Magnetic Monopoles: Quantization and Quasiparticles

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Magnetic monopoles are proposed particles that carry magnetic charge. Their existence can be motivated by two observations: that Maxwell's equations would be made more symmetric, and that the existence of even one magnetic monopole would require electric charge to be quantized. We will present three derivations of a quantization condition upon magnetic charge: one built using the Aharonov-Bohm effect, one using gauge invariance, and one using Landau Levels. Magnetic monopole quasiparticles in certain materials known as spin ices, as well as magnetic monopole detection experiments, will be discussed qualitatively.

I. INTRODUCTION

A. Overview

Magnetic monopoles are a proposed, as-yet-unobserved particle. A magnetic monopole is an isolated magnetic charge, such as a single "north" pole or a single "south" pole. Just as electric charge density gives the divergence of the $\vec{E}$ field, a magnetic charge would give non-zero divergence to the $\vec{B}$ field. Similarly, a current of magnetic monopoles would give a circulating $\vec{E}$ field along that current. When magnetic monopoles are admitted, Maxwell's equations take a particularly symmetric form [9]:

$$\begin{aligned} \nabla \cdot \vec{E} &= 4\pi\rho \\ \nabla \cdot \vec{B} &= 4\pi\rho_m \\ -\nabla \times \vec{E} &= \frac{1}{c}\frac{\partial \vec{B}}{\partial t} + \frac{4\pi}{c}\vec{j}_m \\ \nabla \times \vec{B} &= \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j} \end{aligned} \tag{1}$$

where ρ_m, $\vec{j}_m$ are the magnetic monopole charge density and magnetic monopole current density, respectively. If $\mathcal{E}$ is any electric quantity, such as $\vec{E}$, $\vec{j}$, or ρ, and $\mathcal{M}$ is the corresponding magnetic quantity, then this form of Maxwell's equations are invariant under the transformation $\mathcal{E} \to \mathcal{M}$, $\mathcal{M} \to -\mathcal{E}$. Electricity and magnetism are made completely symmetric.

Magnetic monopoles must have magnetic charge that comes in quanta of size $\frac{\hbar c}{2e}$, as first derived by Dirac in 1931 [6]. This quantization condition can be used to explain electric charge quantization, if magnetic monopoles exist. I will present three derivations of this fascinating result, each offering a different perspective upon magnetic monopoles (Section II). Then, I will discuss spin ice, a condensed matter system which has magnetic monopole like excitations (Section III A). Finally, I will summarize magnetic monopole detection experiments (Section III B)

B. Electromagnetism in Quantum Mechanics

In this section, I will summarize some important features of electromagnetism in quantum mechanics, drawn from [12, pg 130]. The electric and magnetic fields are related to a scalar and vector potential by

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \vec{B} = \nabla \times \vec{A} \tag{2}$$

This introduces a redundancy: if $\vec{A}_1$, ϕ_1 describe $\vec{E}$, $\vec{B}$, then we can perform a gauge transformation using a function of position and time f to produce $\vec{A}_2$, ϕ_2 that describe the same $\vec{E}$ and $\vec{B}$ fields:

$$\phi_2 = \phi_1 - \frac{1}{c}\frac{\partial f}{\partial t} \qquad \vec{A}_2 = \vec{A}_1 + \nabla f \tag{3}$$

Given particular electromagnetic potentials, the Hamiltonian for a particle of electric charge q and mass m is

$$\hat{H} = \frac{1}{2m}\left(\hat{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi \tag{4}$$

If a wave function $|\psi(t)\rangle$ satisfies Schrodinger's Equation with the above Hamiltonian, then the wave function

$$\exp\left(\frac{iqf}{\hbar c}\right)|\psi(t)\rangle \tag{5}$$

satisfies Schrodinger's Equation with the gauge transformed potentials.

Magnetic monopoles introduce regions of non-zero divergence in the magnetic field. This conflicts with the description of the magnetic field as the curl of a vector potential, which necessarily has zero divergence. In this paper, I will describe the fields around magnetic monopoles using vector potentials containing certain singularities.

A magnetic monopole of charge g at the origin produces a radial $\vec{B}$ field $\frac{g}{r^2}\hat{r}$. In spherical coordinates, one possible vector potential is

$$\vec{A} = \frac{g(1-\cos\theta)}{r\sin\theta}\hat{\phi} \tag{6}$$

This vector potential is singular about the line $\theta = \pi$. This singularity is called a "Dirac String;" I will show that any vector potential for a magnetic monopole has these singularities, and give a physical interpretation to them in Section II A.

II. THE QUANTIZATION CONDITION

Magnetic charge must come in quanta of a given size, just as electric charge does. The value of this charge quanta will be derived, with connections to the Aharonov-Bohm effect (Section II A), to gauge invariance (Section II B), and to Landau Levels (Section II C).

A. A Semi-infinite Solenoid

Imagine a solenoid. Each end produces the magnetic field of one of the poles of a magnetic dipole. I will show that, if one end of the solenoid is infinitely far away, we won't see the field of a dipole, but instead that of a monopole. Thus, a magnetic monopole may be thought of as the end of a semi-infinitely long, infintesimally thin solenoid [13]. In order for the solenoid to be indistinguishable from a magnetic monopole, the infinite extent of the solenoid must be completely aphysical, and so undetectable. In particular, one needs no phase shift in an Aharonov-Bohm style experiment. By quantizing the flux through the solenoid, we will obtain the quantization condition upon magnetic monopoles.

I'll model the semi-infinite solenoid as a cylindrical shell extending from the origin downwards to infinity, with cross sectional area S and azimuthal current λ per unit length. Each ring of current in the solenoid between z and $z + \mathrm{d}z$ has magnetic dipole moment [8, pg 244]:

$$\mathrm{d}\vec{m} = \frac{\mathrm{d}IS}{c}\hat{z} = \frac{\lambda \mathrm{d}z S}{c}\hat{z} \tag{7}$$

A magnetic dipole moment $\vec{m}$ at the origin produces the following vector potential:

$$\vec{A}(\vec{r}) = \frac{\vec{m}\times\vec{r}}{r^3} \tag{8}$$

I'll compute the total vector potential everywhere by integrating the contributions from each of these dipole moments, where the dipole moments extend from the origin downward to minus infinity. The coordinate system is illustrated in Fig. 1.

$$\begin{aligned}
\vec{A}(r,\theta,\phi) &= \int \frac{\mathrm{d}\vec{m}\times\vec{r'}(z')}{[r'(z')]^3} \\
&= \int_{-\infty}^{0} \mathrm{d}z' \frac{\lambda S \sin\theta'(z')\hat{\phi}}{c[r'(z')]^2} \\
&= \int_{-\infty}^{0} \mathrm{d}z' \frac{\lambda S \sin\theta r/c\hat{\phi}}{(r^2+(z')^2-2rz'\cos\theta)^{3/2}} \\
&= \left(\frac{\lambda S}{c}\right)\frac{1-\cos\theta}{r\sin\theta}\hat{\phi}.
\end{aligned} \tag{9}$$

Comparing with equation (6) and introducing the magnetic charge $g = \frac{\lambda S}{c}$, one sees that this is just the field of a magnetic monopole of charge g at the origin. However, one may detect the presence of the infintesimally thin solenoid using the Aharonov-Bohm effect [12, pg 136]. An electron double-slit experiment whose paths encircle the solenoid, as shown in Fig. 2, will have its probability amplitudes offset by a quantity that varies depending on the phase

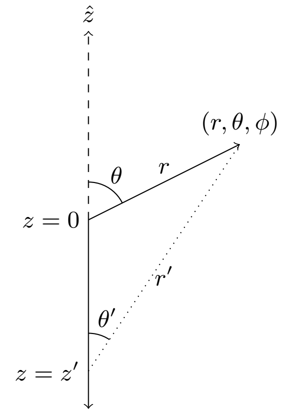

FIG. 1: A magnetic dipole moment at $z'\hat{z}$ contributes to the vector potential at (r,θ,ϕ). Shown projected on to the plane normal to $\hat{\phi}$.

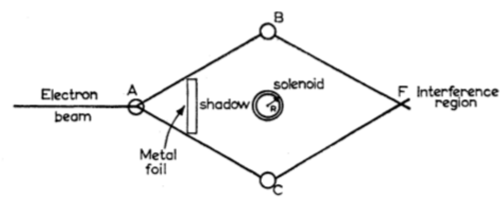

FIG. 2: Detecting the presence of a solenoid via the Aharonov-Bohm effect. The electron trajectories encircle the solenoid, and so the probability amplitudes in the interference region are shifted by an amount dependent on equation (10). Taken from [1]

$$\exp\left(i2\pi\frac{e}{hc}\Phi_B\right) \tag{10}$$

where Φ_B is the flux through the solenoid. So, if Φ_B is an integer multiple of $\frac{e}{hc} = \Phi_0$, known as the fundamental flux quantum, then equation (10) will have the same value as if the Φ_B were zero, and the solenoid will be undetectable. The flux through the solenoid is $\Phi_B = 4\pi\frac{\lambda S}{c} = 4\pi g$, so

$$\begin{aligned}
\Phi_B &= N\Phi_0 \\
4\pi g &= N\frac{2\pi\hbar c}{e} \\
g &= N\frac{\hbar c}{2e}.
\end{aligned} \tag{11}$$

Equation (11) is the celebrated *Dirac Quantization Condition.* It means that, if any magnetic monopoles exist, their charge must come in quanta of a certain size. This size is parameterized by the fine structure constant $\alpha = \frac{e^2}{\hbar c} \approx 1/137$. Magnetic charge must come in lumps of size $\frac{1}{2\alpha}e \approx \frac{137}{2}e$. This proportionality constant means that two magnetic monopoles of unit charge would experience a Coulomb force ≈ 4692.25 times stronger than two electric monopoles of charge e. In [6], Dirac speculates that this large force may explain why isolated magnetic poles are not encountered.

Perhaps even more intriging is the possibility that equation (11) could explain electric charge quantization [6]. If even a single magnetic monopole of charge g exists in our universe, then solving equation (11) for e gives the *electric* charge quantization condition $q = N\frac{\hbar c}{2g}$, where q is any electric charge. The fact that, in our universe, electric charge is quantized hints at the possibility of magnetic monopoles.

Looking back at equations (11) and (6), we see that the vector potential is singular along the line $\theta = \pi$. This line coincides with the extension of the infinitely long solenoid. Thus, we can interpret Dirac strings as a line along which lies an infintesimally thin, infinitely long solenoid. Because this solenoid is infinitely thin and invisible to Aharonov-Bohm style experiments, no experiment could ever detect it; we are free, even, to move it about. In the next section, I'll show that different gauges correspond to moving about the Dirac strings.

B. Gauge Transformations and the Dirac String

It is also possible to derive the quantization condition by looking at the electromagnetic field around a magnetic monopole, and performing gauge transformations so as to move about the Dirac string [13]. When this is done, the quantization condition pops out of the requirement that the gauge transformation be single valued everywhere.

This approach can be motivated by the following result: If we have a magnetic monopole at the origin, then there is no gauge for which the vector potential is non-singular everywhere except the origin [13]. I'll now prove this result.

Establish spherical coordinates, and put a monopole of magnetic charge g at the origin. Consider any closed loop ∂S at fixed θ and r as ϕ goes from 0 to 2π, and integrate $\vec{A}$ along ∂S. Let S be a surface whose boundary is ∂S. If $\vec{A}$ were non-singular, you could rewrite this using Stoke's Theorem:

$$\begin{aligned}\oint_{\partial S} \mathrm{d}\vec{l}\cdot\vec{A} &= \iint_S \mathrm{d}\vec{S}\cdot\vec{B}\\ &= \int_0^\theta d\theta' \int_0^{2\pi} d\phi\, r^2\sin\theta' \frac{g}{r^2}\\ &= 2\pi g(1-\cos\theta) \qquad (12)\end{aligned}$$

But, taking $\theta \to \pi$, one obtains a value of $4\pi g$, a contradiction, because, as $\theta \to \pi$, the loop ∂S shrinks to a point. So, the assumption that $\vec{A}$ was non-singular must be wrong, completing the proof that any vector potential must have a singularity not at the origin.

No matter what gauge, there will be a Dirac string; let's try to patch together two different gauges, each of which is non-singular in the region it is defined. Fix a magnetic monopole of strength g at the origin. The magnetic field is $\vec{B} = \frac{g}{r^2}\hat{r}$. Depending on choice of gauge, a Dirac string will point off in some direction from the origin. One trick to avoid this is to define two overlapping regions, each with their own gauge, such that, in the region it is defined, the vector potential does not have any singularities in its respective gauge.

Define the vector potentials and regions

$$\begin{aligned}\vec{A}_1 &= \frac{g(1-\cos\theta)}{r\sin\theta}\hat{\phi} \qquad R_1: \theta \in [0, \pi/2+\delta)\\ \vec{A}_2 &= \frac{-g(1+\cos\theta)}{r\sin\theta}\hat{\phi} \qquad R_2: \theta \in (\pi/2-\delta, \pi] \quad (13)\end{aligned}$$

These regions overlap in $\pi/2 - \delta < \theta < \pi/2 + \delta$, and in each gauge, the curl of $\vec{A}_i$ gives the correct magnetic field. Place a particle with electric charge e in the presence of the magnetic monopole. Let's solve for the phase factor, $S_{1\to2}$, of the gauge transformation that goes between $\vec{A}_1$ and $\vec{A}_2$; see equation (5).

$$\begin{aligned}\nabla f &= \vec{A}_1 - \vec{A}_2\\ &= \frac{2g}{r\sin\theta}\hat{\phi}\\ f &= 2g\phi\\ \exp\left(\frac{ief}{\hbar c}\right) &= \exp\left(\frac{i2eg\phi}{\hbar c}\right) = S_{1\to2}\\ S_{1\to2}|^{\phi=0} &= S_{1\to2}|^{\phi=2\pi}\\ \frac{2ge}{\hbar c} &= 0, \pm1, \pm2, ... \qquad (14)\end{aligned}$$

which is the quantization condition: g is constrained to be a multiple of $\frac{\hbar c}{2e}$. This derivation does not rely upon the more heuristic reasoning of Section II A, which depended upon building up the magnetic monopole as an infinite solenoid. Rather, this approach must apply to *any* magnetic monopole, no matter how it is constructed.

C. Landau Levels for Magnetic Monopoles

If magnetic monopoles exist, a very natural system to consider is the magnetic analog of Landau Levels [7]. Using this system, we will give a heuristic argument for the quantization condition.

Place a magnetic monopole of charge g and mass m between the plates of a parallel plate capacitor whose plates lie in the xy plane. The capacitor will produce a uniform electric field $\vec{E} = E\hat{z}$. The monopole will have energy $\hbar\omega(n + 1/2)$, where $\omega = \frac{gE}{mc}$ and $n = 0, 1, 2, ...$

Consider a high-energy, semi-classical limit. The monopole will be orbiting in the xy plane. Let r be the radius of its orbit and v its velocity. In the electric field, the monopole will experience a Lorentz force

gEv/c which, when equated with the centripetal force mv^2/r, gives

$$E = \frac{mvc}{rg} \tag{15}$$

Meanwhile, we can equate the kinetic energy of the orbit with the quantized energy of the monopole,

$$\begin{aligned} \frac{1}{2}mv^2 &= \hbar\omega(n+1/2) \\ &= \hbar\frac{gE}{mc}(n+1/2) \\ &= \hbar\frac{v}{r}(n+1/2), \text{ using equation (15)} \\ J_z = mvr &= 2\hbar(n+1/2) \end{aligned} \tag{16}$$

which is a quantization condition upon the angular momentum, J_z, in the $\hat{z}$ direction. We can use this result to obtain a quantization condition upon E by writing equation (15) in terms of J_z:

$$\begin{aligned} E &= J_z\frac{c}{r^2g} \\ &= \frac{\hbar c}{r^2g}(2n+1) \end{aligned} \tag{17}$$

Finally, we'll get the Dirac quantization condition by relating E to the electric charge upon the capacitor plates. The magnitude of the electric field between two plates, each having uniform charge density $\pm\sigma$, is given by $E = 4\pi\sigma$. If $\pm Q$ is the charge on each plate directly above or below the orbit, then $Q = \pi r^2\sigma$. So, we have the condition

$$E = 4\frac{Q}{r^2} \tag{18}$$

Combining equations (15) and (18) yields a quantization condition upon Q:

$$Q = \frac{\hbar c}{2g}(n+1/2) \tag{19}$$

When $n = 0$, there still remains some charge due to the zeropoint energy of the monopole. However, as $r \to 0$, the area on the plate we are considering also shrinks to zero, and therefore so must the charge we observe. In order to enforce the condition that the measured charge goes to zero in the lowest energy state, we subtract out this vacuum effect. The observable charge is

$$Q_{obs} = n\frac{\hbar c}{2g} \tag{20}$$

And so the charge on the plate is quantized in units of $\frac{\hbar c}{2g}$. But we already know that the electric charge is quantized in units of e, so we can conclude that g must also be quantized in units of $\frac{\hbar c}{2e}$. This derivation makes explicit the connection between magnetic charge quantization and electric charge quantization.

III. REAL MAGNETIC MONOPOLES

Although true magnetic monopoles have yet to be discovered, some condensed matter systems, at low temperature, contain quasiparticles resembling magnetic monopoles. Additionally, there have been many searches for monopoles from astrophysical sources and in particle colliders.

A. Spin Ice

Certain materials known as *spin ices* can behave as though they contain a free gas of magnetic monopoles [5]. Examples of spin ices include $Dy_2Ti_2O_7$ and $Ho_2Ti_2O_7$. These materials exhibit magnetic monopole quasiparticles at temperatures below order 1K [3].

A spin ice is a solid composed of a number of sites, each of which is tetrahedral in shape, with an electron at each vertex of the tetrahedron. This configuration is known as a pyrochlore lattice. The electron's spin is constrained to either point directly inwards or directly outwards of the tetrahedron. The crystal, and the axis of each spin, are shown in Fig. 3.

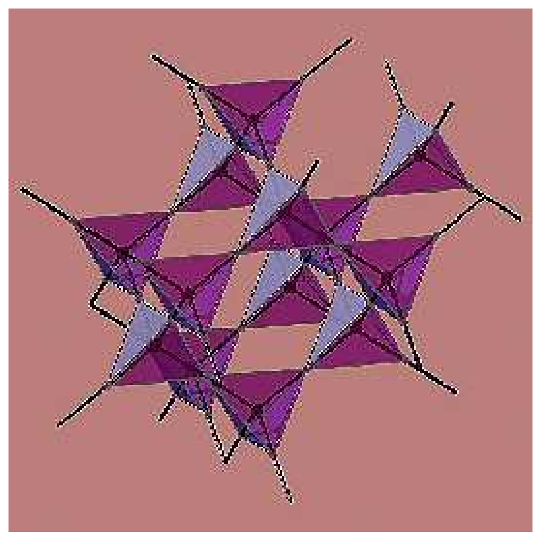

FIG. 3: The pyrochlore lattice of spin ice. Each spin resides along an axis parallel to one of the black lines, which penetrate the tetrahedrons at their vertexes and intersect at the center of the tetrahedral site. Taken from [5]

Each site within a spin ice has four spins pointing in/out of it, one per vertex on the tetrahedron. Each spin lies along the axis connecting the centers of two sites. When most of the spins point inward (or outward), the site looks like a magnetic monopole when a certain approximation is made. The approximation is that of modeling a single spin as a pair of magnetic monopoles attached by a dumbbell. The dumbbell is chosen exactly long enough that each monopole must reside at the center of a site. This approximation is well-illustrated by

Fig. 4 and Fig. 5.

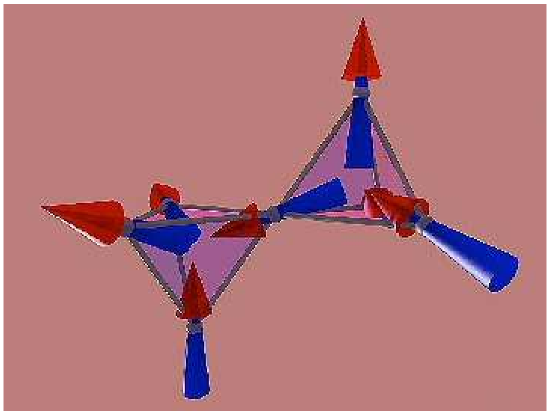

FIG. 4: A pair of tetrahedral sites obeying the ice rule. Spins are indicated via arrows. Taken from [5]

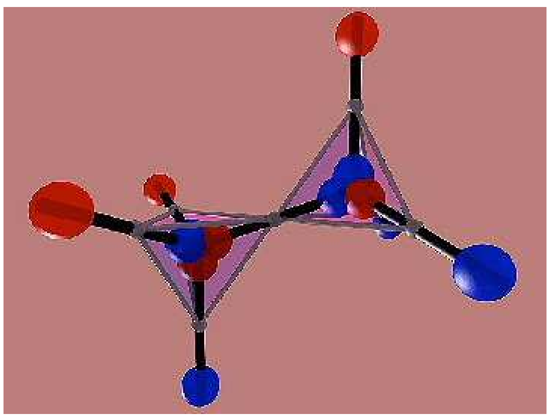

FIG. 5: The approximation used to recover magnetic monopoles. Each spin is modeled as a "dumbbell" with each end of the dumbbell corresponding to a north pole and a south pole. Taken from [5]

The lowest energy configuration of this system occurs when each tetrahedron has two spins pointing inward and two spins pointing outward, known as the *ice rule*. The physical intuition behind this may be understood through the dumbbell approximation: each site in the crystal contributes a Coulomb-like potential term to the Hamiltonian of the crystal, and so the lowest energy state occurs when all sites are neutral. Neutral sites correspond to two spins in, two spins out: the ice rule.

A huge number of configurations follow the ice rule, giving a highly degenerate ground state. Excitations of the system introduce violations of the ice rule. If a single spin (dumbbell) is flipped, then one site in the crystal picks up a positive magnetic charge, and an adjacent site picks up a negative magnetic charge. These charges can then move through the crystal by flipping adjacent spins. This behavior is shown in Fig. 6.

These excitations produce a monopole-antimonopole pair; what makes this phenonenon different from the pair of "monopoles" at the ends of an ordinary solenoid is that the monopoles are deconfined, and so free to move throughout the crystal. They may do so by flipping adjacent spins, creating a string of flipped dipoles. This string of flipped dipoles is equivalent to a Dirac string. Because the Coulomb-like magnetic monopole force falls off like $1/r^2$, while the dipole-dipole interactions fall off like $1/r^3$, one may, to a very good approximation, ignore their contribution to the Hamiltonian [11]. In this approximation, the path of the flipped dipoles doesn't matter, and so this analog of the Dirac String cannot be said to lie upon any one particular path connecting two monopoles. This is analogous to the ambiguity of the location of the Dirac string, which varies with the gauge. Crucially, however, these Dirac strings are not aphysical, and the monopoles in the system do not obey the Dirac quantization condition.

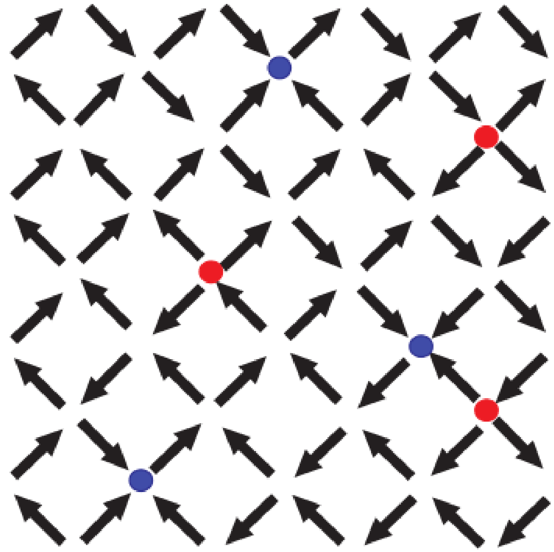

FIG. 6: Magnetic monopoles in a 2D spin ice. Each arrow represents a spin; red/blue dots are positive/negative magnetic monopoles. Paths of flipped dipoles connect the monopoles, analogous to Dirac strings. Taken from [2]

The existence of these magnetic monopole quasiparticles has been experimentally verified. Using the material $Dy_2Ti_2O_7$, Bramwell et al measured a magnetic monopole current, as well as Coulomb-like $1/r^2$ interactions between the monopoles [3]. They also determined the magnetic conductivity of the material, and measured the charge of these emergent monopoles. There is a real, physical system in which we may experimentally study magnetic monopole quasiparticles.

B. Where Are The Monopoles?

Magnetic monopoles have been extensively searched for. One search technique relies upon the fact that, just as an electric current produces a circulating magnetic field, a magnetic current produces a circulating electric field; see equation (1). If a magnetic monopole were to pass through a conducting ring, there would be an induced current from the circulating $\vec{E}$ field. This current would be qualitatively different from that of an ordinary magnetic dipole passing through a conducting ring.

In [4], Blas Cabrera describes one such magnetic monopole detection experiment. The current through a superconducting ring was monitored over the course of 151 days. One candidate detection was observed. The detection was consistent with a magnetic monopole carrying one unit of magnetic charge. This tantilizing result has never been repeated. Consensus indicates that this detection was inconclusive.

A number of accelerator experiments have attempted to create magnetic monopoles. Previous searches at the Tevatron, LEP, and HERA have failed to produce and detect any magnetic monopoles [11]. These negative results suggest that no magnetic monopoles exist of mass less than 1 TeV$/c^2$. Current searches at higher energy are ongoing at the LHC in the MOeDAL experiment [10].

Other experiments have attempted to measure the flux of magnetic monopoles from astrophysical sources [11]. The MACRO experiment used an underground detector running for 11 years. It found no monopoles, and placed an upper bound of 10^{-16} cm^{-2}sec^{-1}sr^{-1} upon the flux of magnetic monopoles. The RICE experiment, also finding no monopoles, improved this bound to 10^{-18} cm^{-2}sec^{-1}sr^{-1}.

Another upper bound upon the flux of magnetic monopoles may be derived from the magnetic field throughout our galaxy [11]. The Milky Way is permeated by an approximately 3 μG magnetic field. If magnetic monopoles are in our galaxy, they would drain energy from this magnetic field, with the rate of dissipation depending on the flux of monopoles. For very heavy magnetic monopoles, their motion would be dominated by gravitational forces, but for lighter monopoles of mass $\lesssim 10^{17}$ GeV$/c^2$, the flux can be bounded above by 10^{-15} cm^{-2}sec^{-1}sr^{-1}.

Despite the theoretical appeal of magnetic monopoles, their physical existence has not yet been verified. If they do exist, they are extraordinarily rare.

IV. DISCUSSION

More relevant physics exists in the literature on magnetic monopoles. A full treatment is beyond the scope of this paper. We quote some of the more important results:

1. There are several more derivations of the magnetic monopole quantization condition. One may heuristically derive the quantization condition using a semi-classical argument based on quantizing the angular momentum carried by electromagnetic fields [8, pg 362]. The electromagnetic field created by a fixed electric monopole and a fixed magnetic monopole carries a certain angular momentum; by saying this angular momentum comes in half-integer multiples of $\hbar$, the Dirac quantization condition is recovered. Dirac originally obtained his result by introducing a nonstandard extension to quantum mechanics, known as *non-integrable phase factors* [6]. Non-integrable phase factors end up uniquely determining the electromagnetic field. The quantization condition results from enforcing the condition that all non-integrable wave functions for a system describe the same magnetic field.

2. Grand Unified Theories predict the existence of magnetic monopoles [9]. Their masses are predicted to be on the order of 10^{16} GeV$/c^2$, far beyond the energies reachable in modern particle accelerators. Some Grand Unified Theories also predict magnetic monopoles with masses on the order of 10 TeV$/c^2$, which could be reached in future colliders. Because of these predictions, along with the fact that even a single magnetic monopole would explain electric charge quantization, string theorist Joseph Polchinski commented that "the existence of magnetic monopoles seems like one of the safest bets that one can make about physics not yet seen"[10].

3. Given our current understanding of the early universe, Grand Unified Theories predict the production of many magnetic monopoles after the Big Bang [11]. The resulting flux of monopoles would be many orders of magnitude higher than existing upper bounds. This is known as the "monopole problem." It helped motivate the theory of inflation, in which the monopole density would be substantially decreased.

4. Magnetic monopoles can be generalized to dyons, particles containing both electric and magnetic charge, first proposed by Schwinger [9]. Dyons obey a quantization condition that generalizes the one for electric and magnetic monopoles. If a dyon has electric charge e_1 and magnetic charge g_1, and any other dyon has electric charge e_2 and magnetic charge g_2, then $e_1 g_2 - e_2 g_1$ must be an integer multiple of $\hbar c$.

In conclusion, magnetic monopoles are completely reasonable particles, but have not yet been observed. Their existence would explain electric charge quantization, and introduce a perfect symmetry between electricity and magnetism in Maxwell's Equations.

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