Scalar Implicatures with Alternative Semantics*

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1 Standard Analysis

One standard account of scalar implicatures considers alternative sentences where the scalar item has been replaced with an item from the same scale, called a Horn Scale after Horn (1972). All such sentences that are stronger are implicated to be false.

1.1 Definitions

(1) $Alt(\phi) = \{\phi' : \phi$ contains scalar item s and ϕ' is the same as ϕ except that s is replaced with an item from the Horn Scale for $s\}$

(2) Implicatures Generated: $\forall \phi' \in Alt(\phi)\ .\ (\phi' \Rightarrow\ \phi) \rightarrow\ \neg\phi'$

1.2 Example

(3) Paul read three books. $\rightsquigarrow$ Paul did not read four books.

(4) Horn Scale for "three": {"one", "two", "three", "four", "five", ...}

(5) $Alt($"Paul read three books"$) =$
$$\left\{ \begin{array}{lll} \text{"Paul read one book"} & , & \text{"Paul read two books"}, \\ \text{"Paul read three books"}, & & \text{"Paul read four books"}, \\ \text{"Paul read five books"} & , & \ldots \end{array} \right\}$$

(6) "Paul read four books." $\Rightarrow$ "Paul read three books."

Therefore by (2) an implicature generated is:

(7) "Paul did not read four books."

2 Challenges

2.1 Comparative Problem

Paul read more than three books. $\not\rightsquigarrow$ Paul did not read more than four books. krifcompafter

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(8) *Alt*("Paul read more than three books") =
$$\left\{\begin{array}{ll} \text{"Paul read more than one book"} & , \text{"Paul read more than two books"}, \\ \text{"Paul read more than three books"}, & \text{"Paul read more than four books"}, \\ \text{"Paul read more than five books"} & , \ \ldots \end{array}\right\}$$

(9) "Paul read more than four books." ⇒
"Paul read more than three books."

Therefore by (2) an (incorrect) implicature generated is:

(10) "Paul did not read more than four books."

2.2 Disjunction Problem

Paul read *The New York Times* or some of the books. ↛
Paul did not read *The New York Times*. chiafter

(11) Horn Scale for "some": {"some", "most", "all"}

(12) *Alt*("Paul read *The New York Times* or some of the books") =
$$\left\{\begin{array}{l} \text{"Paul read } \textit{The New York Times} \text{ or most of the books"} \\ \text{"Paul read } \textit{The New York Times} \text{ or all of the books"} \end{array}\right\}$$

(13) "Paul read *The New York Times* or all of the books." ⇒
"Paul read *The New York Times* or some of the books."

Therefore, by (2):

(14) "Paul did not read *The New York Times* or all of the books."

Therefore, by DeMorgan's Law:

(15) "Paul did not read *The New York Times* and Paul did not read all of the books."

Therefore:

(16) "Paul did not read *The New York Times*."

2.3 Some + Plural Problem

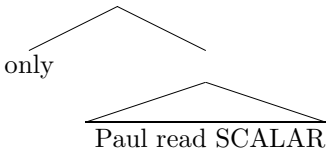
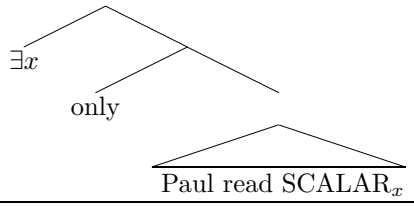
Other scalar implicature systems (Groenendijk and Stokhof 1984, Sauerland 2004, Fox 2006) have the following problem:

(17) Paul read some of the books. ↛ Paul read exactly two books. [1]

[1] This problem was pointed out to me by Danny Fox

3 Intuitive Proposal

Scalar implicatures arise when a sentence contains a silent exhaustive operator (Groenendijk and Stokhof 1984, Krifka 1995, Fox 2004), which scopes below the existential closure of indefinites (Heim 1982, Kratzer and Shimoyama 2002) and disjunction (Rooth and Partee 1982).

Paraphrases of the standard system	Paraphrases where "only" scopes below ∃
	
Paul only read **three** books.	There is a group x of three books : Paul only read x.
Paul only read more than **two** books.	There is a group x of more than two books : Paul only read x.
Paul only read **some** of the books.	There is a group x that is a proper subset of the books : Paul only read x.
Paul only read *War and Peace* **or** *The Brothers Karamazov*.	There's a group of books x which is either *War and Peace* or *The Brothers Karamazov*: Paul only read x.
Paul only read *The New York Times* **or some** of the books.	There is a group x such that x is *The New York Times* or x is a proper subset of the books : Paul only read x.

4 Background

I adapt a system proposed by Kratzer and Shimoyama (2002) where the normal semantic values for many items of type σ (or $\langle \sigma t, t \rangle$) are replaced by sets containing items of type σ (which I will call type $\{\sigma\}$ or $\subseteq D_\sigma$):

(18) $[\![\textbf{a book}]\!] = \{x \in D_e : x \text{ is a book}\}$ (type $\{e\}, \subseteq D_e$)

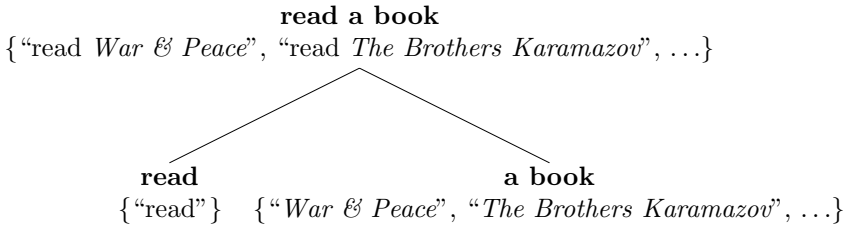
4.1 Hamblin Functional Application

These sets combine pointwise with predicates via the following composition rule:

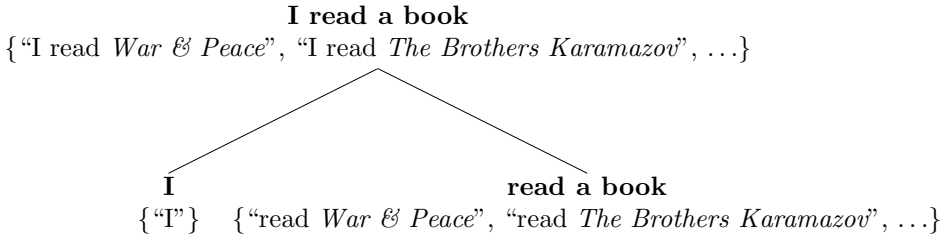
(19)

(20) *Hamblin Functional Application*
If α is a branching node with daughters β and γ, and $[\![\beta]\!] \subseteq D_{\langle\sigma,\tau\rangle}$ and $[\![\gamma]\!] \subseteq D_\sigma$, then $[\![\alpha]\!] = \{a \in D_\tau : \exists b \exists c[b \in [\![\beta]\!] \;\&\; c \in [\![\gamma]\!] \;\&\; a = b(c)]\}$.

(21)



(22)



4.2 Sentential Quantifier

The normal meaning of the sentence "Paul read a book" is obtained by applying the Sentential Quantifier $\exists$ (using the standard Functional Application Rule):

(23) $[\![\exists\alpha]\!] = \{\lambda w' \,.\, \exists p[p \in [\![\alpha]\!] \;\&\; p(w') = 1]\}$ for $[\![\alpha]\!] \subseteq D_{st}$

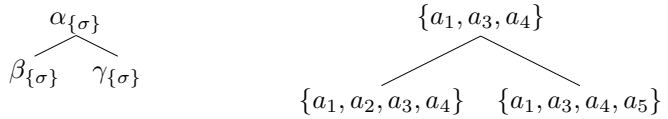
4.3 Plural Individuals

Plural individuals (Link 1983) in D_e have some of the analogous properties of sets (e.g., cardinality, part-of and proper part-of relations, and membership) and the alternatives in the meaning sets of indefinites are closed under conjunction:

(24) a. $[\![\textbf{books}]\!] = \{x \in D_e : x \text{ is one or more books}\}$ (type $\{e\}$)
b. $[\![\textbf{three}]\!] = \{x \in D_e : |x| = 3]\}$ (type $\{e\}$)
c. $[\![\textbf{read}]\!] = \{[\lambda y \in D_e \,.\, \lambda x \in D_e \,.\, \lambda w \in D_s \,.\, x \text{ reads } y \text{ in } w]\}$
(type $\{\langle e, \langle e, st\rangle\rangle\}$)
d. $[\![\textbf{Paul}]\!] = \{Paul\}$ (type $\{e\}$)

Note that the standard Predicate Modification rule (Heim and Kratzer 1998) will work correctly on sets of alternatives:

(25)



(26) *Predicate Modification*
If α is a branching node with daughters β and γ, and $[\![\beta]\!] \subseteq D_{\{\sigma\}}$ and $[\![\gamma]\!] \subseteq D_{\{\sigma\}}$, then $[\![\alpha]\!] = [\![\beta]\!] \cap [\![\gamma]\!]$.

Now we can derive the complete meaning of a simple sentence:

(27) $[\![\exists$ **Paul read three books.**$]\!]$ = A statement in the following set is true: {Paul read x : x is the sum of exactly three books}.

4.4 Context

Every sentence is assumed to be uttered in relation to a subset of D_{st} that I will call the focus set, which can be thought of as the denotation of a question. For instance:

(28) a. What did Paul read last night?
b. {Paul read *War and Peace* last night, Paul read *The Brothers Karamazov* last night, Paul read *The New York Times* last night, Paul read *W&P* $\oplus$ *NYT* last night, ...}

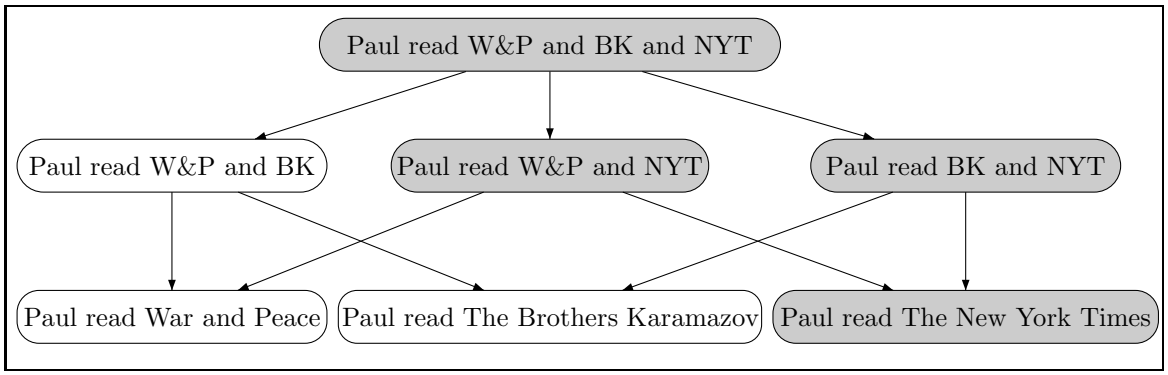
5 Proposal

Definition for Exh (Krifka 1995, Fox 2004):

(29) $[\![Exh]\!] = \{[\lambda p \in D_{st} \,.\, \lambda w' \in D_s \,.\, p(w') \;\&\; \forall f \in F\; [f(w') \rightarrow\; (p \Rightarrow f)]]\}$, where F is the focus set.

(30) a. What did Paul read last night?
b. $\exists Exh$ Paul read *War & Peace* and *The Brothers Karamazov*.
c. A statement from this set is true:
{Paul only read *War & Peace* and *The Brothers Karamazov*}

(31) Entailment diagram for "Paul only read *War & Peace* and *The Brothers Karamazov*}" in relation to the focus set in (28b):



5.1 Numerals

(32) What did Paul read last night?

(33) a. Paul read three books last night.

b. There's a group x of three books : Paul only read x last night.

(34) a. $\exists Exh$ Paul read three books last night.

b. $\exists Exh$ {Paul read x last night : $x \in [\![\textbf{book}]\!]$ & $|x| = 3$}

By Hamblin Functional Application, you combine Exh and the set pointwise:

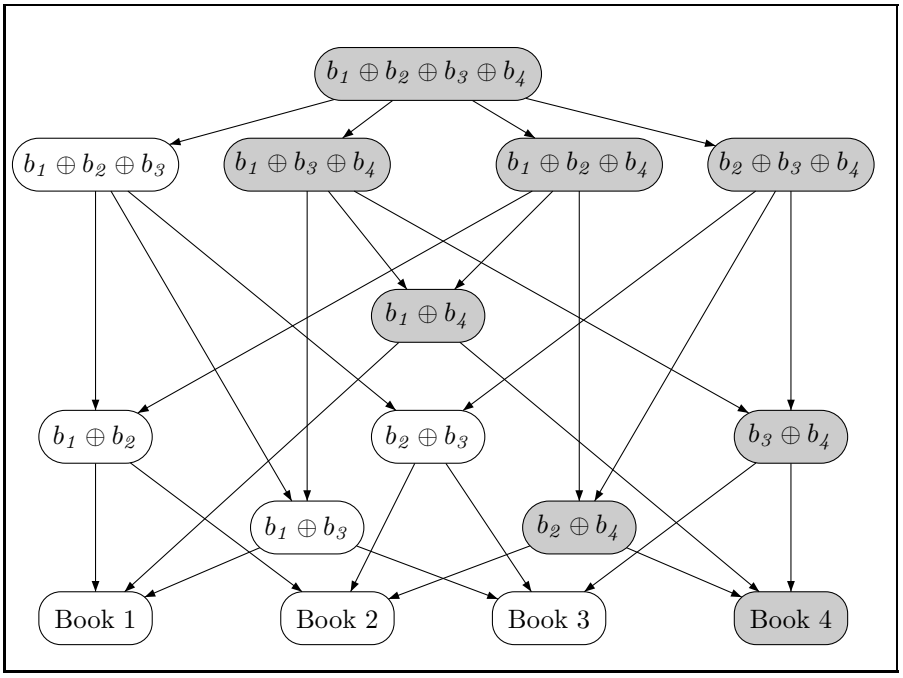
(34) c. $\exists\{Exh$ Paul read x last night : $x \in [\![\textbf{book}]\!]$ & $|x| = 3\}$

Now, if we assume there are only four books under discussion:

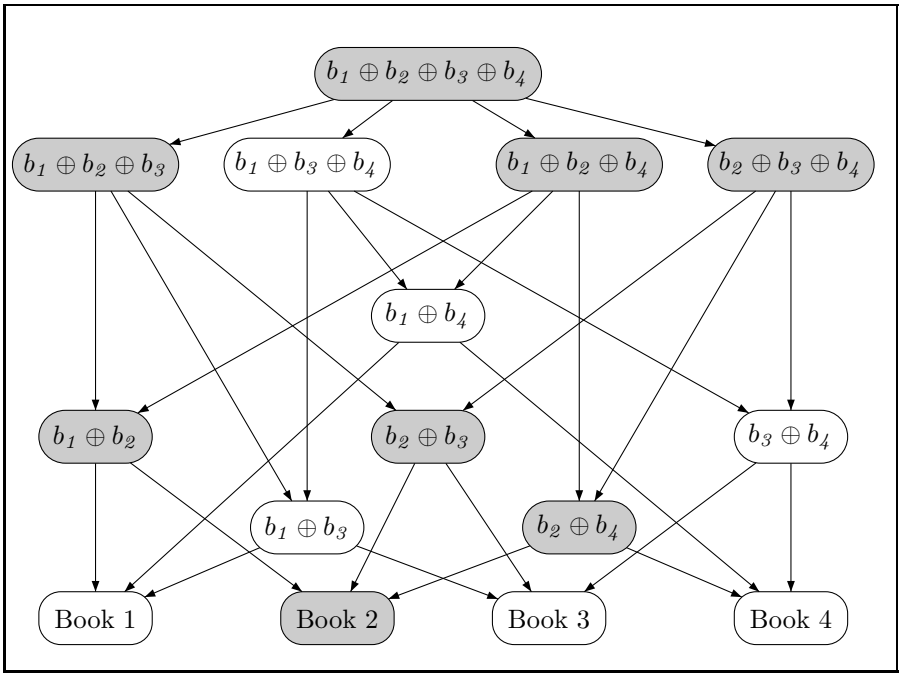
(34) d. One of these statements is true:

$$\left\{\begin{array}{l}\text{Paul only read } b_1 \oplus b_2 \oplus b_3,\\ \text{Paul only read } b_1 \oplus b_3 \oplus b_4,\\ \text{Paul only read } b_1 \oplus b_2 \oplus b_4,\\ \text{Paul only read } b_2 \oplus b_3 \oplus b_4\end{array}\right\}$$

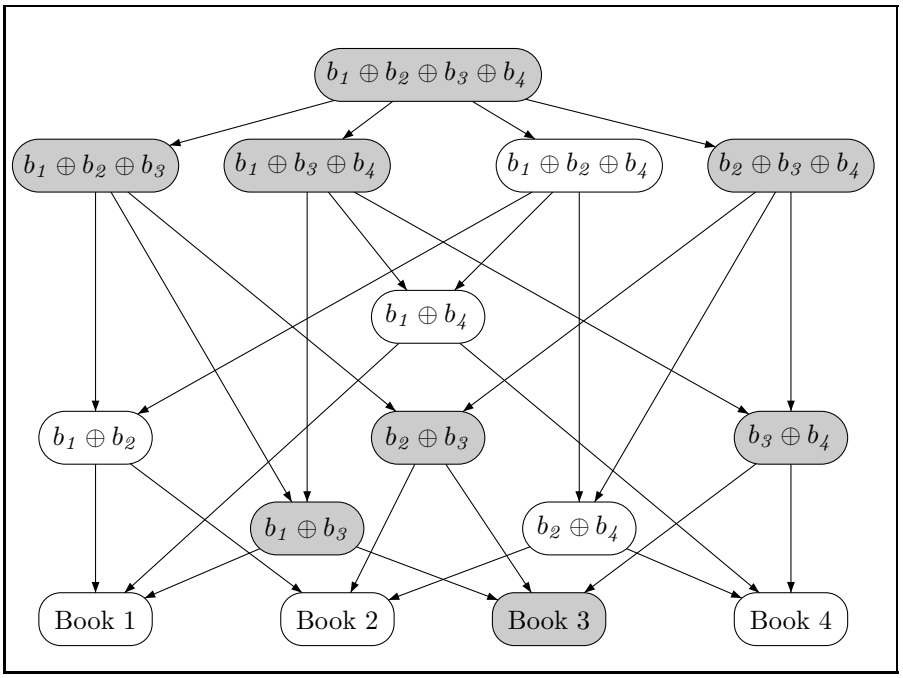
(35) Paul only read $b_1 \oplus b_2 \oplus b_3$.



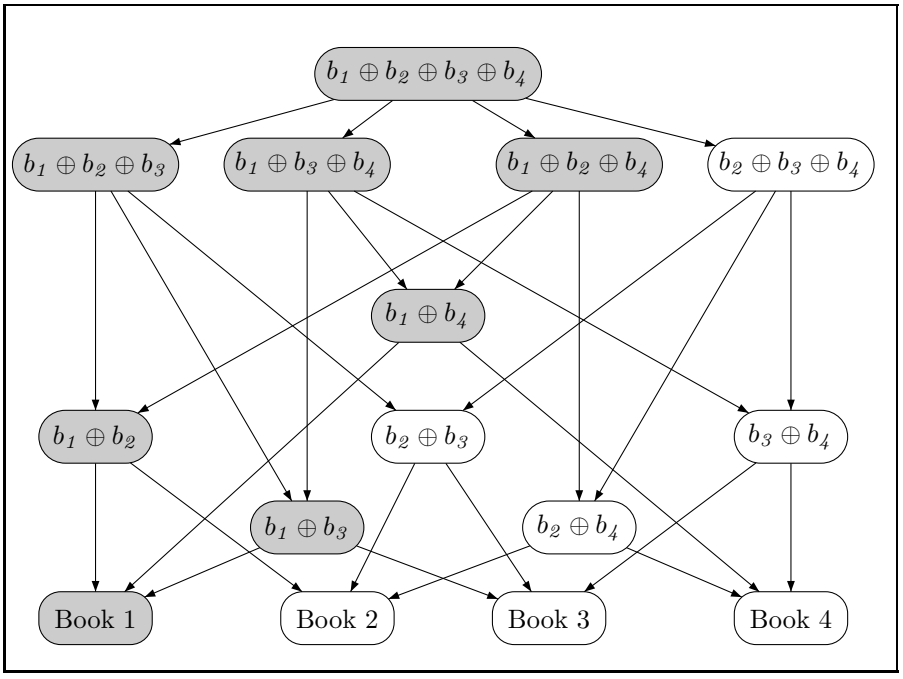
(36) Paul only read $b_1 \oplus b_3 \oplus b_4$



(37) Paul only read $b_1 \oplus b_2 \oplus b_4$



(38) Paul only read $b_2 \oplus b_3 \oplus b_4$



5.2 Some

(39) $[\![\textbf{some of the}]\!] = [\lambda X \subseteq D_e.\{x \in X : \exists y \in X[y \not< x]\}]$
(type $\langle\{e\},\{e\}\rangle$, applied with normal Functional Application)

In our four-book world:

(40) a. $[\![\textbf{books}]\!] =$
$$\left\{\begin{array}{l} b_1 \oplus b_2, b_1 \oplus b_3, b_1 \oplus b_4, b_2 \oplus b_3, b_2 \oplus b_4, b_3 \oplus b_4, \\ b_1 \oplus b_2 \oplus b_3, b_1 \oplus b_3 \oplus b_4, b_1 \oplus b_2 \oplus b_4, b_2 \oplus b_3 \oplus b_4, \\ b_1 \oplus b_2 \oplus b_3 \oplus b_4 \end{array}\right\}$$

b. $[\![\textbf{some of the books}]\!] =$
$$\left\{\begin{array}{l} b_1 \oplus b_2, b_1 \oplus b_3, b_1 \oplus b_4, b_2 \oplus b_3, b_2 \oplus b_4, b_3 \oplus b_4, \\ b_1 \oplus b_2 \oplus b_3, b_1 \oplus b_3 \oplus b_4, b_1 \oplus b_2 \oplus b_4, b_2 \oplus b_3 \oplus b_4 \end{array}\right\}$$

(A richer definition of "some of the books" might include the fact that it only includes groups containing a small proportion of the books.)

(41) a. $\exists$ Paul read some of the books last night.

b. $\exists$ {Paul read x last night: $x \in [\![\textbf{some of the books}]\!]$}
(Doesn't make any claim about the sum of all the books or newspapers, magazines, etc.)

(42) a. $\exists Exh$ Paul read some of the books last night.

b. $\exists \left\{\begin{array}{l} \text{Paul only read } b_1 \oplus b_2, \\ \text{Paul only read } b_1 \oplus b_3, \\ \text{Paul only read } b_1 \oplus b_4, \\ \text{Paul only read } b_2 \oplus b_3, \\ \text{Paul only read } b_2 \oplus b_4, \\ \text{Paul only read } b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_3, \\ \text{Paul only read } b_1 \oplus b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_4, \\ \text{Paul only read } b_2 \oplus b_3 \oplus b_4, \end{array}\right\}$

5.3 Comparatives

(43) $[\![\textbf{more than two}]\!] = \{x \in D_e : |x| > 2\}$ (type $\{e\}$)

In our world with only four books:

(44) a. $\exists Exh$ Paul read more than two books last night.

b. $\exists Exh$ {Paul read x last night : $x \in [\![\textbf{book}]\!]$ & $|x| > 2$ }

c. $\exists\{Exh$ Paul read x last night : $x \in [\![\textbf{book}]\!]$ & $|x| > 2$ }

d. $\exists \left\{\begin{array}{l} \text{Paul only read } b_1 \oplus b_2 \oplus b_3, \\ \text{Paul only read } b_1 \oplus b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_4, \\ \text{Paul only read } b_2 \oplus b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_3 \oplus b_4 \end{array}\right\}$

5.4 Disjunction Problem

(45) $[\![\textbf{or}]\!] = [\lambda X \subseteq D_\sigma \,.\, \lambda Y \subseteq D_\sigma \,.\, X \cup Y]$
(type $\langle\{\sigma\}, \langle\{\sigma\}, \{\sigma\}\rangle\rangle$, applied with normal Functional Application)

(46) a. $\exists$*Exh* Paul read *The New York Times* or *The Boston Globe* last night.

b. $\exists\{$*Exh* Paul read x last night : $x \in \{$*The New York Times*, *The Boston Globe*$\}$

c. $\exists \left\{ \begin{array}{l} \text{Paul only read } \textit{The New York Times} \text{ last night,} \\ \text{Paul only read } \textit{The Boston Globe} \text{ last night} \end{array} \right\}$

(47) a. $\exists$*Exh* Paul read *The New York Times* or some of the books.

b. $\exists \left\{ \begin{array}{l} \text{Paul only read } \textit{The New York Times}, \\ \text{Paul only read } b_1 \oplus b_2, \\ \text{Paul only read } b_1 \oplus b_3, \\ \text{Paul only read } b_1 \oplus b_4, \\ \text{Paul only read } b_2 \oplus b_3, \\ \text{Paul only read } b_2 \oplus b_4, \\ \text{Paul only read } b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_3, \\ \text{Paul only read } b_1 \oplus b_3 \oplus b_4, \\ \text{Paul only read } b_1 \oplus b_2 \oplus b_4, \\ \text{Paul only read } b_2 \oplus b_3 \oplus b_4, \end{array} \right\}$

6 Further Issues

6.1 Order of Operators

The "wrong" order is either the same meaning as the right order...

(48) a. *Exh*$\exists$ Paul read *The New York Times*.

b. Paul only read *The New York Times*.

...or contradictory:

(49) a. $Exh\exists$ Paul read three books.

b. One of these statements is true:

$$\left\{\begin{array}{l}\text{Paul read } b_1 \oplus b_2 \oplus b_3,\\ \text{Paul read } b_1 \oplus b_3 \oplus b_4,\\ \text{Paul read } b_1 \oplus b_2 \oplus b_4,\\ \text{Paul read } b_2 \oplus b_3 \oplus b_4\end{array}\right\}$$

And all of these statements are false:

$$\left\{\begin{array}{l}\text{Paul read } b_1,\\ \text{Paul read } b_2,\\ \text{Paul read } b_3,\\ \text{Paul read } b_4,\\ \text{Paul read } b_1 \oplus b_2,\\ \text{Paul read } b_2 \oplus b_3,\\ \text{Paul read } b_3 \oplus b_4,\\ \text{Paul read } b_1 \oplus b_4,\\ \text{Paul read } b_1 \oplus b_2 \oplus b_3,\\ \text{Paul read } b_1 \oplus b_3 \oplus b_4,\\ \text{Paul read } b_1 \oplus b_2 \oplus b_4,\\ \text{Paul read } b_2 \oplus b_3 \oplus b_4\\ \text{Paul read } b_1 \oplus b_2 \oplus b_3 \oplus b_4\end{array}\right\}$$

7 Conclusions / Extensions

- Uses focus set (Rooth 1992) as alternatives instead of Horn Scales; Uses existential closure (Heim 1982, Kratzer and Shimoyama 2002) to get scope; Uses "contradiction avoidance" to motivate scope.
- Solves Comparative Problem & Disjunction Problem
- Solves Some+Plural problem that other solutions to the Disjunction Problem have.
- Extend to modals in the "how many" case?
- Extend to free choice cases using innocent exclusion?
- Extend to lumping cases?

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