Possible connections between GPDs and TMDs

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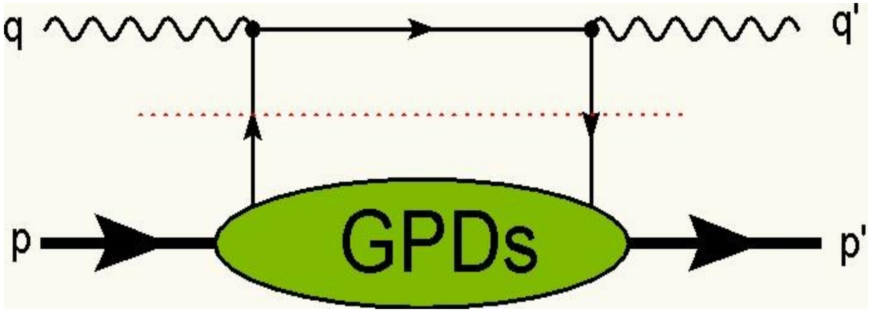
Content of the talk

- Generalized Parton Distributions (GPDs)
- Tranverse Momentum Dependent Parton Distributions (TMDs)
- Possible Relations between GPDs and TMDs
- “Mother distributions”

Generalized Parton Distributions

- Exclusive processes (DVCS, meson production, ...):



$$P = \frac{1}{2}(p + p') \qquad \Delta = p' - p$$

Skewness-parameter:

$$\Delta^+ = -2\xi P^+$$

- GPDs $\longrightarrow$ "off-diagonal" matrix elements of quark-quark operator:

$$F_{ij}(x, \xi, \vec{\Delta}_T) = \int \frac{dz^-}{2(2\pi)} \, e^{ixP^+z^-} \langle p' | \bar{\psi}_j(-\tfrac{z^-}{2}) \left[-\tfrac{z^-}{2} \,;\, \tfrac{z^-}{2} \right] \psi_i(\tfrac{z^-}{2}) | p \rangle$$

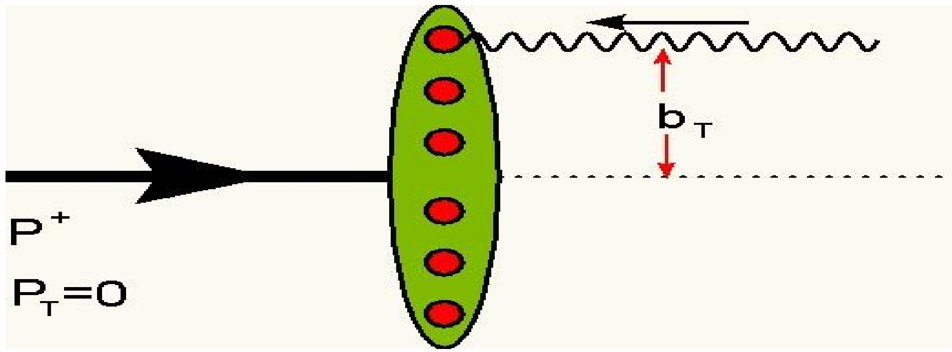
- Project out quark polarizations:

unp.: $\mathrm{Tr}\left[F\,\gamma^+\right] \longrightarrow (H\,,\,E)(x, \xi, t)$ long.: $\mathrm{Tr}\left[F\,\gamma^+\gamma_5\right] \longrightarrow (\tilde{H}\,,\,\tilde{E})$

transv. [chiral-odd]: $\mathrm{Tr}\left[F\,\sigma^{\perp +}\right] \longrightarrow (H_T\,,\,E_T\,,\,\tilde{H}_T\,,\,\tilde{E}_T)$

Impact Parameter Space

- Impact Parameter Space: (ξ=0, P_T=0) [M. Burkardt, PRD62, 071503]



- Impact parameter b_T and transv. momentum transfer $\Delta_T \longrightarrow$ FT

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2}\, \mathrm{e}^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, F_{ij}(x,0,\vec{\Delta}_T)$$

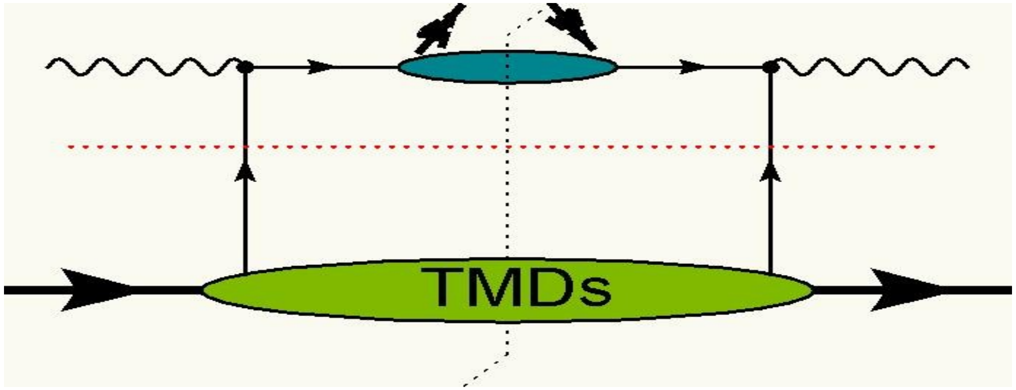
- Impact parameter space $\longrightarrow$ “diagonal” matrix element $z_{1/2} = \mp\frac{z^-}{2} n_- + b$

$$\mathcal{F}_{ij}(x,\vec{b}_T) = \int \frac{dz^-}{2(2\pi)}\, \mathrm{e}^{ixP^+z^-} \langle P^+;\vec{0}_T|\bar{\psi}_j(z_1)\,[z_1;z_2]\,\psi_i(z_2)|P^+;\vec{0}_T\rangle$$

- (ξ=0, P_T=0): density interpretation.

Transverse Momentum Dependence (TMD)

- Semi-inclusive processes (SIDIS, Drell-Yan)



$$d\sigma \sim f \otimes D$$

- TMDs $\longrightarrow$ "diagonal" matrix element, but k_T-dependence.

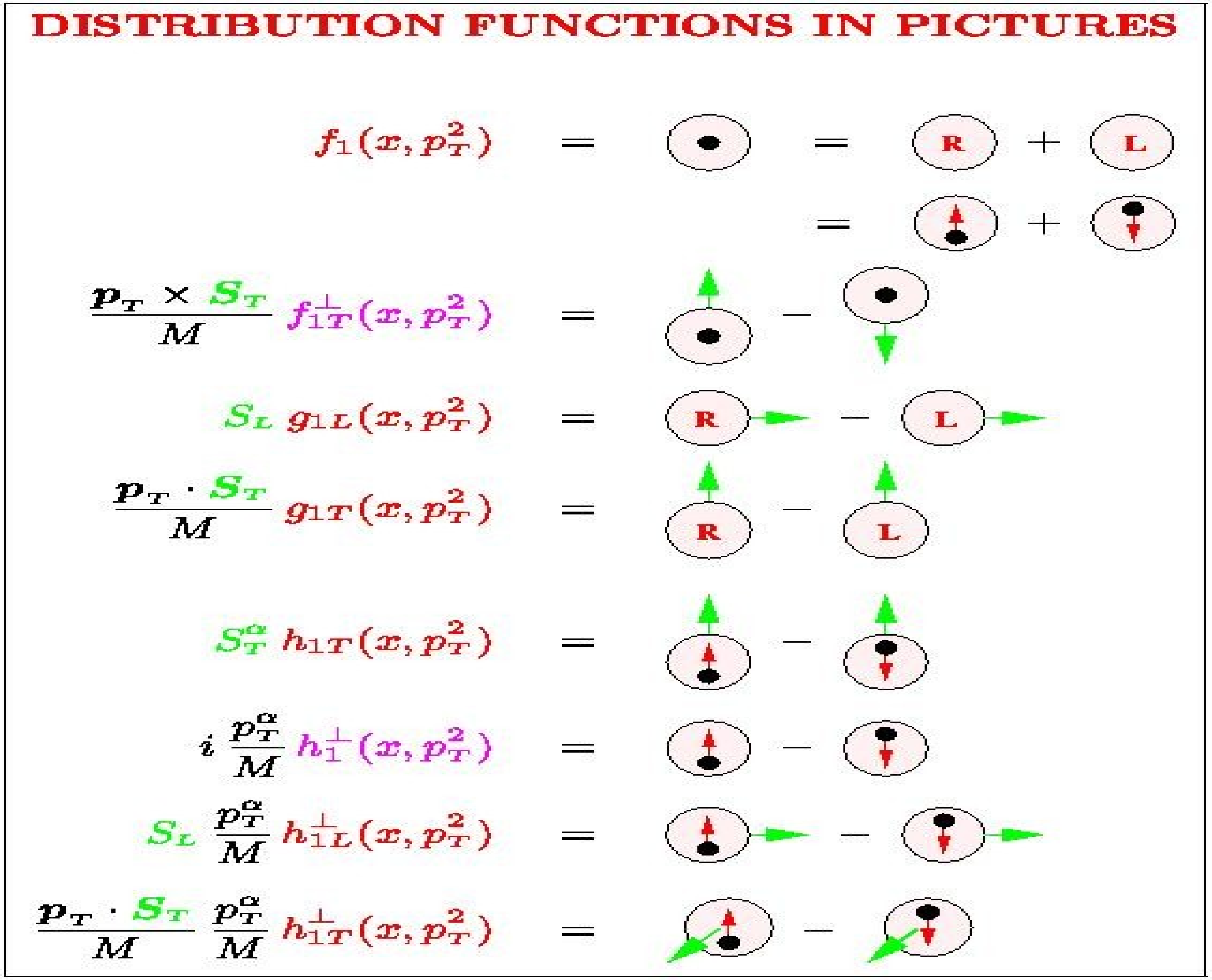
$$\Phi_{ij}(x,\vec{k}_T;S) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3}\, e^{ik\cdot z} \langle P,S|\, \bar{\psi}_j(0)\mathcal{W}_{\mathrm{SIDIS/DY}}[0,z]\,\psi_i(z)\,|P,S\rangle\Big|_{z^+=0}$$

- Project out polarizations:

unp.: $\mathrm{Tr}\left[\Phi\,\gamma^+\right] \longrightarrow (f_1\,,\, f_{1T}^{\perp})(x,\vec{k}_T^2)$ long.: $\mathrm{Tr}\left[\Phi\,\gamma^+\gamma_5\right] \longrightarrow (g_{1L}\,,\, g_{1T})$

transv. [chiral-odd]: $\mathrm{Tr}\left[\Phi\,\sigma^{\perp +}\right] \longrightarrow (h_1\,,\, h_{1T}^{\perp}\,,\, h_{1L}^{\perp}\,,\, h_1^{\perp})$

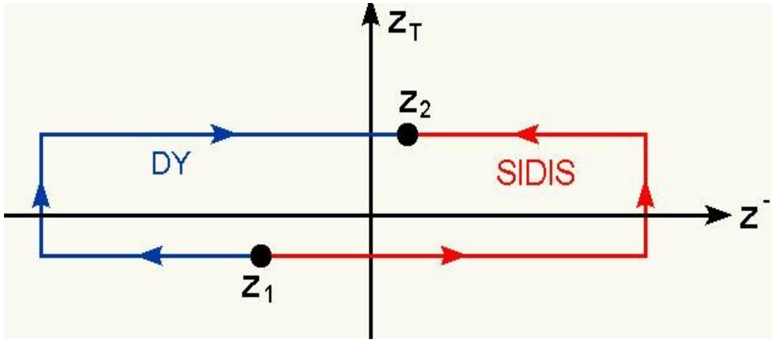
TMDs in pictures



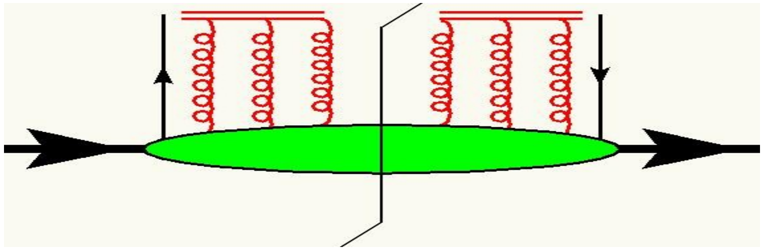
Gauge link for TMDs

- kT-dependence $\longrightarrow$ more complicated gauge link

$$\mathcal{W}[z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$



- Describes *Initial (DY)* and *Final (SIDIS)* State Interactions



- Time-reversal: switches Wilson-lines ISI $\longleftrightarrow$ FSI

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY} \qquad h_1^{\perp}\Big|_{DIS} = -h_1^{\perp}\Big|_{DY}$$

Trivial relations

Trivial Relations are well-known:

$$f_1(x) \; = \; H(x,0,0) \; = \int d^2k_T \, f_1(x,\vec{k}_T^2) \; = \int d^2b_T \mathcal{H}(x,\vec{b}_T^2)$$

$$g_1(x) \; = \; \tilde{H}(x,0,0) \; = \int d^2k_T \, g_{1L}(x,\vec{k}_T^2)$$

$$h_1(x) \; = \; H_T(x,0,0) \; = \int d^2k_T \, h_1(x,\vec{k}_T^2)$$

→ model-independent, integrated relations

also for twist-3 PDFs e(x), $g_T(x)$, ...

Non-trivial Relations

Non-trivial relations for "T-odd" parton distributions:

M. Burkardt [Nucl.Phys. A735, 185], [PRD66, 114005]

Step 1: Average transverse of unpolarized partons in a transversely polarized nucleon:

$$\langle k_T^i \rangle_T(x) = \int d^2k_T \, k_T^i \, \tfrac{1}{2}\left[\Phi^{[\gamma^+]}(\vec{S}_T) - \Phi^{[\gamma^+]}(-\vec{S}_T)\right] \propto f_{1T}^{\perp,(1)}(x)$$

Step 2: Impose parity and time reversal:

$$\Phi(x, \vec{k}_T; -\vec{S}_T) = \mathcal{FT}\left[\langle P, -S_T | \, \bar{\psi}\gamma^+ \mathcal{W}_{\mathrm{SIDIS}} \psi \, | P, -S_T \rangle\right]$$

$$\longrightarrow \quad \mathcal{FT}\left[\langle P, +S_T | \, \bar{\psi}\gamma^+ \mathcal{W}_{\mathrm{DY}} \psi \, | P, +S_T \rangle\right]$$

Non-trivial Relations

Step 3: Derivatives of gauge links:

$$\langle k_T^i \rangle_T(x) \propto \int d^2k_T \int d^2z_T\, k_T^i \mathrm{e}^{ik\cdot z} \langle \bar\psi\, \gamma^+ \left(\mathcal{W}_{\text{SIDIS}} - \mathcal{W}_{\text{DY}}\right) \psi \rangle$$

$i\partial_T^i$

$$i\partial_T^i \left(\mathcal{W}_{\text{SIDIS}} - \mathcal{W}_{\text{DY}}\right)\Big|_{z_T=0} \propto \int dy^- [-\tfrac{z^-}{2}; y^-]\, gF^{+i}(y^-)\, [y^-; \tfrac{z^-}{2}]$$

$$\equiv 2[-\tfrac{z^-}{2}; \tfrac{z^-}{2}]\, I^i(\tfrac{z^-}{2})$$

$$\langle k_T^i \rangle(x) = \int \frac{dz^-}{2(2\pi)} \mathrm{e}^{ixP^+z^-} \langle P, S_T | \bar\psi(-\tfrac{z^-}{2}) \gamma^+ [-\tfrac{z^-}{2}; \tfrac{z^-}{2}]\, I^i(\tfrac{z^-}{2})\, \psi(\tfrac{z^-}{2}) | P, S_T \rangle$$

collinear “soft gluon pole” matrix element

Non-trivial Relations

Step 4: Impact parameter space: $z_{1/2} = \mp \frac{z^-}{2} n_- + b_T$

$$\langle k_T^i \rangle(x) = \int d^2 b_T \int \frac{dz^-}{2(2\pi)} e^{ixP^+ z^-} \langle P^+; \vec{0}_T; S_T | \bar{\psi}(z_1) \gamma^+ [z_1; z_2] I^i(z_2) \psi(z_2) | P^+; \vec{0}_T; S_T \rangle$$

→ Impact parameter representation for GPD E

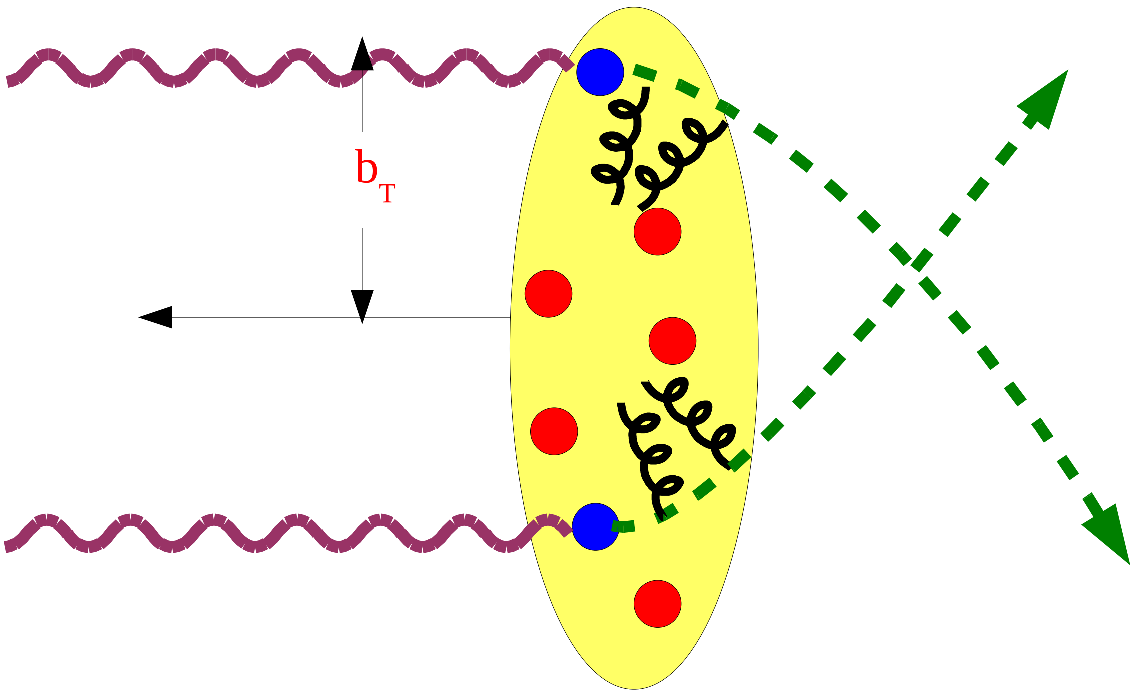
Assume factorization of final state interactions and spatial distortion:

$$\langle k_T^i \rangle = -M \epsilon_T^{ij} S_T^j f_{1T}^{\perp,(1)}(x) \simeq \int d^2 b_T \, \mathcal{I}^i(x, \vec{b}_T) \frac{\vec{b}_T \times \vec{S}_T}{M} \frac{\partial}{\partial b_T^2} \mathcal{E}(x, \vec{b}_T^2)$$

$\mathcal{I}^i(x, \vec{b}_T^2)$: Lensing Function = net transverse momentum

Physical picture of the Relation

Intuitive picture of the Final State Interactions:



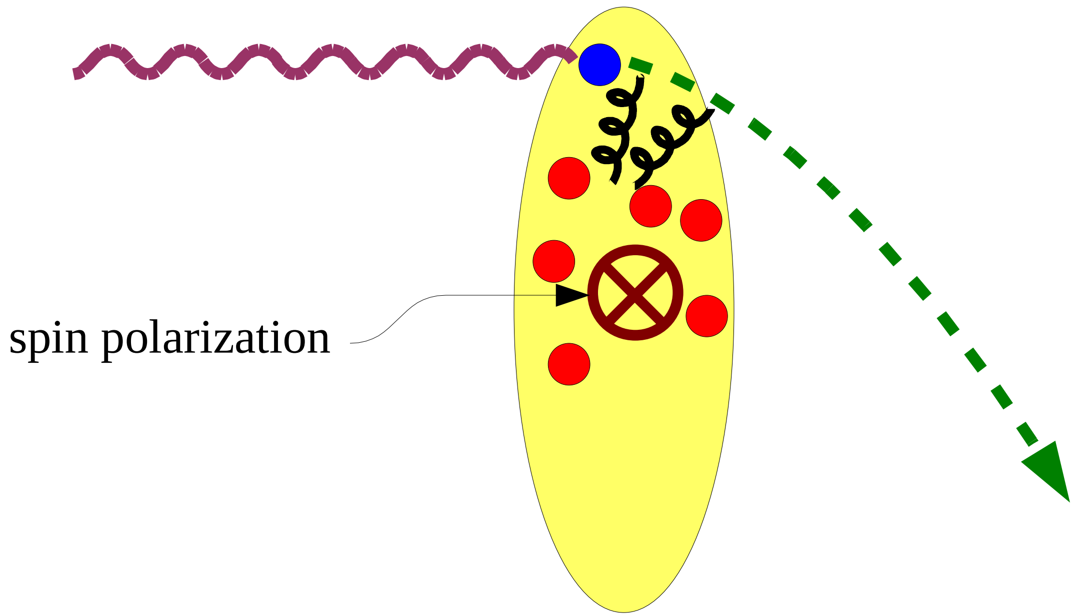
Final State interactions are assumed to be attractive

→ Lensing!

Physical picture of the Relation

Intuitive picture of the Sivers asymmetry:

Spatial distortion in the transverse plane due to polarization!



Mechanism leads to non-zero Sivers asymmetry!

Predictions

Intuitive picture seems to work "numerically":

Distortion effect given by flavor dipole moment:

$$d^{q,i} = \int dx \int d^2 b_T \, b_T^i \, \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx E^q(x,0,0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

with flavor dipole moment $\kappa^{u/p} \simeq 1.7 \quad \kappa^{d/p} \simeq -2.0$

$$f_{1T}^{\perp,(1)}(x) \propto \int d^2 b_T \, \mathcal{I}(x, \vec{b}_T) \frac{\vec{b}_T \times \vec{S}_T}{M} \mathcal{E}'(x, \vec{b}_T^2)$$

Predicts opposite signs of u- and d- Sivers functions.

- in agreement with large-N_c prediction [Pobylitsa, 2003]
  model calculations in spectator models, MIT-bag model, etc.

Predictions

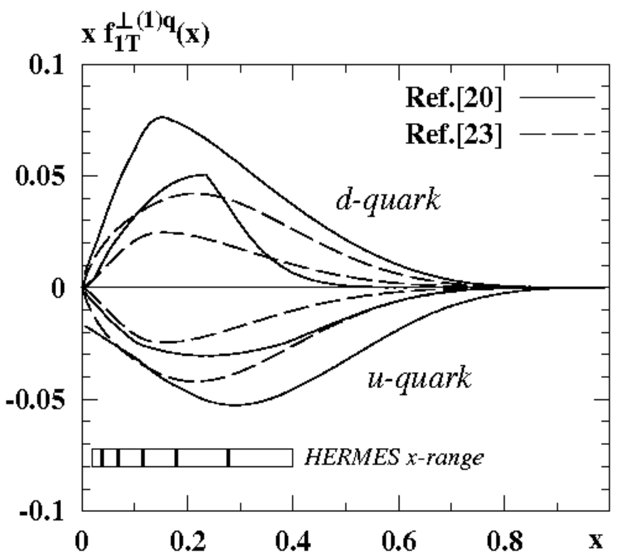
Intuitive picture also predicts the absolute sign

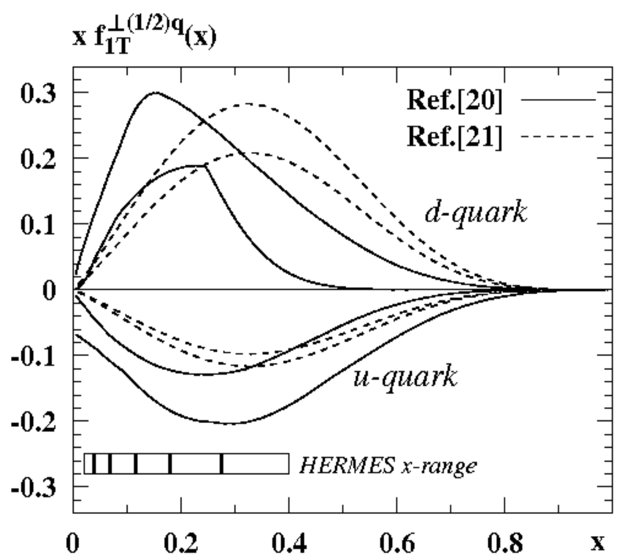
if:

→ Final state interactions are attractive, $\mathcal{I}(x, \vec{b}_T) < 0$

$f_{1T}^{\perp,u} < 0$ $f_{1T}^{\perp,d} > 0$

Confirmed by HERMES, COMPASS data:





Fits taken from:
[20] Anselmino et al., PRD72 (05)
[21] Vogelsang, Yuan, PRD72 (05)
[23] Collins et al., hep-ph/0510342

Chiral-odd Relation

- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

$$\langle k_T^i \rangle^j(x) \;=\; \int d^2k_T\, k_T^i\, \frac{1}{2}\left(\Phi^{[i\sigma^{i+}\gamma^5]}(S) + \Phi^{[i\sigma^{i+}\gamma^5]}(-S)\right)$$

$$-2M^2\, h_1^{\perp,(1)}(x) \simeq \int d^2b_T\, \vec{b}_T \cdot \vec{\mathcal{I}}(x,\vec{b}_T)\, \frac{\partial}{\partial b_T^2}\left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)(x,\vec{b}_T^2)$$

- Spatial distortion in transv. plane of transv. pol. quarks quantified by

$$\kappa_T \;=\; \int dx \left(E_T + 2\tilde{H}_T\right)(x,0,0)$$

- <u>Lattice QCD, const. quark model:</u> $\kappa_T^u > 0$ and $\kappa_T^d > 0$

Boer-Mulders function negative for u- and d-quarks!

[in agreement with large-N_c, models.]

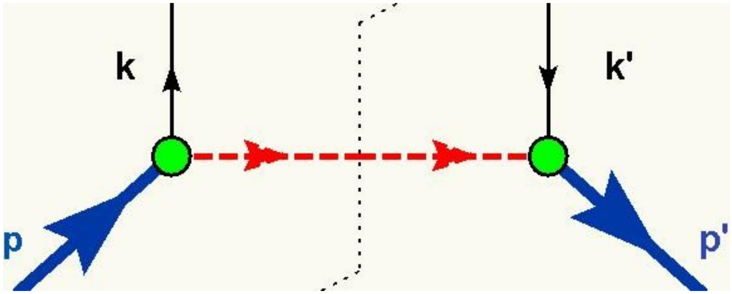
Relations in Spectator Models

Explicit checks of relations in a diquark spectator model:

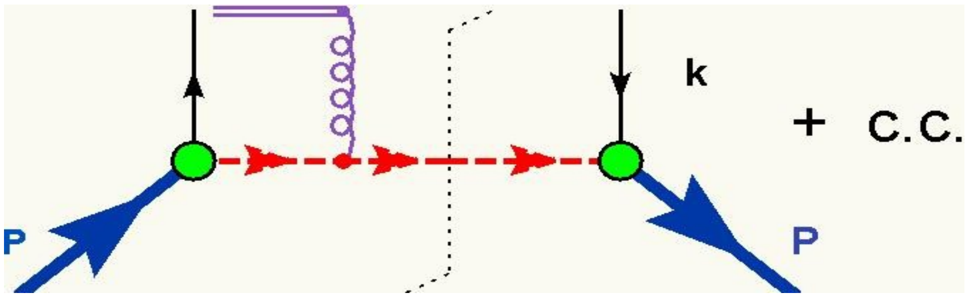
[Burkardt, Hwang, PRD69, 074032], [Meissner, Metz, Goeke, PRD76, 034002]

Lowest order calculations:

GPDs:



(T-odd) TMDs:



Non-trivial relations are *exactly* fulfilled!

$$-M\epsilon^{ij}S_T^j\, f_{1T}^{\perp,(1)} = \int d^2b_T\, \mathcal{I}^i\, \frac{\vec{b}_T\times\vec{S}_T}{M}\,\mathcal{E}'$$

$$-2M\, h_1^{\perp,(1)} = \int d^2b_T\, \frac{\vec{b}_T\cdot\vec{\mathcal{I}}}{M}\left(\mathcal{E}_T + 2\tilde{\mathcal{H}}_T\right)'$$

Relations in Spectator Models

In the diquark-spectator model:

- Relations between *arbitrary* moments:

$$f_{1T}^{\perp,(n)}(x) \propto E^{(n)}(x)\,,\ 0 \leq \mathrm{n} \leq 1$$

- TMD: $f^{(n)}(x) \sim \int d^2k_T\, (\vec{k}_T^2)^n f(x, \vec{k}_T^2)$

- GPD: $E^{(n)}(x) \sim \int d^2\Delta_T\, (\vec{\Delta}_T^2)^{n-1} E(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2})$

- Relation between GPDs and *T-even* TMDs:

$$h_{1T}^{\perp,(n)}(x) \sim \tilde{H}_T^{(n)}(x)$$

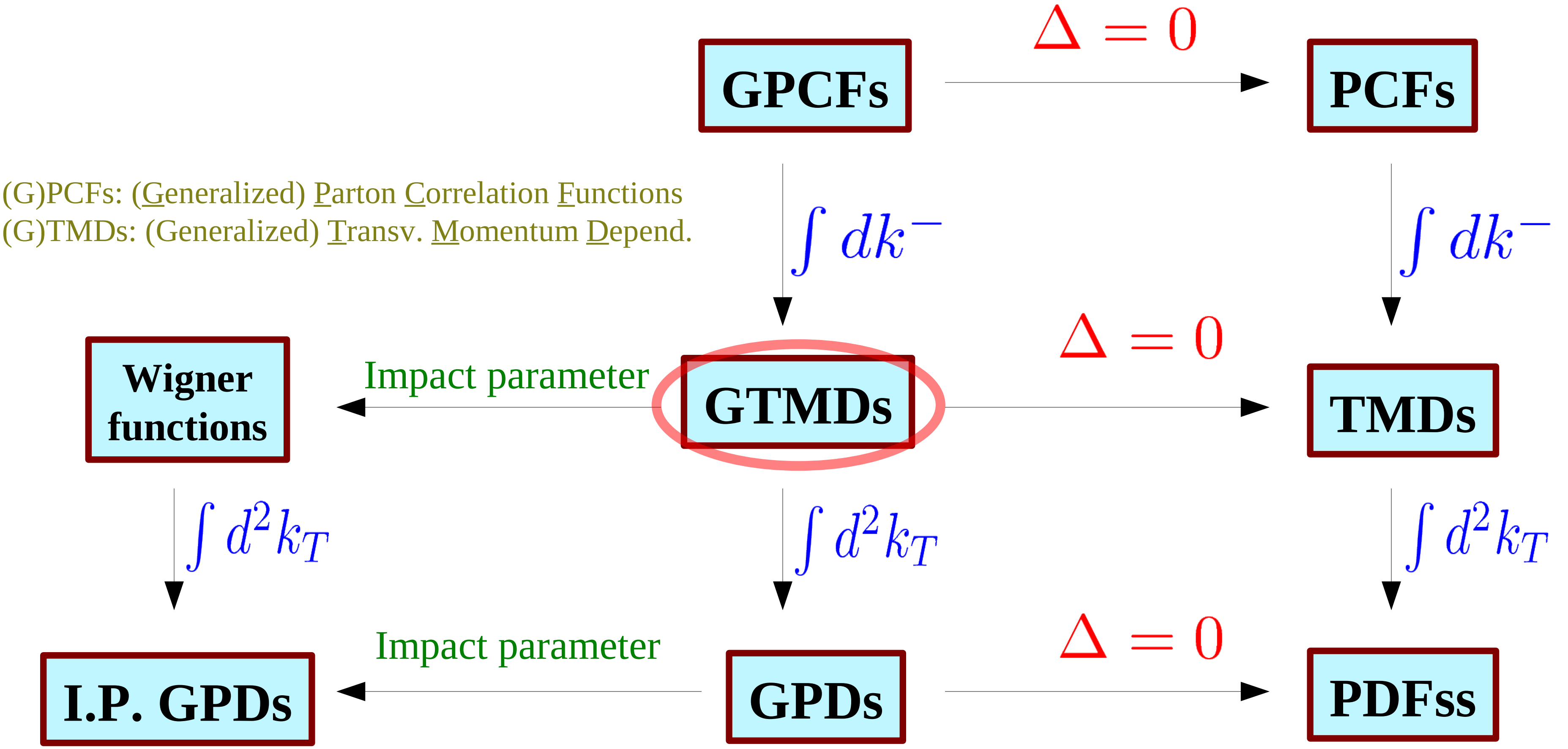
→ No FSI / Lensing function needed!

- Relations also for gluon-GPDs and gluon-TMDs.

- Relations are likely to be broken for higher order diagrams.

Mother functions

Relations between functions:



Which GPDs and TMDs have the same mother functions?

Classification of GPCFs

[Goeke, Meissner, Metz, M.S., soon to be published, spin-0]

[Goeke, Meissner, Metz, M.S., in preparation, spin-1/2]

Generalized, fully unintegrated quark-quark correlator:

$$W_{ij}(P,k,\Delta,N;\eta) = \int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z} \langle p' \,|\bar{\psi}_j(-\tfrac{z}{2})\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2}\,|\,n)\psi_i(\tfrac{z}{2})\,|p\rangle$$

- Wilson line chosen 'by hand': TMDs in *SIDIS / DY* can be recovered.
- Encodes the maximum amount of information on two-parton structure.
- W depends on a *light cone direction N*, $N^\mu = n^\mu M^2/(P\,n)$,
  Parameter $\eta = \text{sign}(n^0) \in \{-1,1\} \Longrightarrow$ ISI / FSI

Classification of GPCFs

Decomposition of correlator W into Generalized Parton Correlation Functions (GPCFs):

- Restriction by *parity* and *epsilon-identity*:

$$g^{\alpha\beta}\epsilon^{\mu\nu\rho\sigma} = g^{\mu\beta}\epsilon^{\alpha\nu\rho\sigma} + g^{\nu\beta}\epsilon^{\mu\alpha\rho\sigma} + g^{\rho\beta}\epsilon^{\mu\nu\alpha\sigma} + g^{\sigma\beta}\epsilon^{\mu\nu\rho\alpha}$$

- Spin-0 hadron: Decomposition *very easy* $\longrightarrow$ 16 GPCFs A_i:

$$W(P,k,\Delta,N;\eta) = M A_1 + \not{P} A_2 + \not{k} A_3 + \not{\Delta} A_4 + \ldots$$

- GPCFs are complex-valued scalar functions X(P,k,Δ,N;η).
- *Time-reversal* and *Hermiticity* determine behavior of η:

$$X(\eta) = \Re[X] + i\eta\Im[X]$$

Classification of GPCFs

- Spin 1/2: Decomposition *very tedious*:

Param. traces of correlator W and Dirac-matrices $\Gamma=1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}$

$$\frac{1}{2}\mathrm{Tr}\left[W_{\lambda,\lambda'}(P,k,\Delta,N;\eta)\Gamma\right] = \bar{u}(p',\lambda')\,\mathcal{G}(\Gamma)\,u(p,\lambda)$$

- Parameterization restricted by *parity*, *eps.-id.* and *Gordon-identities*
  $\longrightarrow$ eliminate all terms of the form $\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5$

$$\frac{1}{2}\mathrm{Tr}[1\,W] = \bar{u}'\left[E_1 + \frac{i\sigma^{k\Delta}}{M^2}E_2 + \frac{i\sigma^{kN}}{M^2}E_3 + \frac{i\sigma^{\Delta N}}{M^2}E_4\right]u \longrightarrow \text{4 GPCFs}$$

$\gamma_5 \longrightarrow 4$, $\gamma^\mu \longrightarrow 16$, $\gamma^\mu\gamma_5 \longrightarrow 16$, $\sigma^{\mu\nu} \longrightarrow 24$

- $\Delta = 0$: GPCFs $\longrightarrow$ PCFs [Goeke, Metz, M.S., PLB 618, 90]

Generalized TMDs

- Integration over a light cone direction (k^-)

  $\longrightarrow$ *GTMDs:*

$$\tilde{W} = \int dk^- \, W$$

For spin-0: Classification with respect to P^+

$$\mathrm{Tr}[\tilde{W}\,\gamma^+] = 2\,F_1(x,\xi,\vec{\Delta}_T^2,\vec{k}_T^2,\vec{\Delta}_T\cdot\vec{k}_T;\eta)$$

[unpol. quark]

$$\mathrm{Tr}[\tilde{W}\,\gamma^+\gamma_5] = 2\,\frac{\vec{k}_T\times\vec{S}_T}{M^2}\,\tilde{G}_1$$

[long. pol. quark]

$$\mathrm{Tr}[\tilde{W}\,i\sigma^{i+}\gamma_5] = -\frac{2i}{M}\left(\epsilon_T^{ij}k_T^j\,H_1^k + \epsilon_T^{ij}\Delta_T^j\,H_1^\Delta\right)$$

[transv. pol. quark]

- GTMDs are complex-valued of the form (Time-reversal)

$$X(\eta) = \Re[X] + i\eta\Im[X]$$

Generalized TMDs

- Phenomenological aspects:

  Are there processes where GTMDs might appear?

Limiting cases of Generalized TMDs

- TMDs in terms of GTMDs: $\Delta = 0$

$$f_1(x, \vec{k}_T^2) = \Re[F_1](x, 0, 0, 0, \vec{k}_T^2)$$

$$h_1^\perp(x, \vec{k}_T^2) = \eta \Im[H_1^k](x, 0, 0, 0, \vec{k}_T^2)$$

- GPDs in terms of GTMDs: $\int d^2 k_T$

$$F_1^\pi(x, \xi, t) = \int d^2 k_T \, \Re[F_1](x, \xi, \vec{\Delta}_T^2, \vec{\Delta}_T \cdot \vec{k}_T, \vec{k}_T^2)$$

$$H_1^\pi(x, \xi, t) = \int d^2 k_T \left[\frac{\vec{\Delta}_T \cdot \vec{k}_T}{\vec{\Delta}_T^2} \Re[H_1^k] + \Re[H_1^\Delta] \right]$$

- Trivial relation fulfilled.
- No *exact* model-independent relations. Approximate relations?
- Spin-1/2: No exact model-ind. relations except the *trivial* ones.

Summary

- Non-trivial relations between GPDs and (T-odd) TMDs were suggested on the basis of a separation between:

  distortion of parton distribution in the *transverse plane*

  \+ *Final state interactions*

- Relations seems to work "numerically"

  $\longrightarrow$ predicts signs of u- and d-quark *Sivers*- and *BM*-function.

- Relations are exact in lowest-order spectator models.

  Also relations between T-even TMDs and GPDs.

- GTMD-investigation: does not support *model-independent, exact relations*. Does not rule out "approximate" relations...