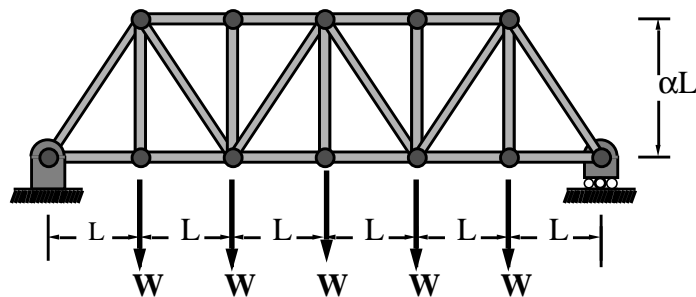


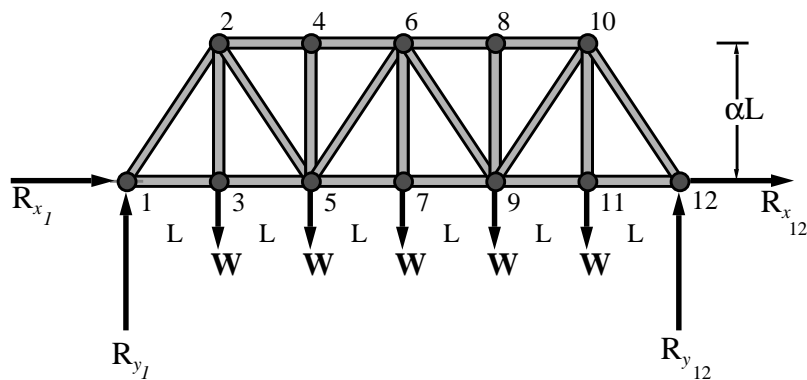
CHAPTER 3 Internal Forces and Moments

3.1 The Truss

Construct a procedure for calculating the forces in all the members of the *statically determinate truss* shown below. In this take $\alpha = \sqrt{3}$



1. We begin with an isolation of the entire structure:



demo

2. Then we determine the reactions at the supports.

This is not always a necessity, as it is here, but generally it is good practice. Note all of the strange little circles and shadings at the support points at the left and

right ends of the structure. The icon at the left end of the truss is to be read as meaning that:

- the joint is frictionless and
- the joint is restrained in both the horizontal and vertical direction, in fact, the joint can't move in any direction.

The icon at the right shows a frictionless pin at the joint but it itself is sitting on more frictionless pins. The latter indicate that the joint is free to move in the horizontal direction. This, in turn, means that the horizontal component of the reaction force at this joint, $R_{x_{12}}$ is zero, a fact crucial to the *determinancy* of the problem. The shading below the row of circles indicates that the joint is *not* free to move in the vertical direction.

From the symmetry of the applied loads, the total load of $5W$ is shared equally at the supports. Hence, the vertical components of the two reaction forces are

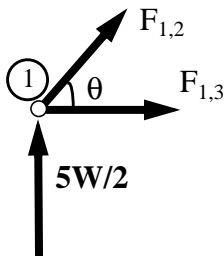
$$R_{y_1} = R_{y_{12}} = 5W/2.$$

Both of the horizontal components of the reaction forces at the two supports must be zero if one of them is zero. This follows from the requirement of force equilibrium applied to our isolation.

$$R_{x_1} = R_{x_{12}} = 0.$$

3. *Isolate a joint at which but two member forces have yet to be determined and apply the equilibrium requirements to determine their values.*

There are but two joints, the two support joints that qualify for consideration this first pass through the procedure.



I choose to isolate the joint at the left support. Equilibrium of force of node # 1 in the horizontal and vertical direction yields the two scalar equations for the two unknown forces in members 1-2 and 1-3. In this we again assume the members are in tension. A negative result will then indicate the member is in compression. The proper

way to speak of this feature of our isolation is to note how “the members in tension pull on the joint”.

Equilibrium in the x direction and in the y direction then requires:

$$F_{1,2} \cdot \cos\theta + F_{1,3} = 0$$

$$F_{1,2} \cdot \sin\theta + (5/2) \cdot W = 0$$

where the $\tan\theta = \alpha$ and given $\alpha = \sqrt{3}$ so $\sin\theta = \sqrt{3}/2$ and $\cos\theta = 1/2$. These yield

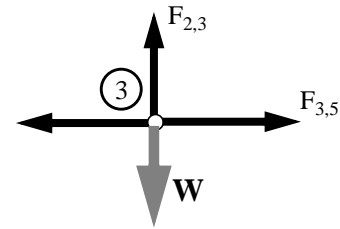
$$F_{1,2} = -(5/\sqrt{3}) \cdot W$$

$$F_{1,3} = (5/2\sqrt{3}) \cdot W$$

The negative sign indicates that member 1-2 is in compression.

4. Repeat the previous step in the procedure.

Having found the forces in members 1,2 and 1,3, node, or joint, # 3 becomes a candidate for isolation.



It shows but two unknown member forces intersecting at the node. Node # 12 remains a possibility as well. I choose node # 3. Force equilibrium yields

$$-F_{1,3} + F_{3,5} = 0$$

and

$$+F_{2,3} - W = 0$$

Note how on the isolation I have, according to convention, assumed all member forces positive in tension. $F_{1,3}$ acts to the left, pulling on pin # 3. This force vector is the equal and opposite, internal reaction to the $F_{1,3}$ shown in the isolation of node 1. With $F_{1,3} = (5/2\sqrt{3})W$ we have

$$F_{3,5} = (5/2\sqrt{3}) \cdot W$$

and

$$+F_{2,3} = W$$

These equations are thus, easily solved, and we go again, choosing either node # 2 or # 12 to isolate in the next step.

5. *Stopping rule: Stop when all member forces have been determined.*

This piece of machinery is called *the method of joints*. Statically determinate truss member forces can be produced using other, just as sure-fire, procedures. (See problem 3.1) The main point to note is that all the member forces in a truss can be determined *from equilibrium conditions alone* using a judiciously chosen sequence of isolations of the nodes **if and only if** the truss is statically determinate. That's a circular statement if there ever was one but you get the point¹.

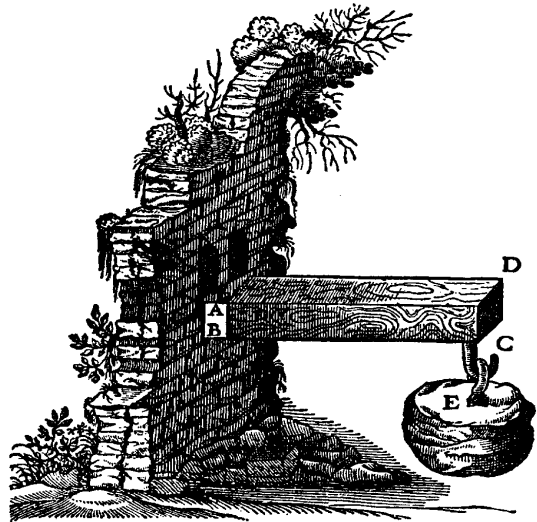
3.2 Internal Forces and Moments in Beams

The Cantilever according to Galileo

The figure shows a seventeenth century *cantilever beam*. It appears in a book written by Galileo, his *Dialogue Concerning Two New Sciences*.

Galileo wanted to know when the cantilever beam would break. He asked: What weight, hung from the end of the beam at C, would cause failure?

You might wonder about Galileo's state of mind when he posed the question. From the



1. Note how, if I were to add a redundant member connecting node #3 to node #4, I could no longer find the forces in the members joined at node #3 (nor those in the members joined at nodes #2 and #5). The problem would become equilibrium indeterminate

looks of the wall it is the latter whose failure he should be concerned with, not the beam.

According to Galileo, the beam will fail when $W_{failure} = \frac{1}{2}(h/L)F_{AB}$

Where, the end load, W , to cause failure of the member acting as a cantilever is much less than the load, F_{AB} , the load which causes failure of the member when loaded axially, as a truss member (by the factor of $(1/2)h/L$).

- This is an important result. If one has determined the value of F_{AB} for a specimen in tension, then inserting this into the equation above, we can compute the end load a cantilever beam will support before failure.
- Galileo has done all of this without drawing an isolation, or free-body diagram!
- He is wrong, precisely **because he did not draw an isolation.**

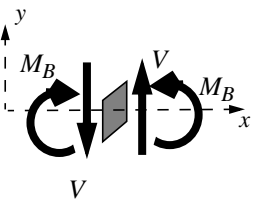
We fault Galileo for not recognizing that there must be a vertical, reaction force at the root of the cantilever. But maybe he just ignored it because he knew from his analysis that it was small relative to the internal forces acting normal to the cross section at AB . Here is his achievement: he saw that the mechanism responsible for providing resistance to bending within a beam is the tension (and compression) of its longitudinal fibers.

Shear Force and Bending Moment in Beams

We will be bold and state straight out, as conjecture informed by our study of Galileo's work, that failure of a beam in bending will be due to an excessive bending moment. When confronted with a beam, we must determine *the bending moment distribution* that is, how it varies along the span so that we can ascertain the section where the maximum bending moment occurs.

Our convention for shear force and bending moment can be stated as follows:
A positive shear force acts on a positive face in a positive coordinate direction or on a negative face in a negative coordinate direction.

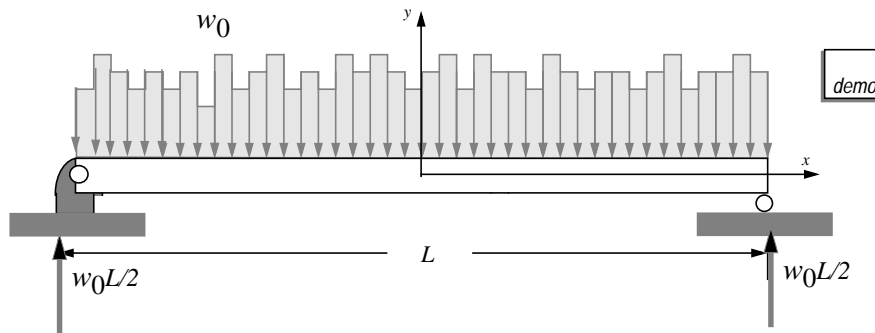
A **positive face** is short for a **face whose outward normal is in a positive coordinate direction**. A positive bending moment acts on a positive face in a positive coordinate direction or on a negative face in a negative coordinate direction. *Warning:* Other textbooks use other conventions. It's best to indicate your convention on all exercises, including in your graphical displays the sketch to the right.



Exercise 3.1

Determine the shear force and bending moment distribution for a uniformly loaded, simply supported beam.

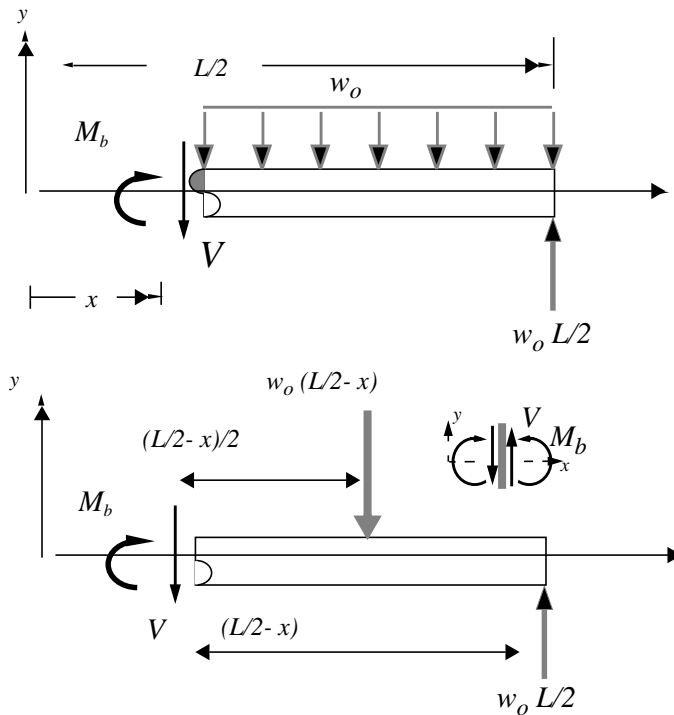
We first determine the reactions at the supports at the left and right ends of the span.



We isolate a portion of the beam to the right of some arbitrarily chosen station x .

We have replaced the load w_0 distributed over the portion of the span x to $L/2$, by an equivalent system.

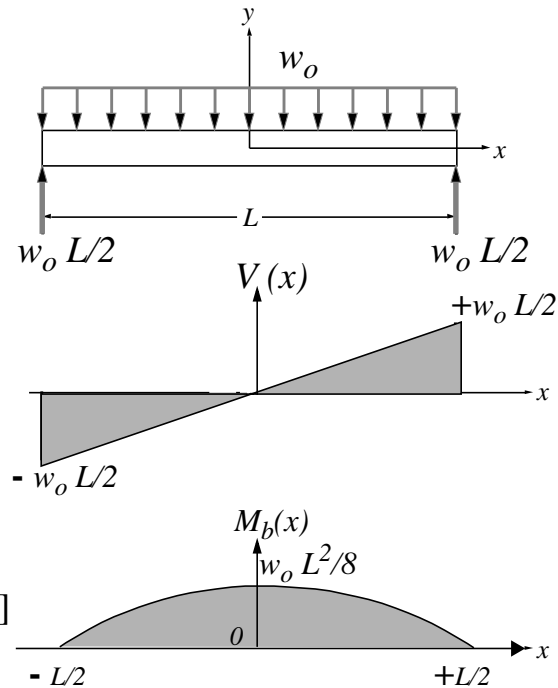
Applying force equilibrium to the isolation and moment equilibrium about point x yields the shear force and bending moment distributions shown next page:



$$w(x) = w_0$$

$$V(x) = w_0 x$$

$$M_b(x) = (w_0/2)[(L/2)^2 - x^2]$$



Exercise 3.2

Construct shear force and bending moment diagrams for the simply-supported beam shown below. How do your diagrams change as the distance a approaches zero while, at the same time, the *resultant* of the distributed load, $w_0(x)$ remains finite and equal to P ?

