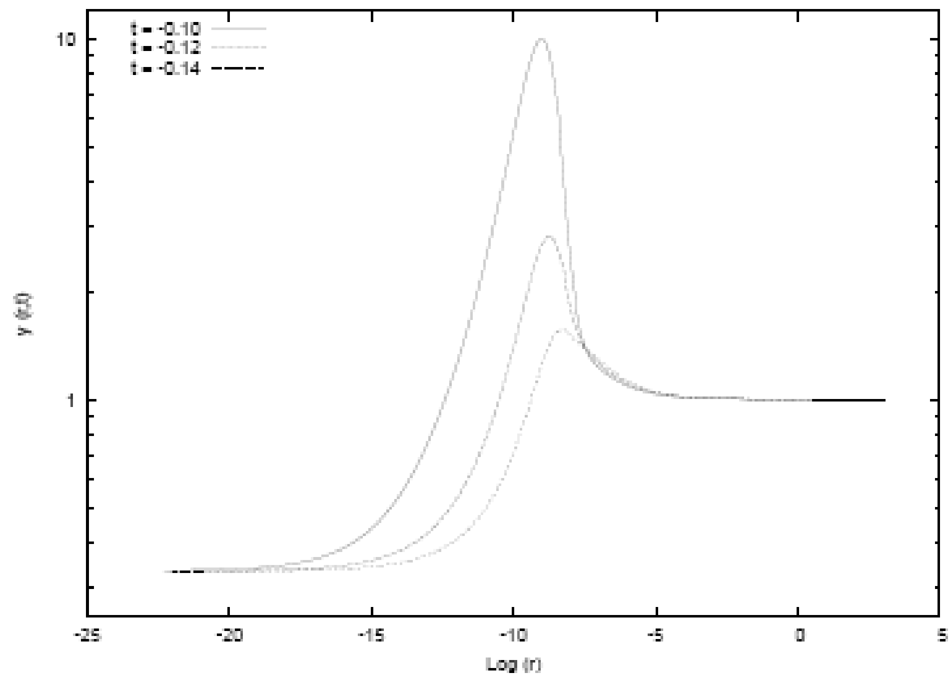


Then we look for an unstable mode in a Liapunov expansion:

$$Z(\tau, z) = Z(z)\left[1 + \epsilon\, e^{\lambda \tau}\, Z_1(z) + \ldots\right]$$

This mode breaks CSS





The Liapunov's mode coincides with Choptuik's critical exponent

$$r_{\mathrm{BH}} \sim \ell_0 \left|p - p^*\right|^{1/\lambda_{\mathrm{BH}}}$$

The one of interest to us is the case of conformal fluid and dimension five.

$$k = 1/4,\ \lambda = 2.58$$

Main features of critical gravitational collapse:

1. Flat/black hole transition
2. Scaling symmetry
3. Critical exponent 2.58
4. Gravity/kinetic competition

k	$\lambda_{d=4}$	$\lambda_{d=5}$	$\lambda_{d=6}$	$\lambda_{d=7}$
0.01	8.747	4.435	3.453	3.026
0.02	8.140	4.288	3.376	2.974
0.03	7.617	4.152	3.302	2.924
0.04	7.163	4.027	3.233	2.876
0.05	6.764	3.911	3.169	2.831
0.06	6.412	3.804	3.107	2.788
0.07	6.099	3.703	3.049	2.746
0.08	5.818	3.609	2.993	2.706
0.09	5.565	3.521	2.940	2.668
0.10	5.334	3.438	2.890	2.631
0.11	5.124	3.360	2.841	2.595
0.12	4.932	3.286	2.795	2.561
0.13	4.756	3.216	2.751	2.527
0.14	4.593	3.149	2.708	2.494
0.15	4.442	3.086	2.667	2.464
0.16	4.301	3.026	2.627	2.433
0.17	4.170	2.968	2.589	2.414
0.18	4.048	2.913	2.552	2.377
0.19	3.933	2.860	2.517	2.348
0.20	3.825	2.809	2.482	2.321
0.21	3.723	2.760	2.449	2.297
0.22	3.627	2.713	2.417	2.272
0.23	3.536	2.668	2.386	2.246
0.24	3.449	2.625	2.355	2.224
0.25	3.367	2.583	2.325	2.202

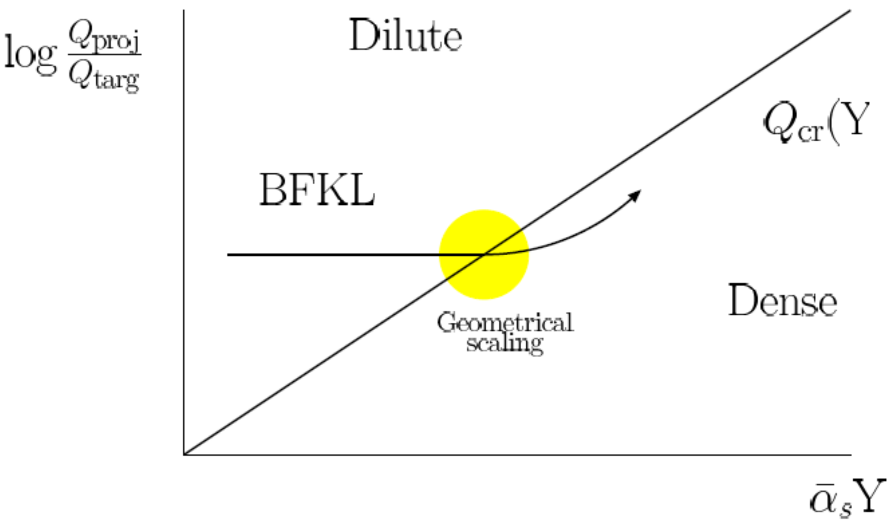
4. Saturation/Black hole holography?

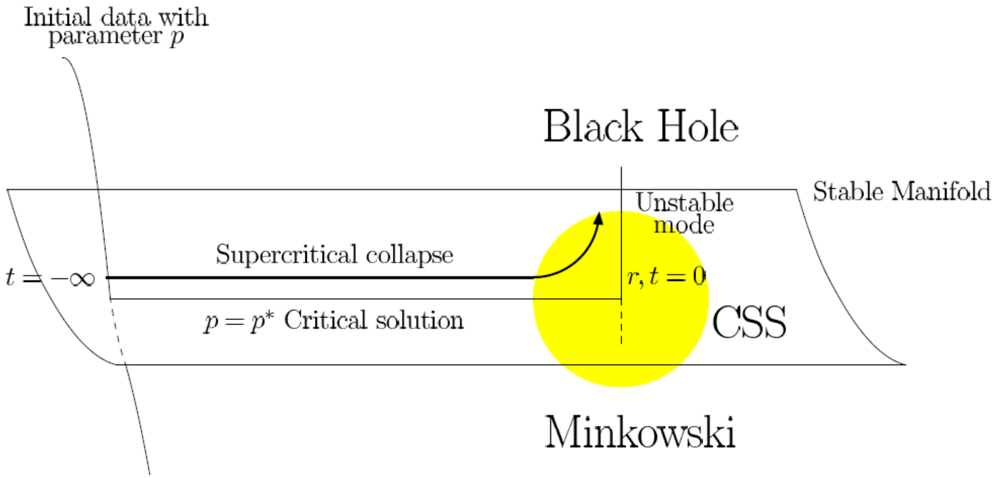
4d Perturbative QCD

1. Dilute/dense transition
2. Geometric scaling
3. Critical exponent 2.44
4. IR/UV competition

5d Tiny Black hole

1. Flat/black hole transition
2. CSS
3. Critical exponent 2.58
4. Gravity/kinetic competition





5. Open questions & future work

We did not need supersymmetry or AdS … Kinematics is the key …

What is the space-time geometry corresponding to the BFKL kernel?

This is probably the local gravity dual picture of perturbative saturation, can we define the correct scattering set up?

Is the final stage of evolution, something like the color glass condensate, dual to a tiny black hole? Can we map entropy flows?

Corrections to the semiclassical gravity picture should correspond to higher order corrections in the gauge theory side.

Is it really correct that unitarity of QCD at high energies is related to the formation of higher dimensional tiny black holes?

Can we learn something new about gravity from experiments at colliders?