Journal of Law, Economics, and Organization Advance Access published December 3, 2008

Disagreement and the Allocation of Control

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This article studies the allocation of control when there is disagreement—in the sense of differing priors—about the right course of action. People then value control rights since they believe that their decisions are better than those of others. More disagreement (due to, e.g., fundamental uncertainty) increases the value that players attach to control. The article shows that all income and control of a project should then be concentrated in one hand: income rights should go more to people with more control since such people value income higher (because they have a higher opinion of the decisions made); control rights should go more to people with more income since they care more (and believe that they make better decisions). Different projects may be optimally "owned" by different people. Furthermore—with residual income exogenously allocated—complementary decisions should be more co-located, whereas substitute decisions should be more distributed. Confident people with a lot at stake should—in a wide range of settings—get more control. (*JEL* D7, D8, L2, M1)

1. Introduction

The allocation of control is a key decision in institutional design. Within firms, organization design is to no small extent concerned with the vertical and horizontal allocation of control. Between firms, the allocation of control plays a key role in, for example, VC financing of entrepreneurs or in the design of joint ventures and alliances.

The allocation of control only matters, however, when different people would make different decisions. One reason for such different decisions is that people may openly and fundamentally disagree on the right course of action, in the sense of differing priors, an assumption that I discuss in more detail at the end of Section 2. Assuming differing priors captures a situation where people may have different intuition or different mental models, so that despite

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This article benefited a lot from extensive conversations with Bob Gibbons. I also thank Roberto Rigobon for his help and support and the participants in the MIT Organizational Economics Lunch for their comments and suggestions. I finally thank the co-editor (Pablo Spiller) and two anonymous referees for making this a much better article. An earlier version of this article circulated as "On the Allocation of Control under Differing Priors" (2004).

The Journal of Law, Economics, & Organization
doi:10.1093/jleo/ewn020

identical information they may still disagree. How does such open disagreement affect the allocation of control?

To analyze this issue, I study a stylized setting in which two players are involved in a project but may disagree (in the sense of differing priors) on what decisions will lead to a project success. Although decisions themselves are not contractible, the control rights over these decisions are, an assumption that I will discuss in more detail. At the start of the game, the players allocate control (and income, if applicable) through Nash bargaining. I study the equilibrium allocation both with the income shares exogenously given and with income shares determined in the bargaining.

I first derive a useful preliminary result: players—in this stylized setting—prefer to control decisions themselves (since they may disagree with the decisions of others) and value such control more when disagreement is more likely. An empirical implication is that in settings with more disagreement—such as new industries, early ventures, or industries and firms in transition—there will be more control fights, and control rights will be used more often as a form of compensation (e.g., in exchange for capital).

I then consider the case with endogenous income rights and derive a new mechanism for the co-location of control and contractible income. In particular, I show that—in equilibrium—all income and control will be concentrated in one hand due to the following intuitive self-reinforcing mechanism: as a player gets more control rights, she values income rights more (because she believes that she makes better decisions) so that it is optimal to shift income to that player; as a player gets more income rights, she values control rights more (since she has more at stake and believes that she makes better decisions) so that it is optimal to shift control to that player. Note that this is a different argument than the traditional "internalizing externalities" argument of agency theory. One implication of the current theory is that income and control are more likely to be co-located when there is more potential for open disagreement.

For the case that income rights are exogenously determined, I first derive the following two results on complements and substitutes:

- Complementary decisions, that is, decisions for which the value of getting one decision correct increases in the other one being correct, should be more concentrated with one person.[1]
- Substitute decisions, that is, decisions for which the value of getting one decision correct decreases in the other one being correct, should be distributed among different people.

To see the intuition for these results, consider the case that two decisions are perfect substitutes: the project is a success for sure if at least one of the decisions is correct. If a player now controls the first decision and is very confident about the right course of action, then he believes that the second decision is irrelevant, so that he values that control right very low. The other player may disagree with the first decision and thus value control rights over the second decision much

1. Complements and substitutes are discussed in more detail in Section 5.1.

higher. The converse intuition works for the case where two decisions are complements, as in a project that is a success only if both decisions are correct. Although this result on complements has a superficial resemblance to the oft-cited result in Hart and Moore (1990) that complementary assets should be owned together, the results are actually quite different. In particular, the current result (vs. Hart-Moore) is about control rights (vs. assets), and about complementarity with respect to the *level* (vs. the *slope*) of revenue as a function of decisions (vs. as a function of personal firm-specific human capital investments). Whinston (2003) has pointed out that the levels versus slope distinction has important empirical ramifications.

I finally also show that—for a broad class of projects—control rights should be allocated more to people with a larger share in the project's income and to people who are confident about the right course of action. The first group of people is more sensitive to project success, whereas the second is more convinced that they make better decisions than others, both of which make a player value control more in this context.

I also discuss the role of differing priors in these results. In particular, apart from doing comparative statics on the probability of disagreement, I compare the results of this article to the results of two related models with, respectively, private information and private benefits as the source of potential conflict. The private information model is identical to the differing priors model except for the fact that the belief differences now originate in private information that the players cannot communicate. The private benefits model, on the other hand, follows articles such as Aghion and Bolton (1992), Aghion and Tirole (1997), Hart and Holmstrom (2002), Prendergast (2002), or Baker et al. (2004). These articles study settings that are similar to this article (i.e., a project that generates income which depends on decisions) but where the agency problem originates in private benefits instead of in differing priors. I show that both the private information model and the private benefits model give some very different results from the differing priors model in this article, and I discuss some intuition for these differences. One driver of the differences is the fact that in the private information and private benefits models—unlike in the differing priors model—the players value residual income in expectation identically. A second driver is that in the models with private information and private benefits, the players agree on what kind of decisions will maximize the project revenue. If maximizing project revenue is optimal (as opposed to, e.g., maximizing private benefits), it is then a matter of which allocation of control is most likely to generate these optimal decisions. With differing priors, on the contrary, players disagree on what decision will maximize the project revenue and each player wants control to ensure that whatever he thinks is right for the project indeed gets done.[2]

2. Since this comparison is focused on particular models, it is not (meant to be) conclusive on whether the results of this article can only obtain in a differing priors model. It just shows, and gives intuition for, some important differences with these (closely related) private information and private benefits models.

1.1 Contribution

By studying the allocation of control under open disagreement, this article makes three contributions. First, it shows why control rights may be valued, even in the absence of private benefits or ex-ante investments, and how this value co-varies with the potential for disagreement. Second, the article derives a new—and intuitive—self-reinforcing mechanism for the co-location of control and contractible income. Third, the article also shows that complementary decisions should be more co-located, whereas substitute decisions should be more distributed. From an empirical perspective, a useful feature of the model is that all the players' benefits are derived from the project's income stream, which is more easily measurable than, say, players' private benefits or costs.[3]

1.2 Literature

The issue of the optimal allocation of control appears under many different guises throughout the economic literature. Even the first welfare theorem deals with the question how delegating control over production and consumption affects efficiency. Rather than taking this broad view, I will focus here on the literature that is directly related to the analysis in this article.

There is an extensive literature on the allocation of control with private benefits or private information, including team theory (Marschak and Radner 1972) and the literature on delegation, centralization, control with decision-externalities, and hierarchies (Aghion and Bolton 1992; Aghion and Tirole 1997; Dessein 2002; Hart and Holmstrom 2002; Zábojník 2002; Aghion et al. 2004; Hart and Moore 2005; Marino and Matsusaka 2005; Baker et al. 2006; Alonso and Matouschek 2007). Although all these articles deal with the optimal allocation of control among players, most focus on issues or settings that are very different from the current one. In terms of setup, the current article is in fact closest to Hart and Holmstrom (2002), who study two models in which decisions are noncontractible, decision rights can be allocated contractually, and managers have private preferences over the decisions. The focus of their article is very different from the current one, however: it shows that firm boundaries matter rather than deriving general principles how control should be allocated. In terms of results, the closest article is Baker et al. (2006), who study the allocation of control when players have private benefits. They argue that, first, in a static model decision rights should be allocated to players who use the decision rights in a way that maximizes the joint utility of all players, but, second, that even such allocation will typically not result in first-best decisions. They then study a repeated-game setting and show that the optimal allocation of control is the one that minimizes the maximum aggregate temptation to renege. It is, however, unclear how to empirically relate or compare their results based on inalienable private benefits to the current context

3. Just like private information or private benefits, a player's beliefs are difficult to measure directly. Empirical tests will thus often rely on comparative statics along other dimensions. However, the empirical literature has suggested some useful proxies for disagreement such as the stage of development of an industry or disagreement among analyst estimates.

where all benefits derive from one, potentially contractible, income stream. In particular, it seems that if all private benefits derive from one income stream, either directly or through reputation, then that would eliminate all decision conflicts in Baker et al. (2006). Second, although their article focuses on the basic allocation principle (which in this case is to allocate control to the player who values it most), the current article focuses on more particular results and predictions, such as these on the co-location of income and control or on complements versus substitutes.

Finally, this article is closely related to two parallel articles that study control under differing priors. The first article, Van den Steen (2007a), studies the costs of incentives under disagreement. A first cost of incentives under disagreement is that—when the principal retains control over the direction of the project but gives the agent incentives for effort—pay-for-performance may allocate residual income to a player who undervalues that residual income relative to the principal. This first result is thus a flip-side to the concentration of income and control derived in the current article. A second, and potentially more important, cost is that pay-for-performance (and intrinsic motivation, for that matter) may make it more difficult for a principal to exert authority over an agent, which leads—on the one hand—to fixed wages for employees who are subject to authority, and—on the other hand—to intrinsically motivated people with strong beliefs becoming independent entrepreneurs. The second article, Van den Steen (2006b), shows that motivation and coordination impose conflicting demands on the allocation of authority, leading to a trade-off between the two.

The next section describes the model of the article. Section 3 derives some preliminary results, mainly regarding the value of control. Section 4 derives the co-location of income and control when income shares can be chosen endogenously, whereas Section 5 derives the allocation principles when income shares are exogenously given. Section 6 considers the role of differing priors in the model, whereas Section 7 concludes. The appendix contains the basic calculations and a comparison to a setting where control rights are allocated through an auction.

2. The Model

The model in this article captures the situation of two players, P_1 and P_2, who are both involved in one and the same project but who may openly disagree on the optimal decisions for that project. The decisions themselves are not contractible—neither ex-ante nor ex-post—but the control rights over these decisions are, an assumption that I will discuss in more detail. The key issue is how these control rights will be allocated between the two players.

A project is formally defined as a revenue stream R that depends on a set of N decisions with typical element D_n.[4] The project will be either a success or

4. Although most of the analysis considers only one project, the results easily extend to a context with multiple projects. In particular, Section 4 explicitly studies multiple projects for the case when income is endogenous. It is fairly straightforward to use that extension throughout the full analysis. The one added insight, that the results also hold with multiple projects, did not seem to warrant the substantial increase in notational complexity.

a failure, with respective payoffs 1 and 0 and with probability of success Q (which thus depends on the decisions). Each decision D_n consists of a choice between two alternatives, denoted X_n and Y_n, of which one and only one is correct. Which of the two alternatives is correct depends on the state variable $S_n \in \{X_n, Y_n\}$: decision D_n is correct if and only if it fits the corresponding state. Formally, let d_n be the indicator function that decision D_n is correct, then $d_n = 1$ iff $D_n = S_n$. The probability of success strictly increases in decisions being correct, i.e., $\frac{\partial Q}{\partial d_n} > 0, \forall n$.

The states S_n are unknown, but each player P_i has a subjective belief $\mu_{i,n}$ that $S_n = X_n$. The state realizations are independent—and that fact is common knowledge—so that P_i's belief that $S_k = X_k$ and $S_l = X_l$ equals $\mu_{i,k}\mu_{i,l}$. A key assumption is that (it is common knowledge that) players have differing priors, that is, they can disagree in their beliefs about S_n even though no player has private information about S_n. I will discuss this differing priors assumption in more detail at the end of this section. The fact that players may have differing priors about S_n and that there is no private information implies that players will not update their beliefs when they notice that someone else has a different belief: they simply accept that people sometimes disagree.

I will assume that the $\mu_{i,n}$ are independent draws from nondegenerate and symmetric distributions $F_{i,n}$ with support [0, 1]. Let now $v_{i,n} = \int_0^1 \max(u, 1 - u)\, dF_{i,n}(u)$ denote the expected strength of belief, or confidence, of player P_i about decision D_n, i.e., how strongly he believes (on average) in the decision he believes in most. The assumptions on $F_{i,n}$ imply that $v_{i,n} \in (0.5, 1)$. The players are risk-neutral, so that each player's utility equals her expected income from the project minus any net financial transfers.

The timing of the model is indicated in Figure 1. In stage 1, the two players bargain over their control rights (and income rights, when applicable), using Nash bargaining with outside options equal to zero and with bargaining power $(\lambda, (1 - \lambda))$ for $\lambda \in (0, 1)$. When a decision is not allocated to any player, it will be decided upon by chance, with X_n and Y_n equally likely. To denote the allocation of control, let $\beta_{i,n} \in \{0, 1\}$ be the indicator function that player P_i will decide on decision D_n, with $\sum_i \beta_{i,n} \le 1,\ \forall n$ so that no decision can be allocated to more than one player. Let $\alpha_i \in [0, 1]$ denote the income share of player P_i, with $\sum_i \alpha_i = 1$. The restriction $\alpha_i \in [0, 1]$ can be interpreted as a non-wagering condition: without this restriction, risk-neutral players with differing priors would bet on the state and generate infinite utility. As argued in Van den Steen (2007a), however, this restriction can easily be endogenized by giving each agent the possibility to "sabotage" the project (i.e., to guarantee its failure). In that case, whenever $\alpha_i \notin [0, 1]$ some player has a reason to sabotage the project, making this allocation suboptimal. To keep the analysis simple and general, however, I impose it here directly as an assumption. With $\boldsymbol{A}=(\alpha_1,\alpha_2)$ denoting the vector of income shares, $\boldsymbol{B}_i=(\beta_{i,1},\ldots,\beta_{i,N})$ denoting the vector of player P_i's control rights, and $\boldsymbol{B}=(\boldsymbol{B}_1,\boldsymbol{B}_2)$ denoting the control allocation matrix for both players, a full allocation of income and control is then $\boldsymbol{L}=(\boldsymbol{A},\boldsymbol{B})$.

1	2	3	4
Allocation of Control The players bargain over control (and income) rights, using Nash bargaining.	Belief Realizations Beliefs $\mu_{i,n}$ get realized.	Actions The players simultaneously make their decisions.	Payoff Payoffs get realized.

Figure 1. Time Line of the Game.

In period 2, beliefs get realized.[5] In period 3, the players simultaneously make their decisions. As mentioned before, these decisions are noncontractible. For each decision, the player who has the control right always makes that decision. For definiteness, I will assume that a player who is indifferent in terms of payoffs chooses the action that he believes is most likely to be correct. This corresponds to an assumption that—apart from any share in the project revenue—each player has an arbitrarily small private benefit from success, $\gamma \sim U[0, \bar{\gamma}]$ with $\bar{\gamma} \downarrow 0$. One could imagine, for example, that people feel slightly more satisfied when their decisions turn out to be correct. In period 4, finally, the payoffs are realized.

Through the use of the Nash bargaining solution in this setting with transferable utility, the analysis will derive the allocation of control rights (and income rights) that maximizes the sum of expected utilities of the players, with a player's utility being his expected revenue. Appendix B, however, also considers—for a simple case—the allocation that would result from auctioning off the decision rights, which is one way to capture how a power-struggle would allocate control.

I will use hatted variables to denote the equilibrium solution. Note that the α_i could be interpreted to include nonfinancial benefits from success, such as the effects of being held responsible for the outcome (although that may imply some further restrictions on the α_i). As mentioned above, the analysis will maximize the joint utility for the subgame starting in period 2 since Nash bargaining (in settings with transferable utility) picks out the utility-maximizing allocation and then reallocates total utility via up-front transfers. I will denote player P_i's utility for the subgame that starts in period 2 by U_i and the corresponding joint utility by $U = U_1 + U_2$.

2.1 Variation B[6]

To study how the likelihood of disagreement may affect the equilibrium allocation, it is useful to consider a variation on the main model that parameterizes the probability of disagreement and that introduces a cost of reallocating control.

To that purpose, Figure 2 shows how stage 1 changes under this variation. The game starts from some randomly picked allocation of control (and residual

5. It does not matter whether belief realizations are public or private.

6. There will be only one variation in this article. The reason to denote this as Variation B is simply to facilitate referencing back to it later in the article.

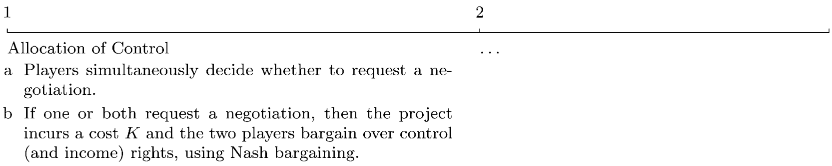

Figure 2. Variation with Costly Bargaining.

income, if applicable) $\check{\boldsymbol{L}}$. The two players now have the *opportunity* to bargain over control (and income, if applicable). In particular, in stage 1a either player can request a negotiation. If either or both players request a negotiation then the project incurs a bargaining cost $K > 0$ and the two players bargain over control rights—and income rights, when applicable—using Nash bargaining with bargaining power $(\lambda, (1 - \lambda))$ for $\lambda \in (0, 1)$. The outside option or disagreement point is now that the control (and income, if applicable) reverts to the starting allocation $\check{\boldsymbol{L}}$. Note that this disagreement point differs from that in the basic model, where both players had outside option 0. This difference is only for expositional reasons and does not affect the results.[7]

To vary the degree of disagreement in a tractable way, assume that with probability $\rho \in [0, 1)$ the players always agree on the optimal actions, whereas with complementary probability $(1 - \rho)$ their beliefs are independently distributed. (Even when the players agree, a player's beliefs are assumed to be symmetric and independently distributed across states as before, that is, $\mu_{i,n}$ and $\mu_{i,m}$ are independently distributed.) The marginal distribution $F_{i,n}$ of $\mu_{i,n}$ is identical whether players agree or disagree.[8]

2.2 The Differing Priors Assumption

The model assumes that people can openly disagree in the sense of differing priors.[9] Whereas Section 6 considers how this assumption affects the results, I discuss it here from a more general perspective.

7. In particular, all propositions hold for both outside options (as long as in Variation B each player's starting utility is also adjusted to equal the utility from the outside option), although the proofs for the results on Variation B differ slightly. The disagreement point in the original model is chosen to keep that model as simple as possible, whereas the disagreement point for Variation B is chosen to fit the spirit/story of that model.

8. A simple distribution that satisfies this is the following. Take a set of values $\nu_{i,n} \in (0.5, 1)$. With probability ρ, each couple $(\mu_{1,n}, \mu_{2,n})$ is drawn from a binary distribution with values $(\nu_{1,n}, \nu_{2,n})$ and $((1 - \nu_{1,n}), (1 - \nu_{2,n}))$ that are equally likely. With complementary probability $(1 - \rho)$, the $\mu_{i,n}$ are independent draws with each $\mu_{i,n}$ an independent draw from a binary distribution with values $\nu_{i,n}$ and $(1 - \nu_{i,n})$ that are equally likely.

9. Obviously, the assumption in this article that players have absolutely no private information is extreme and made for analytical convenience. If players had both differing priors and private information, they would update their beliefs when encountering someone with whom they disagree, but disagreement would remain (Morris 1997). For a more conceptual discussion of differing priors, see Morris (1995). Note that differing priors is *not* the same as private information that is impossible to communicate.

The differing priors assumption, although not mainstream, has actually a long tradition in economics. Earlier articles that assumed differing priors include, among others, Arrow (1964), Wilson (1968), Harrison and Kreps (1978), Leland (1980), Varian (1989), Harris and Raviv (1993), Morris (1994, 1997), Daniel et al. (1998), and more recently Scheinkman and Xiong (2003), Yildiz (2003, 2004), Van den Steen (2004), Brunnermeier and Parker (2005), Boot et al. (2006), and Guiso et al. (2006). There has been a rapid rise in recent years, in part due to the growing popularity of behavioral economics which often implicitly assumes differing priors. There is also a burgeoning empirical literature such as Chen et al. (2002) or Landier and Thesmar (2007). Furthermore, Hong and Stein (2007) argue that "disagreement models (..) represent the best horse on which to bet [as the future consensus model for behavioral finance]."

The assumption in this article of unbiased differing priors captures the fact that people may have different intuition or different "mental models." Such different mental models or different intuition may lead people with identical data to draw very different conclusions. Consider, for example, one's belief whether to trust a particular person or group of people, or one's belief to trust intuition over data, or one's belief whether punishment is a good motivator, or whether we will be watching television on our cell phones. People often have strong beliefs about such issues without any concrete evidence.[10] Such beliefs have immediate implications for business decisions. Whether to delegate a set of decisions to assembly line workers depends on whether you trust these workers. Product design and R&D investment decisions for cell phones depend critically what you believe people will be using cell phones for 5 years down the road. These kinds of issues are repeated many times over in organizations. People disagree on how to design an organization, on how to deal with a difficult employee, on whether to trust a supplier, etc. (Although the model in this article is written as being about one issue, it is not difficult to rewrite it as being about a finite succession of smaller issues.) Open disagreement is thus an issue in both strategic and day-to-day decision making. In effect, the fundamental role of "belief systems" or "mental models" in organizations has been stressed by academic studies of managers and managerial decision making (Donaldson and Lorsch 1983; Schein 1985).

An important question is why—if the decision is important—players do not simply discuss and collect more data until they reach agreement. The answer is that the choice whether to rely on persuasion is a time and cost trade-off, and in many cases persuasion is just not the right option. In particular, many of these beliefs are deeply engrained and difficult to change, whereas further data collection may be costly and time consuming. Consider, for example, the case that you disagree on whether to trust a group of employees sufficiently to delegate

10. Most of us, including me, take it as a fact that the Earth orbits around the Sun, but there was a time that people believed exactly the opposite. Very few people have actually seen any first-hand evidence one way or the other. We hold these rather strong beliefs "on authority." The website www.fixedearth.com shows that there are actually even today people who openly disagree with this view.

certain decisions to them. It is very difficult to rationally persuade someone to really trust another person when they are not already inclined to do so. Moreover, further data collection to resolve this disagreement is complex and probably very time consuming. Finally, the process of convergence of beliefs is more complex than it may seem at first sight.[11] It may then be much more efficient to simply give control over the decision to one person and let that person decide. To see this from another perspective, imagine the deadlock if a CEO (or a Dean) needed to completely persuade all his subordinates every time of the correctness of his judgment before making any decision, or if a board could only decide by *true and honest* unanimity of opinions! Overall then, the possibility of persuasion will not eliminate the need to study disagreement, and how to optimally allocate control when people disagree.

A final question is where such differing priors would come from in a Bayesian framework? There are two ways to think about this. Since the prior for this game is a posterior from earlier updating, bounded rationality (which the player does not fully take into account) will often lead to differing priors for this game, even when starting originally (long before this game) from a common prior. Unknowingly forgetting some of the data used to update beliefs, for example, would do.[12] A second—more philosophical and more controversial—argument is that people may actually be born with differing priors: in the absence of information there is no reason to agree. From that perspective, differing priors would be perfectly consistent with a fully rational Bayesian paradigm: priors are just primitives of a model. In this article, I am completely agnostic about the source of the disagreement. I just believe that open disagreement, as captured by differing priors, can be an important force in organizations and explore its potential consequences.

2.3 Contractible Decision Rights but Noncontractible Decisions?

A central assumption of this analysis is that decisions are not contractible, but decision rights are. Since it is such a central assumption, it is useful to clarify this further. There are actually two distinct ways to interpret this assumption.

11. Although more data nearly always eventually lead to convergence, this is not necessarily guaranteed, especially not in the short term (which may be the more relevant here). There are indeed both empirical (Lord et al. 1979; Plous 1991; McHoskey 1995) and theoretical (Diaconis and Freedman 1986; Acemoglu et al. 2006) reasons why that may not be the case. Acemoglu et al. (2006) show, for example, how potential disagreement over the interpretation of new information is sufficient to prevent convergence. The psychology literature on polarization shows empirically how differential reading of identical information may sometimes lead to divergence. This does not mean that convergence will not happen, only that it is a more difficult process than often imagined. This will particularly be the case when the disagreement derives from different "mental models" or "world views," since these often imply different interpretations of the data.

12. A close alternative, favored by Aumann (1976), is that people may have some bias at the time of forming their beliefs. Since these so formed beliefs are the priors for this game, the players effectively start from differing priors.

The first interpretation is quite literal. Take, for example, trading stocks. I can easily contract with my broker that he gets the right to sell my stocks, that is, I give him the control right over when to sell my stocks. The particular decision to sell these stocks "at the right time," however, is nearly impossible to contract. Note what is causing the problem here: the action itself (to sell or not) is actually easy to verify, but the state on which I want it to be conditioned (the right time) is not. So although the control right over the decision is contractible, the particular decision is not. This literal interpretation, however, applies only to a limited number of situations.

The second—and much broader—interpretation is that this is a reduced form for a situation where the decision rights themselves are not contractible, but the means to effectively control the player who executes the decision *are* contractible. It is, for example, difficult to contract with a sales manager that she gets the control right over how her salespeople should greet customers. Instead, that control right gets indirectly allocated by giving the manager the right (or not) to monitor her salespeople when they meet customers and the right (or not) to hire and fire at will. In this second interpretation, the assumption that control rights are contractible is obviously a reduced form for a more elaborate game. This reduced form approach is useful for two reasons. First of all, the enforcement is often so straightforward that this is a very close approximation. Second, even if it is not, this approach allows to separate the questions of optimality and feasibility. Although there is an obvious interaction between the two, the separation may make the analysis much more transparent. In fact, the enforcement issues were explicitly stripped from this article in order to focus better on the forces that drive the optimal allocation. Van den Steen (2007a, 2007b) study such enforcement issues.

3. A Preliminary Result

When people openly disagree—in the sense of differing priors—each player believes that her own decisions are better than these of others (Van den Steen 2004): since the player chose this particular decision and could have chosen any of the other decisions, it must be that she believes that this decision is (weakly) better than all these other decisions. From the perspective of the current article, two important things then happen.

A first implication—and the main force in this article—is that players prefer to have control and value control rights directly. Since this effect is so central to this article, I derive it here formally. Consider therefore the subgame starting in period 2, and let all but one decision right be allocated. The following proposition then captures how much a player would be willing to pay in order to acquire that one control right from the other player.

Proposition 1. When $\alpha_i > 0$, the value to player P_i of controlling decision D_k (in the subgame starting in period 2, for a given allocation of the other D_n, and relative to the other player controlling D_k) is always strictly positive, strictly increasing in his share of residual income α_i, strictly increasing in his confidence about this decision $\nu_{i,k}$, and (under Variation B) strictly increasing in the probability of disagreement $(1 - \rho)$.

Proof. As in Appendix A, let $\boldsymbol{d} = (d_1,\ldots,d_N)$ denote a vector of d_n, let $\boldsymbol{\mathcal{D}}$ denote the set of all such vectors, let $\boldsymbol{d}_{-k}$ denote the vector $\boldsymbol{d}$ excluding the kth element and $\boldsymbol{d} = (\mathbf{d}_{-k},\mathrm{d}_k)$ again the full vector. Let $\boldsymbol{N}$ denote the set $\{1, 2, \ldots, N\}$ and $\boldsymbol{N}_{-k} = \{1,\ldots,k - 1,k + 1,\ldots,N\}$. The calculations in Appendix A imply that, for any given allocation of income and control (except for D_k) and for $\rho \in [0, 1)$, player P_i's gain from controlling decision D_k is independent from who would control D_k otherwise and equals

$$(1 - \rho)\alpha_i \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_{n\in\boldsymbol{N}_{-k}} \left(\frac{1}{2} + (2d_n - 1)\beta_{i,n}(\nu_{i,n} - \frac{1}{2}) \right) \right] (2d_k - 1)(\nu_{i,k} - \frac{1}{2})R(\boldsymbol{d}).$$

For $\alpha_i > 0$, this expression is strictly positive following an argument identical to the argument in Appendix A that U is increasing in $\beta_{i,n}$. The expression is then also strictly increasing in α_i, $\nu_{i,k}$, and in $(1 - \rho)$. ■

Apart from its role in the further analysis, this result is also of independent interest. Since, for example, a control right is valued higher when there is more disagreement, it will be used more as "compensation in bargaining" (and thus be allocated inefficiently from other perspectives, such as incentives or access to information) when disagreement is likely. This suggests that such inefficient allocations of control will occur more frequently in early stage ventures, new industries, or industries in turmoil—where fundamental uncertainty creates a lot of room for disagreement—than in late stage ventures and more mature industries.[13] This is related to the broader issue how parties will trade off residual income against residual control when one of them is capital constrained, which should have empirical implications for alliances and venture capital financing. These issues await further research.

Note also that this makes the following prediction, which—I believe—is both distinctive and testable (at least experimentally): if one particular control right gets allocated, for example, through an auction, each player's willingness to pay for that control right increases in his share of the project's

13. This prediction is in line with the finding of Lerner and Merges (1998) that—against their expectations—the control allocation in early-stage ventures is more driven by capital constraints and less by efficiency consideration compared to late-stage ventures. Other potential explanations for this result have been raised by Dessein (2005) and by Baker et al. (2006). The empirical distinction between these different explanations must come either from matching the assumptions (differing priors versus private information versus symmetric uncertainty) or from other predictions. Predictions of the current theory that could be useful in this sense are the predictions on complements or the prediction that a particular control right will be valued relatively more by those with strong beliefs about that particular decision.

residual income, that is, in his financial stake in the project.[14] A useful experimental context may be investment decisions over a jointly owned stock portfolio.

An immediate implication of Proposition 1 for the analysis in this article is that no control right will remain unallocated in equilibrium:

Fact 1. In equilibrium, each control right is allocated to a player. (Formally, $\beta_{1,n} + \beta_{2,n} = 1 \ \forall n$.)

Proof. The result is part of the calculations in Appendix A. ■

The intuition is obviously that players attach a positive value to control and not allocating control is wasting value. Henceforth, I will often use $\beta_n = \beta_{1,n}$ and $\beta_{2,n} = 1 - \beta_n$.

The second implication—of the fact that players believe that they make better decisions than others—is that players will value project income more highly when they have more control. Van den Steen (2007a) derived this effect in the context of incentives. It will be an important force in the next section to which I turn now for the analysis proper of the allocation of control (and income).

4. The Co-location of Control and Contractible Income

The article's first result on the allocation of control is a new mechanism for the co-location of control and contractible income. I show, in particular, that—when both the control rights $\beta_{i,n}$ and the income rights α_i are contractible—it is strictly optimal to concentrate all income and control of one project in one hand, due to the afore-mentioned self-reinforcing cycle between the allocation of control rights and the allocation of income rights:

- When moving control rights to a player, that player will value the project more highly (because he believes the decisions will improve). So it becomes more attractive to also move income rights to that player.
- When moving income rights to a player, that player will value control rights more highly (since he has more at stake and believes that he makes better decisions). So it becomes more attractive to also move control rights to that player.

14. At first sight, a similar prediction seems to obtain in a model where players can make private investments to collect information: as a player with a larger stake has more incentives to collect information, he also believes that he will make better decisions and thus seems to want more control. Although it would be fairly easy to exclude such investments in an experimental context, note also that this result actually does not hold for a player whose stake is smaller than that of some other player: such player knows that that other player has even more incentives than himself and will thus be a better decision maker. As a consequence, his willingness to pay for control may actually be negative and decrease as his stake increases (up to the point where he becomes the largest stakeholder).

This result is important for two reasons. First, the co-location of income and control (or, analogously, of authority and responsibility) is a widely observed and widely accepted principle, though with important exceptions. Understanding the underlying mechanisms not only deepens our understanding of the principle but also allows us to determine the costs of deviating from it and to make predictions as to when such deviations are likely. For example, in as far as the mechanism in this article is the one that is the driving force, shared responsibility will be more likely when the players tend to agree more on the right course of action. Second, from a more formal perspective, this particular mechanism turns out to be a first-order effect in many contracting models with differing priors.

To see this result formally, consider the basic model. When the α_i are contractible, the Nash bargaining solution will—as always in a setting with transferable utility—select the allocation $\boldsymbol{L} = (\boldsymbol{A},\boldsymbol{B})$ that maximizes the joint expected utility U (and then reallocate that utility by up-front transfers). Let $\bar{\boldsymbol{L}}_i$ denote the allocation in which all control rights and all income rights are in the hands of player P_i, that is, $\boldsymbol{L} = \bar{\boldsymbol{L}}_1$ if $\alpha_1 = 1$ and $\beta_{1,n} = 1 \; \forall \; n$, and analogous for $\bar{\boldsymbol{L}}_2$. Let furthermore $U(\boldsymbol{L}) = \alpha E_1 R(\boldsymbol{L}) + (1-\alpha) E_2 R(\boldsymbol{L})$, where $\alpha = \alpha_1$ and $E_i R(\boldsymbol{L})$ is the project's expected revenue according to player P_i when the allocation of income and control is $\boldsymbol{L}$. The following proposition then says not only that the optimal allocation concentrates all income and control rights with one person but also that this allocation *strictly* dominates any intermediate allocation.

Proposition 2. For any $\boldsymbol{L} \notin \{\bar{\boldsymbol{L}}_1, \bar{\boldsymbol{L}}_2\}$, $\max(U(\bar{\boldsymbol{L}}_1), U(\bar{\boldsymbol{L}}_2)) > U(\boldsymbol{L})$.

Proof. I show first that, at the optimum, for each player i either $\hat{\beta}_{i,n} = 0, \forall n$ or $\hat{\beta}_{i,n} = 1, \forall n$. Take any allocation $\boldsymbol{L}$ where that is not the case, and assume wlog. that player P_1 has the highest expected value, that is, $E_1 R(\boldsymbol{L}) \geq E_2 R(\boldsymbol{L})$. I now argue that $U(\bar{\boldsymbol{L}}_1) > U(\boldsymbol{L})$. By the assumption that $E_1 R(\boldsymbol{L}) \geq E_2 R(\boldsymbol{L})$, U clearly increases as we increase α (and thus decrease $(1 - \alpha)$ proportionally). Once $\alpha = 1$, $U = E_1 R(\boldsymbol{L})$. By the fact that players disagree with strictly positive probability and that P_1 thinks his own decisions are strictly better when they disagree, it then further follows that U strictly increases as we increase each and every $\beta_{1,n}$. It thus follows that $U(\bar{\boldsymbol{L}}_1) > U(\boldsymbol{L})$.

Assume next that $\hat{\beta}_{1,n} = 1, \forall n$, but $\alpha \neq 1$. If $E_1 R(\boldsymbol{L}) > E_2 R(\boldsymbol{L})$, then it follows that $U(\bar{\boldsymbol{L}}_1) = E_1 R(\boldsymbol{L}) > \alpha E_1 R(\boldsymbol{L}) + (1 - \alpha) E_2 R(\boldsymbol{L}) = U(\boldsymbol{L})$. If, on the contrary, $E_2 R(\boldsymbol{L}) \geq E_1 R(\boldsymbol{L})$, then $E_1 R(\boldsymbol{L}) \leq E_2 R(\boldsymbol{L}) < E_2 R(\bar{\boldsymbol{L}}_2)$ by the fact that players disagree with strictly positive probability and that P_2 thinks his own decisions are strictly better when they disagree. And thus

$$U(\boldsymbol{L}) = \alpha E_1 R(\boldsymbol{L}) + (1 - \alpha) E_2 R(\boldsymbol{L}) < \alpha E_1 R(\boldsymbol{L}) + (1 - \alpha) E_2 R(\bar{\boldsymbol{L}}_2)$$
$$< E_2 R(\bar{\boldsymbol{L}}_2) = U(\bar{\boldsymbol{L}}_2). \quad \blacksquare$$

Note that the co-location of income and control here obtains in the absence of any private benefits or externalities (beyond the effect of the residual income).

An important restriction, however, is the assumption—implicit in the Nash bargaining solution—that players are not capital constrained. This can be an interesting source of empirical variation, as suggested in Section 3.

4.1 The Effect of Disagreement

To study the effect of disagreement, consider Variation B—introduced in Section 2—with variable disagreement and costly bargaining. The following proposition shows that—in that case—control and income are more likely to be concentrated when there is more disagreement.

Proposition 3. Consider Variation B. The set of parameters for which income and control are (always) concentrated in the hands of one person increases in the probability of disagreement $(1 - \rho)$.

Proof. Assume, wlog., that $U(\bar{\boldsymbol{L}}_1) \geq U(\bar{\boldsymbol{L}}_2)$. Let the starting allocation be some allocation $\check{\boldsymbol{L}}$ with shares $(\check{\alpha}, \check{\beta}_1, \check{\beta}_2)$. The players will request a negotiation if the change in utility exceeds the bargaining cost, that is, if $U(\bar{\boldsymbol{L}}_1) - U(\check{\boldsymbol{L}}) > K$. It thus suffices to show that $U(\bar{\boldsymbol{L}}_1) - U(\check{\boldsymbol{L}})$ increases in $(1 - \rho)$. Following the calculations in Appendix A, the utility change equals

$$
\begin{aligned}
U(\bar{\boldsymbol{L}}_1) - U(\check{\boldsymbol{L}}) = \rho\check{\alpha} \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ & \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{1,n} - \frac{1}{2})\right)\right] \\
& - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{1,n} - \frac{1}{2})\right)\right] \Bigg\} R(\boldsymbol{d}) \\
& + \rho(1 - \check{\alpha}) \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{1,n} - \frac{1}{2})\right)\right] \\
& - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{2,n} - \frac{1}{2})\right)\right] \Bigg\} R(\boldsymbol{d}) \\
& + (1 - \rho)\check{\alpha} \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{1,n} - \frac{1}{2})\right)\right] \\
& - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\check{\beta}_{1,n}(v_{1,n} - \frac{1}{2})\right)\right] \Bigg\} R(\boldsymbol{d}) \\
& + (1 - \rho)(1 - \check{\alpha}) \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(v_{1,n} - \frac{1}{2})\right)\right] \\
& - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\check{\beta}_{2,n}(v_{2,n} - \frac{1}{2})\right)\right] \Bigg\} R(\boldsymbol{d}),
\end{aligned}
$$

so that

$$\frac{\mathrm{d}(U(\bar{\boldsymbol{L}}_1) - U(\check{\boldsymbol{L}}))}{\mathrm{d}\rho} = -(1-\check{\alpha}) \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{2,n} - \frac{1}{2}) \right) \right] - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\check{\beta}_{2,n}(\nu_{2,n} - \frac{1}{2}) \right) \right] \Bigg\} R(\boldsymbol{d}) - \check{\alpha} \sum_{\boldsymbol{d}\in\mathcal{D}} \Bigg\{ \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{1,n} - \frac{1}{2}) \right) \right] - \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\check{\beta}_{1,n}(\nu_{1,n} - \frac{1}{2}) \right) \right] \Bigg\} R(\boldsymbol{d}).$$

Since $\sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\beta_{i,n}(v_{i,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d})$ strictly increases in $\beta_{i,n}$ (as shown in Appendix A), this whole expression is negative. This proves the proposition. ■

For empirical purposes, this proposition implies that the forces for co-location are strong when there is a lot of potential for disagreement, for example, in new ventures or new product categories.

4.2 Who Gets Control?

The identity of the person in control is endogenous here. Can we say anything about who that person will be? To answer this question, consider again the original model formulation. The following proposition then says that an increase in a player's confidence about any decision makes it more likely that the player will get control.

Proposition 4. The set of parameters for which all income and control is concentrated in equilibrium with player P_i increases in $\nu_{i,n}$.

Proof. In the optimal allocation, control goes to the player with the highest $E_j R(\bar{\boldsymbol{L}}_j)$. So it suffices to show that $E_j R(\bar{\boldsymbol{L}}_j)$ increases in $\nu_{j,n}$. Following the calculations in Appendix A, the joint expected utility when all income and control is allocated to player 1 equals

$$U(\bar{\boldsymbol{L}}_1) = \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\ (\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}),$$

so that

$$\frac{\mathrm{d}U(\bar{\boldsymbol{L}}_1)}{\mathrm{d}v_{1,k}} = \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_{n\in \boldsymbol{N}_{-k}} \left(\frac{1}{2} + (2d_n - 1)\ (v_{1,n} - \frac{1}{2}) \right) \right] (2d_k - 1) R(\boldsymbol{d}),$$

so that $U(\bar{\boldsymbol{L}}_1)$ indeed increases in $\nu_{1,k}$, following an argument analogous to that for U increasing in $\beta_{i,n}$ in Appendix A. ■

This result has a very intuitive implication: people who are confident about the right course of action will be in charge and become the leader or manager of the project. This is in line with the empirical literature on entrepreneurship, which shows that entrepreneurs are often people with strong beliefs (Cooper et al. 1988; Landier and Thesmar 2007).

4.3 Multiple Projects

Models on the allocation of control can sometimes lead to the problematic logical conclusion that all projects should be owned by one person (Coase 1937; Williamson 1985; Hart 1995). This is obviously an important issue to check.

For the model in this article, the key observation in this respect is that it may well be different people who are most confident about different projects. Whenever that is the case, control over such projects will then also be spread out among these different people.

To capture this formally, consider first the case of two independent projects. In particular, I will assume that each project depends on a set of decisions that has no overlap with the set of decisions of the other project. Let $\mathcal{D}_1$ and $\mathcal{D}_2$ denote the sets of decisions for, respectively, project 1 and project 2.

Proposition 5. There exists a subset of the parameter space with non-empty interior where it is strictly optimal for the two projects to be controlled (and their respective residual income owned) by different players.

Proof. The Nash bargaining solution will maximize the joint expected utility over all projects. Since the projects are independent, we can solve this project by project. Note now that whenever $\nu_{1,n} > \nu_{2,n}$ for all $D_n \in \mathcal{D}_1$, while $\nu_{1,k} < \nu_{2,k}$ for all $D_k \in \mathcal{D}_2$, the equilibrium allocation will be that all income and control of project 1 is concentrated with player P_1 and conversely for project 2 and player P_2. This proves the proposition. ■

Note that in that case each project is completely under the control of one person but different projects are under the control of different people.

The results for projects with overlapping decisions are very similar. In particular, Van den Steen (2006a) showed the following for that case.

1. Even with overlapping decisions, it remains true that all income and control of each project should be maximally concentrated with one person.
2. It also remains true that control over different projects will sometimes be allocated to different people.

An interesting conjecture for further research was that more overlap between two projects would make it more likely that the same person will be in charge of both.

5. Exogenous Income Allocation: Complements, Substitutes, and Personal Characteristics

I now turn to the case where the income shares α_i are exogenous, which covers some important situations. A first situation with exogenous α_i is when the outcome and its resulting benefits are simply noncontractible. This will, for example, be the case when the benefits are mainly of a reputational nature. A second situation is when the income agreements are open to renegotiation at a later implementation stage. Finally, the most important situation is when the monetary benefits are so large that one of the parties is capital constrained. Important examples of this are VC financing of entrepreneurs, alliances between firms of very different size and the case when an employee makes very critical decisions.

With the α_i exogenously given, two groups of results can be distinguished: those that depend on the decisions' characteristics and those that depend on the players' characteristics. I will first consider the results for decision characteristics, in particular for how decisions interact, which are the most interesting.

5.1 Decision Characteristics: Complements versus Substitutes

Since complements or substitutes (in the monotone comparative statics sense) are central to this analysis, let me concisely review what these concepts represent in the current context. Two variables, x and y, are complements with respect to some objective function R if R has increasing differences in x and y, that is, for $\underline{x} < \bar{x}$ and $\underline{y} < \bar{y}$, $R(\bar{x},\bar{y}) - R(\underline{x},\bar{y}) \geq R(\bar{x},\underline{y}) - R(\underline{x},\underline{y})$, or if its cross-partial derivative is non-negative (Milgrom and Roberts 1994). In the current context, two decisions will be complements if the returns from getting one decision correct are higher when you also get the other decision correct. The simplest example of complementary decision rights is when a project is a success if and only if both decisions on which the project depends are correct.

Conversely, the variables x and y are substitutes with respect to R if R has decreasing differences in x and y, that is, for $\underline{x} < \bar{x}$ and $\underline{y} < \bar{y}$, $R(\bar{x},\bar{y}) - R(\underline{x},\bar{y}) \leq R(\bar{x},\underline{y}) - R(\underline{x},\underline{y})$ or its cross-partial derivative is nonpositive. Two decisions will thus be substitutes if the returns from getting one decision correct are lower when you already got the other decision correct. The simplest example of substitute decision rights is when the project is a success if either of two decisions is correct. Backup projects are an important example of substitute decisions.

The key results of this section then are:

- As decisions become more complements, the corresponding control rights should be more concentrated.
- As decisions become more substitutes, the corresponding control rights should be more distributed.

I will discuss the intuition and some further implications below, after the formal proposition.

To study the effect of substitutes and complements formally, consider the model with two players and two decisions, that is, $N = 2$. In this case, R is completely characterized by its four values for $R(d_1, d_2)$. Let now $\underline{R}$, δ_1, δ_2, Δ be defined as follows:

$$R(0,0) = \underline{R},$$

$$R(1,0) = \underline{R} + \delta_1,$$

$$R(0,1) = \underline{R} + \delta_2,$$

$$R(1,1) = \underline{R} + \delta_1 + \delta_2 + \Delta.$$

Since $R(1, 1) - R(1, 0) - R(0, 1) + R(0, 0) = \Delta$, we have the following fact:

Fact 2. R has increasing (resp. decreasing) differences in d_1 and d_2 iff $\Delta \geq 0$ (resp. $\Delta \leq 0$).

In other words, whether D_1 and D_2 are complements or substitutes depends completely and only on Δ. I will therefore use Δ as a measure for the degree to which D_1 and D_2 are complements or substitutes. I will also say that two decisions are pure complements when $\delta_1 = \delta_2 = 0$ and $\Delta > 0$ and that two decisions are pure substitutes if $-\delta_1 = -\delta_2 = \Delta < 0$. Pure complements thus captures the case where the project is a success if and only if both decisions are correct, whereas pure substitutes captures the case where the project is a success for sure if at least one decision is correct.

The following proposition says that as the two decisions are more complements (as measured by Δ), they are—in equilibrium—more likely to be concentrated, whereas as the two decisions are more substitutes, they more likely to be distributed. It also says that when players are ex-ante symmetric (in beliefs and income shares), pure complements are always co-located, whereas pure substitutes are always distributed.

Proposition 6a. The set of parameters for which one person controls both decisions increases in the degree of complementarity Δ between the decisions. When the players are ex-ante symmetric ($\nu_{1,n} = \nu_{2,n}, \forall n$), decisions that are pure complements are always co-located. When the players are ex-ante symmetric ($\nu_{1,n} = \nu_{2,n}, \forall n$) and have equal shares of residual income ($\alpha_1 = \alpha_2$), decisions that are pure substitutes are always distributed.

Proof. Let $\hat{U}$ and $\check{U}$ be the joint utilities for $(\hat{\beta}_1, \hat{\beta}_2)$ and $(\check{\beta}_1, \check{\beta}_2)$, respectively. The difference in joint utilities then equals (following Appendix A)

$$
\begin{aligned}
\hat{U}-\check{U}= & \\
& \alpha\frac{1}{4}(\hat{\beta}_1-\check{\beta}_1)(2\nu_{1,1}-1)[2\delta_1+\Delta]+\alpha\frac{1}{4}(\hat{\beta}_2-\check{\beta}_2)(2\nu_{1,2}-1)[2\delta_2+\Delta] \\
& +\alpha\frac{1}{4}(\hat{\beta}_1\hat{\beta}_2-\check{\beta}_1\check{\beta}_2)(2\nu_{1,1}-1)(2\nu_{1,2}-1)\Delta \\
& +(1-\alpha)\frac{1}{4}(\check{\beta}_1-\hat{\beta}_1)(2\nu_{2,1}-1)[2\delta_1+\Delta] \\
& +(1-\alpha)\frac{1}{4}(\check{\beta}_2-\hat{\beta}_2)(2\nu_{2,2}-1)[2\delta_2+\Delta] \\
& +(1-\alpha)\frac{1}{4}[(1-\hat{\beta}_1)(1-\hat{\beta}_2)-(1-\check{\beta}_1)(1-\check{\beta}_2)](2\nu_{2,1}-1)(2\nu_{2,2}-1)\Delta.
\end{aligned}
$$

The above expression can be rewritten as

$$
\begin{aligned}
\hat{U}-\check{U}= & \frac{1}{4}(\hat{\beta}_1-\check{\beta}_1)[\alpha(2\nu_{1,1}-1)-(1-\alpha)(2\nu_{2,1}-1)][2\delta_1+\Delta] \\
& +\frac{1}{4}(\hat{\beta}_2-\check{\beta}_2)[\alpha(2\nu_{1,2}-1)-(1-\alpha)(2\nu_{2,2}-1)][2\delta_2+\Delta] \\
& +\frac{1}{4}(\hat{\beta}_1\hat{\beta}_2-\check{\beta}_1\check{\beta}_2)[\alpha(2\nu_{1,1}-1)(2\nu_{1,2}-1) \\
& +(1-\alpha)(2\nu_{2,1}-1)(2\nu_{2,2}-1)]\Delta+(1-\alpha)\frac{1}{4}[\check{\beta}_1+\check{\beta}_2-\hat{\beta}_1-\hat{\beta}_2] \\
& \times(2\nu_{2,1}-1)(2\nu_{2,2}-1)\Delta.
\end{aligned}
$$

It follows that

$$
\begin{aligned}
U(1,1)-U(1,0)= & \frac{1}{4}[\alpha(2\nu_{1,2}-1)-(1-\alpha)(2\nu_{2,2}-1)][2\delta_2+\Delta] \\
& +\frac{1}{4}\alpha(2\nu_{1,1}-1)(2\nu_{1,2}-1)\Delta,
\end{aligned}
$$

so that

$$
\begin{aligned}
\frac{\mathrm{d}(U(1,1)-U(1,0))}{\mathrm{d}\Delta}= & \frac{1}{4}[\alpha(2\nu_{1,2}-1)-(1-\alpha)(2\nu_{2,2}-1)] \\
& +\frac{1}{4}\alpha(2\nu_{1,1}-1)(2\nu_{1,2}-1).
\end{aligned}
$$

I will now show that if $\frac{\mathrm{d}(U(1,1)-U(1,0))}{\mathrm{d}\Delta}<0$ then $U(1,1)-U(1,0)<0$ for all possible values of δ_n and Δ so that this derivative is irrelevant to the proposition. Note that R increasing in its arguments requires

$$
R(1,1)-R(1,0)=\Delta+\delta_1+\delta_2+\underline{R}-\delta_1-\underline{R}=\Delta+\delta_2\geq 0,
$$

and analogously $R(1,1)-R(0,1)=\Delta+\delta_1\geq 0$. Note further that if $\frac{\mathrm{d}(U(1,1)-U(1,0))}{\mathrm{d}\Delta}<0$ then $[\alpha(2\nu_{1,2}-1)-(1-\alpha)(2\nu_{2,2}-1)]+\alpha(2\nu_{1,1}-1)$

$(2\nu_{1,2} - 1) < 0$ and thus also $[\alpha(2\nu_{1,2} - 1) - (1-\alpha)(2\nu_{2,2} - 1)] < 0$ since the second term is always positive.

If $\Delta \geq 0$ then we can write

$$U(1,1) - U(1,0) = \frac{1}{4}[\alpha(2\nu_{1,2} - 1) - (1-\alpha)(2\nu_{2,2} - 1)]2\delta_2 + \frac{1}{4}\big[\alpha(2\nu_{1,2} - 1) - (1-\alpha)(2\nu_{2,2} - 1) + \alpha(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\big]\Delta$$

so that $U(1, 1) - U(1, 0) < 0$. If $\Delta < 0$ then we can write

$$U(1,1) - U(1,0) = \frac{1}{4}[\alpha(2\nu_{1,2} - 1) - (1-\alpha)(2\nu_{2,2} - 1)][2\delta_2 + \Delta] + \frac{1}{4}\alpha(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\Delta,$$

so that $U(1, 1) - U(1, 0) < 0$ since $2\delta_2 + \Delta > 0$.

Since the numbering of players and actions is arbitrary, permutation gives the same results for $\frac{d(U(1,1)-U(0,1))}{d\Delta}$, $\frac{d(U(0,0)-U(0,1))}{d\Delta}$, and $\frac{d(U(0,0)-U(1,0))}{d\Delta}$. This proves the first part of the proposition.

I now turn to the second part of the proposition. I will do the calculations for the case of $\rho > 0$ since that will imply immediately the proof of Proposition 6b. Note that the fact that players are ex-ante symmetric implies that $\nu_{1,n} = \nu_{2,n} = \nu_n$.

Consider now first the case of pure complements, so that $\delta_n = 0$ whereas $\Delta > 0$. In that case,

$$\begin{aligned} U(1,1) - U(1,0) &= (1-\rho)\alpha\frac{1}{4}(2\nu_2 - 1)\Delta + (1-\rho)\alpha\frac{1}{4}(2\nu_1 - 1) \\ &\quad \times (2\nu_2 - 1)\Delta - (1-\rho)(1-\alpha)\frac{1}{4}(2\nu_2 - 1)\Delta \\ &= (1-\rho)\frac{1}{4}[(2\alpha - 1)(2\nu_2 - 1) + \alpha(2\nu_1 - 1)(2\nu_2 - 1)]\Delta. \end{aligned}$$

If $\alpha \geq 0.5$, then (since $\Delta > 0$ for pure complements) $U(1, 1) > U(1, 0)$. Permuting the decisions implies also that $U(1, 1) > U(0, 1)$ so that decisions will indeed be concentrated (with one player or the other) when $\alpha \geq 0.5$. Permuting the players implies that $U(0, 0) > \max(U(1, 0), U(0, 1))$ when $(1-\alpha) \geq 0.5$ and thus implies the proposition for pure complements. Note also that the difference in utilities is proportional to $(1-\rho)$. That will imply the result on complements in Proposition 6b.

Consider next the case of substitutes, that is, $\Delta = -\delta_1 = -\delta_2 < 0$. Since the proposition now only considers symmetric players and shares, we get that $\nu_{1,n} = \nu_{2,n} = \nu_n$ and $\alpha = (1-\alpha) = 0.5$. The difference in utilities then becomes

$$\hat{U} - \check{U} = (1 - \rho)\frac{1}{8}[2\hat{\beta}_1\hat{\beta}_2 - \hat{\beta}_1 - \hat{\beta}_2 + \check{\beta}_1 + \check{\beta}_2 - 2\check{\beta}_1\check{\beta}_2](2\nu_1 - 1)(2\nu_2 - 1)\Delta,$$

so that

$$U(1,1) - U(1,0) = (1 - \rho)\frac{1}{8}(2\nu_1 - 1)(2\nu_2 - 1)\Delta < 0.$$

Permuting the players gives $U(0, 1) > U(0, 0)$ and thus proves the proposition. Note, for further reference that the differences in utilities are again proportional to $(1 - \rho)$. This finalizes the proof. ■

To see the intuition behind this result, it is easiest to start from a situation where two decisions are substitutes, as in $R = d_1 + d_2 - d_1 d_2$. If a player controls decision D_1 and is very sure about the right course of action, then he does not care about control over decision D_2 since he expects a success no matter what happens to D_2. A player who disagrees with him on the right course of action, on the other hand, will value control over D_2 highly since he believes D_1 will likely be wrong. It follows that it is optimal to distribute substitute decision rights among different people.

To see the converse for complementary decision rights, consider $R = d_1 d_2$, so that R is a success if and only if both decisions are correct. If a player controls D_1 and is sure about the right course of action, then he will care a lot about also controlling D_2: since he thinks D_1 is likely correct, getting D_2 correct makes all the difference. If, on the other hand, a player does not control D_1 and disagrees with the person who does control D_1, then he would not care much about control over D_2 since he believes it is unlikely to matter anyways. It follows that it is optimal to concentrate complementary decision rights in one hand.

The basic empirical implications of these results are straightforward: more complementary decisions are more likely to be co-located whereas more substitute decisions are more likely to be distributed.

One potential application of these results is the observation that all decisions that affect one particular project are often under the control of one and the same player, with different projects controlled by different people. For the current theory to help explain that observation, an important question is whether a project's decisions are typically complements or substitutes. Both Rosen (1982) and Kremer (1993) have argued that project decisions tend to be complements and found evidence supporting this. One reason is that many projects are of the type that all decisions have to be right for the project to succeed. If that holds true then this theory may indeed help to explain that observed concentration of control.

Other potential implications come from the observation that urgency often eliminates "second chances" for a decision and that such second chances may provide substitutes, thus weakening the complementarity between this decision and any other.[15] Although more formal analysis is necessary to understand whether this effect is actually general and robust, it seems that urgency may thus create or exacerbate complementarities, whereas slack may create substitutes. This would then suggest the prediction that more urgent projects would be under more centralized control.

The result on substitutes has some further implications for when parts of projects should be controlled by different people. One implication is, for example, that—all else equal (especially abilities and access to information)—backup plans should be developed by a different team than the team that developed the main plan. Another implication is that projects that compete for funds for implementation are more likely to be controlled by different people. The reason is that even if the projects' success probabilities are completely independent, the competition for funds will make them substitutes in terms of profitability and thus make it optimal to allocate the projects to different people.

Note that these allocation mechanisms will compete with allocation mechanisms based on access to information or incentives for investments. As in the case of Lerner and Merges (1998), this trade-off can be leveraged empirically by looking at variation in uncertainty as a proxy for variation in disagreement. Although such analysis would likely rely more directly on a variation of Proposition 1, the following proposition can also be useful in that context. It says that co-location of pure complements, resp. distribution of pure substitutes, is more likely when disagreement is more likely.

Proposition 6b. Consider Variation B. The set of parameters for which control is co-located in the case of pure complements (and symmetric players), respectively distributed in the case of pure substitutes (and symmetric players and shares), increases in the probability of disagreement $(1-\rho)$.

Proof. From the proof of Proposition 6a, it follows that all differences in utilities for the case of pure complements and pure substitutes are directly

15. To see this a bit more formally, take, for example, a model with two decisions that are pure complements ($R = d_1 d_2$) and consider the following variation on the main model. In period 4, it is first revealed whether the project will be a success or not. If the project—with the current decisions—will be a success, then payoffs get realized and the game is over. If the project would be a failure, on the other hand, then with probability π the person controlling D_1 has another chance to make (and thus potentially change) that decision. I will denote that second decision as D_1' and its outcome as d_1'. The payoff is then realized after that player has made her second decision D_1'. The overall project revenue for this modified game thus equals $R' = [d_1 + (1-d_1)\pi d_1']d_2$. Consider now the optimal choice of D_1'. Since the project was a failure, it must be that at least either D_1 or D_2 were wrong. If D_2 was wrong then it does not matter what D_1' is: the project will fail again. If D_2 was right, then it must have been that D_1 was wrong so that it is optimal to choose the opposite action from D_1. Overall, choosing D_1' to be the opposite of D_1 dominates choosing the same action again. In equilibrium, it must therefore be that $d_1' = 1 - d_1$ so that $R' = [d_1 + \pi(1-d_1)(1-d_1)]d_2 = [(1-\pi)d_1 + \pi(1 - d_1 + {d_1}^2)]d_2$ or $R' = [\pi + (1-\pi)d_1]d_2$. An increase in π thus indeed decreases the degree of complementarity (as defined above) between d_1 and d_2. This is obviously just a first stab at formalizing this result and requires more research to understand its robustness.

proportional to $(1 - \rho)$. That implies the proposition following an argument analogous to the proof of Proposition 3. ■

5.2 Players' Characteristics

I now turn to the players' characteristics. I will show, in particular, for a wide class of projects, the following two results:

1. Control rights will be allocated more to a player with a larger share in the residual income.
2. Control rights will be allocated more to a player with more confidence.

The intuition derives from Proposition 1: a player with a larger share of the residual income and with stronger beliefs is more sensitive—in terms of utility—to the decision being correct (from his perspective) and thus values control rights higher.

The following proposition derives these two results for a project in which the decisions either do not interact or are complementary in the sense of supermodularity.

Proposition 7. When R is additively separable or R is supermodular in the decisions, then player P_i's control right over any particular decision, $\hat{\beta}_{i,n}$, increases in his confidence $\nu_{i,n}$ and in his share of residual income α_i.

Proof. I will first show that if R is supermodular in the decisions d_n, then U is supermodular in control rights β_n and has increasing differences in both $(\beta_{1,n}, \alpha)$ and $(\beta_{1,n}, \nu_{1,n})$. The results then follow by symmetry for P_2.

To this purpose, note that (following the calculations in Appendix A) joint expected utility equals

$$U = \alpha \sum_{\boldsymbol{d} \in \mathcal{D}} \left[\prod_{n} \left(\frac{1}{2} + (2d_n - 1)\beta_n(\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d})$$
$$+ (1 - \alpha) \sum_{\boldsymbol{d} \in \mathcal{D}} \left[\prod_{n} \left(\frac{1}{2} + (2d_n - 1)(1 - \beta_n)(\nu_{2,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}).$$

Using again $\tilde{\beta}_n$ for the continuous version of β_n—as in the calculations of Appendix A—the partial derivative for, say, $\tilde{\beta}_l$ equals then

$$\frac{\partial U}{\partial \tilde{\beta}_l} = \alpha \sum_{\boldsymbol{d} \in \mathcal{D}} \left[\prod_{n \in N_{-l}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_n(\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_l - 1)(\nu_{1,l} - \frac{1}{2}) R(\boldsymbol{d})$$
$$- (1 - \alpha) \sum_{\boldsymbol{d} \in \mathcal{D}} \left[\prod_{n \in N_{-l}} \left(\frac{1}{2} + (2d_n - 1)(1 - \tilde{\beta}_n)(\nu_{2,n} - \frac{1}{2}) \right) \right] (2d_l - 1)$$
$$\times (\nu_{2,l} - \frac{1}{2}) R(\boldsymbol{d}).$$

The cross-partial for $(\tilde{\beta}_l, \tilde{\beta}_m)$ then equals

$$\frac{\partial^2 U}{\partial\tilde{\beta}_l \partial\tilde{\beta}_m} = \alpha \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l,m}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_n (\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_l - 1)(\nu_{1,l} - \frac{1}{2})$$
$$\times (2d_m - 1)(\nu_{1,m} - \frac{1}{2}) R(\boldsymbol{d}) + (1 - \alpha)$$
$$\times \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l,m}} \left(\frac{1}{2} + (2d_n - 1)(1 - \tilde{\beta}_n)(\nu_{2,n} - \frac{1}{2}) \right) \right] (2d_l - 1)$$
$$\times (\nu_{2,l} - \frac{1}{2})(2d_m - 1)(\nu_{2,m} - \frac{1}{2}) R(\boldsymbol{d})$$

Let $\boldsymbol{d}_{-(l,m)}$ denote the subvector of $\mathbf{d}$ without the elements d_l and d_m and $(\boldsymbol{d}_{-(l,m)}, d_l, d_m)$ again the full vector. It then suffices to show that conditional on any particular $\tilde{\boldsymbol{d}}_{-(l,m)}$,

$$\sum_{(d_l, d_m) \in \{00,01,10,11\}} (2d_l - 1)(2d_m - 1) R(\tilde{\boldsymbol{d}}_{-(l,m)}, d_l, d_m) \geq 0,$$

or, with $\check{R}(d_l, d_m) = R(\tilde{\boldsymbol{d}}_{-(l,m)}, d_l, d_m)$,

$$\check{R}(1,1) - \check{R}(1,0) - \check{R}(0,1) + \check{R}(0,0) \geq 0,$$

which follows from supermodularity of R in the d_n.

It follows that if R is supermodular in the d_n then U is supermodular in the β_n. All that remains to be shown is that both (α, β_n) and $(\nu_{1,l}, \beta_n)$ have increasing differences (which imply the rest by permutation of players and decisions). For the first note that

$$\frac{\partial^2 U}{\partial\tilde{\beta}_l \partial\alpha} = \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_n (\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_l - 1)(\nu_{1,l} - \frac{1}{2}) R(\boldsymbol{d})$$
$$+ \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l}} \left(\frac{1}{2} + (2d_n - 1)(1 - \tilde{\beta}_n)(\nu_{2,n} - \frac{1}{2}) \right) \right] (2d_l - 1)$$
$$\times (\nu_{2,l} - \frac{1}{2}) R(\boldsymbol{d}),$$

which is positive by an argument similar to that in Appendix A for U increasing in β_l. For the second, note that

$$\frac{\partial^2 U}{\partial\tilde{\beta}_l \partial\nu_{1,l}} = \alpha \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_n (\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_l - 1) R(\boldsymbol{d}),$$

is positive by an argument similar to that in Appendix A for U increasing in β_l, while

$$\frac{\partial^2 U}{\partial \tilde{\beta}_l \partial \nu_{1,k}} = \alpha \sum_{d \in \mathcal{D}} \left[\prod_{n \in N_{-l,k}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_n(\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_k - 1)\tilde{\beta}_k (2d_l - 1) \\ \times (\nu_{1,l} - \frac{1}{2}) R(\boldsymbol{d}),$$

is positive by an argument similar to that for supermodularity. The cross-partials for $\nu_{2,l}$ and $\nu_{2,k}$ are similar. This concludes the proof. ■

To see the role of the independence (or supermodularity) condition, consider the following simple example. Let the project have two decisions that are pure substitutes, so that $R = d_1 + d_2 - d_1 d_2$. Let both players have identical confidence $\nu = 0.6$ for both decisions. Let, finally, player 1 have 99% of the residual income. In that case, it is optimal to give control over both decisions to player 1.[16] But consider now what happens when P_1's confidence for decision 1 is increased from 0.6 to 1. In that case, it becomes optimal to give control over decision 1 to player 1 and control over decision 2 to player 2: since control over decision 1 makes player 1 sure of a success, he does not care any more about decision 2, whereas player 2 still gets positive value from control over decision 2.[17] In this case, an increase in player 1's confidence thus moved a control right to player 2. This is caused by an interaction of decision characteristics and player characteristics.

This observation obviously raises the question whether a project's decisions—if not independent—tend to be complements, that is, whether the project tends to be supermodular. As mentioned before, both Rosen (1982) and Kremer (1993) have argued that project decisions tend to be complements and found evidence supporting this. One reason is that many projects are of the type that all decisions have to be right for the project to succeed. In general, however, this is not guaranteed and implies that care has to be taken when applying these comparative statics in cases where there are multiple decisions that are not independent. On the other hand, however, I do conjecture that this result does hold in fact as long as the cross-partial derivatives are not too negative.

This effect is also less of an issue for the comparative static with respect to the player's share of residual income. In particular, the following proposition shows that for a project with two decisions, a player's total control always increases in his share of residual income.

Proposition 8. When $n = 2$, the number of control rights allocated to P_i increases in his share of residual income.

16. In particular, the joint expected utility when P_1 controls both decisions is $0.99(0.6 + 0.6 - 0.6*0.6) + 0.01(0.5 + 0.5 - 0.5*0.5) = 0.839$, whereas it is only $(0.99 + 0.01)(0.6 + 0.5 - 0.6*0.5) = 0.8$ if each player controls one decision.

17. In particular, the joint expected utility when P_1 controls both decisions is $0.99 + 0.01(0.5 + 0.5 - 0.5*0.5) = 0.9975$, whereas it increases to $0.99 + 0.01(0.6 + 0.5 - 0.6*0.5) = 0.998$ when control over decision 2 is shifted to player 2.

Proof. The number of decision rights can only decrease either when going from (1, 1) to any other state or when going from any other state to (0, 0). Consider first the possibility of going from (1, 1) to (0, 0). The change in joint utility equals (using the equations from Appendix A)

$$\begin{aligned} U(1,1) - U(0,0) = {} & \alpha\frac{1}{4}(2\nu_{1,1} - 1)[2\delta_1 + \Delta] + \alpha\frac{1}{4}(2\nu_{1,2} - 1)[2\delta_2 + \Delta] \\ & + \alpha\frac{1}{4}(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\Delta - (1-\alpha)\frac{1}{4}(2\nu_{2,1} - 1) \\ & \times [2\delta_1 + \Delta] - (1-\alpha)\frac{1}{4}(2\nu_{2,2} - 1)[2\delta_2 + \Delta] \\ & - (1-\alpha)\frac{1}{4}(2\nu_{2,1} - 1)(2\nu_{2,2} - 1)\Delta, \end{aligned}$$

so that the partial derivative for α then is

$$\begin{aligned} \frac{\partial(U(1,1) - U(0,0))}{\partial\alpha} = {} & \frac{1}{4}(2\nu_{1,1} - 1)[2\delta_1 + \Delta] + \frac{1}{4}(2\nu_{1,2} - 1)[2\delta_2 + \Delta] \\ & + \frac{1}{4}(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\Delta + \frac{1}{4}(2\nu_{2,1} - 1)[2\delta_1 + \Delta] \\ & + \frac{1}{4}(2\nu_{2,2} - 1)[2\delta_2 + \Delta] + \frac{1}{4}(2\nu_{2,1} - 1)(2\nu_{2,2} - 1)\Delta, \end{aligned}$$

which is positive using the fact that $\delta_n > 0$ and $\delta_n + \Delta > 0$ and recombining $2\delta_n + \Delta$ with Δ. This concludes that part of the proof.

Consider next the possibility for going from, say, (1, 1) to (1, 0). The change in joint utility then equals

$$\begin{aligned} U(1,1) - U(1,0) = {} & \alpha\frac{1}{4}(2\nu_{1,2} - 1)[2\delta_2 + \Delta] + \alpha\frac{1}{4}(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\Delta \\ & - (1-\alpha)\frac{1}{4}(2\nu_{2,2} - 1)[2\delta_2 + \Delta], \end{aligned}$$

so that the partial derivative for α then is

$$\begin{aligned} \frac{\partial(U(1,1) - U(1,0))}{\partial\alpha} = {} & \frac{1}{4}(2\nu_{1,2} - 1)[2\delta_2 + \Delta] + \frac{1}{4}(2\nu_{1,1} - 1)(2\nu_{1,2} - 1)\Delta \\ & + \frac{1}{4}(2\nu_{2,2} - 1)[2\delta_2 + \Delta], \end{aligned}$$

which is again positive using the fact that $\delta_n > 0$ and $\delta_n + \Delta > 0$ and recombining $2\delta_n + \Delta$ with Δ. A permutation of decisions and of players then concludes the proof. ■

Note that this result that control should be allocated more to players with a larger stake in the outcome favors unified control over all decisions that affect one particular project, since—by definition—the person with the largest stake in the project is the same for all decisions affecting the same project.

Another practical implication of these results is that people who are held responsible for the outcome should also be given control rights. This is a well-known principle in management. In particular, it is often said that giving "responsibility without control" can only generate dissatisfaction.

An issue that often gets raised in this context is how the result that people with more confidence should get more control differs from the standard result that people with better information should get more control. I return to this issue in Section 6. The short answer is that with private information, all players prefer the player with the best information to have control. In other words, a player with worse information can value control negatively. That is never the case with differing priors: each player wants control and it is only when they get compensated financially that players with low confidence are willing to cede control to those with more confidence.

6. The Role of Differing Priors

An important issue is obviously the role that differing priors play in the results of this article. The analysis of Variation B already gave some indication of that role by varying the amount of disagreement. It showed that more disagreement makes players value control more highly and also makes income and control more likely to be concentrated.

Another useful way to clarify the role of differing priors is to compare the results of this article to what similar models with private information or private benefits imply for the allocation of a project's residual income and control. Such comparison may sharpen the insights in the underlying mechanisms of this article and can hopefully also give some intuition for how differing priors may differ from, for example, private information or private benefits.[18]

In what follows, I will thus consider models that capture the basic setting of this article—a project with an income stream that depends on a set of decisions—but where the potential conflict now arises from either private information or private benefits, rather than from differing priors. The question is how the allocation of control (and of residual income, when applicable) would compare to the results of this article. The key conclusion of this section is that the alternative settings that I consider do lead to some very different predictions. Moreover, the results also give some useful intuition on the differences.

6.1 Models with Private Information

Among models with private information, there is a fairly obvious and logical comparison point for differing priors. In particular, a question that is often raised is how differing priors relate to the same belief differences caused by private information that cannot be communicated. The comparison point is thus the model of Section 2 with one change: the players start with a common

18. However, since the comparison is necessarily limited in scope and focused on particular models, it is not (meant to be) conclusive as to whether differing priors are actually necessary to obtain the results in this article. The purpose of the comparison is to give intuition for the differences. I will also disregard here models with state-dependent utility since the notion of beliefs, and thus differing priors, is not unambiguous any more.

prior but each player P_i gets a (private and independent) signal which implies that $S_n = X_n$ with probability $\mu_{i,n}$. As before, the $\mu_{i,n}$ are independent draws from nondegenerate and symmetric distributions $F_{i,n}$ with support [0, 1]. Let again $\nu_{i,n} = \int_0^1 \max(u, 1-u)\, \mathrm{d}F_{i,n}(u)$. In this setting, the distribution of beliefs at the end of period 2 is identical to that in the model of Section 2, but the beliefs are derived from different primitives.

Although this model with private information may seem, at first sight, very similar to the one with differing priors, it actually turns out to give some drastically different results (for which I will explain the mechanisms in more detail below). First, a player's valuation of control (relative to the other player controlling the decision, as in Proposition 1) may actually be negative and may decrease in her share of residual income. Second, as long as each player has some share of the residual income, the joint expected utility in the private information model is independent of the allocation of residual income. Third, concentration of all income and control in the hands of one person can be strictly suboptimal. Fourth, the degree of complements and substitutes does not affect (in itself) the optimal allocation of control.

The key to understand all these results are two observations about the private information model:

1. Both players in the private information model agree on who of them has (on average) the best information. They both prefer that control over a decision is allocated to the player with the best information on that decision.
2. Both players in the private information model value residual income (in expectation) identically. There is thus no gain from reallocating residual income (beyond making sure that each player has some stake in the project to ensure that she tries to maximize the project revenue).

To see how these two observations lead to the predictions above, take, for example, the result that a player's valuation may be negative and that it may decrease as the player's share of residual income increases. Consider to that purpose a setting with two players (P_1 and P_2), one decision, and $R = d$. Note that when player P_i controls the decision then the project's expected revenue (at the start of period 2) is ν_i according to both players. The change in P_1's utility when control over the decision shifts from P_2 to P_1 is then $\alpha_1(\nu_1 - \nu_2)$. This is also P_1's valuation of the control right. It is indeed negative when $\nu_2 > \nu_1$ and then becomes more negative when α_1 increases. The joint utility when P_1 controls the decision is $\alpha_1\nu_1 + \alpha_2\nu_1 = \nu_1$ and is thus independent of the allocation of residual income. In a setting with two decisions, it may be uniquely optimal to give each decision to a different player—when the player with the best information is different for the two decisions—so that concentrating all income and control in the hands of one person is then strictly suboptimal. Finally, the fact that the decisions are complements affects how important it is to make the right decision as a function of the other decisions being right or wrong, but it does not affect who optimally controls that decision.

Another way to see the difference is to note that there is in fact no agency problem in the private information model: both players want to maximize expected revenue and agree on the best way to get there. In a differing priors context, in contrast, each player believes that he is right and thus—by definition—the other is wrong when they hold different beliefs. This open disagreement creates a real agency problem since the players' objectives differ.

This comparison also provides an answer to the following issue that is sometimes raised: how does the result in this article that control will be allocated more to players with more confidence differ from the standard result that control should be allocated more to players with more information? One important difference was pointed out above: with private information all players prefer the same allocation of control, whereas with differing priors each player wants control for himself. Another difference is that with differing priors it is the *combination* of strong beliefs with a large share of residual income that makes it optimal to allocate control to a player (so that it may be optimal to give control to a player with low confidence but a high share over a player with high confidence and a low share), whereas the share of the residual income plays no role in this private information model.

6.2 Models with Private Benefits

For private benefits as the source of conflict, there does not seem to be such an obvious comparison point as there was for private information. It is nevertheless interesting to compare the model in this article to private benefits models that have been used to study the allocation of a project's residual income and control. Aghion and Bolton (1992), Aghion and Tirole (1997), Hart and Holmstrom (2002), Prendergast (2002), and Baker et al. (2004) all have studied settings that are similar to this article (i.e., a project that generates income which depends on decisions) but where the agency problem originates in private benefits instead of in differing priors.[19] Along the dimensions that matter for this article, these articles all have a similar setup. There is a project that generates an income stream (that may be contractible), with the level of income depending on a set of decisions. Apart from a share in this residual income (i.e., the common benefit), the decision makers also derive some private benefits or private costs directly from the decisions. These models differ, among other things, on whether the residual income is contractible and whether or how decision rights can be moved around.

To translate this setup to the current context, consider again the model of Section 2 but with the following changes. Assume that for each decision D_n, it is common knowledge that the right choice is X_n (i.e., $S_n = X_n$) with probability

19. Private benefits models that consider only inalienable and noncontractible private benefits, such as Baker et al. (2006), are difficult to compare to the current model. The reason is that there is no obvious match between, on the one hand, such (conflicting) inalienable, incontractible private benefits from decisions and, on the other hand, residual income from a project that depends in a non-trivial way on these same decisions: when private benefits are interpreted as being derived from residual income, then it seems that the players' private benefits should be aligned.

$\theta > 0.5$.[20,21] Apart from their share of the residual income ($\alpha_i R$), the players also get private benefits from particular actions. In particular, each player P_i always privately prefers one action (either X_n or Y_n) for each decision D_n and gets $B_{i,n} \geq 0$ from that privately preferred action (and 0 from the other action). Which of the two actions (X_n or Y_n) P_i privately prefers is determined at random in period 2, with both actions equally likely. To say this a bit more formally, let $Z_{i,n}$ denote P_i's privately preferred action for decision D_n, then the $Z_{i,n}$ are drawn randomly from $\{X_n, Y_n\}$ with both actions being equally likely. One particular issue in this case is that—when decisions interact, as with complements and substitutes—there may be multiple equilibria in stage 3. To get a unique equilibrium, I will assume that in that case the players choose their actions sequentially with each player equally likely to choose first.[22]

The results for this private benefits model differ in important respects from the results of the differing priors model (and I will again explain the mechanisms behind each result in more detail below). First, a player's valuation of a control right (relative to the other player controlling the decision, as in Proposition 1) may actually be decreasing in her share of residual income. Second, an increase in the level of a player's private benefits ($B_{i,n}$) may make it strictly optimal to shift control over that decision D_n *away* from that player. Third, with both income and control contractible, it may be weakly optimal to distribute control rights and income rights over different players. Fourth, in a project with perfectly complementary decisions and symmetric players, it may be strictly optimal to distribute the control rights among different players, whereas in a project with perfect substitutes and symmetric players, it may be strictly optimal to concentrate both decisions with one player.

To see why a player's valuation of control may be decreasing in her share of residual income, consider a setting with one decision (a choice between X and Y) where $R = d$ and players P_1 and P_2 derive private benefits B_1 and B_2 from their preferred choice (drawn for each randomly and independently from $\{X, Y\}$). Consider more specifically the case that $\alpha_1(2\theta - 1) < B_1$ and $\alpha_2(2\theta - 1) > B_2$ so that P_1 will choose her preferred action whereas P_2 will choose X. Player P_1's utility equals $\alpha_1\theta + B_1\frac{1}{2}$ when P_2 controls the decision and $\alpha_1\frac{1}{2} + B_1$ when she controls the decision herself. Her valuation for the control right (relative to P_2 controlling the decision) is thus $B_1\frac{1}{2} - \alpha_1(\theta - \frac{1}{2})$ which indeed decreases in α_1.

20. In the articles mentioned, this is immediately modeled as R deterministically depending on the decisions with $R(\ldots, X_n, \ldots) > R(\ldots, Y_n, \ldots)$. Such formulation would work perfectly fine in this context and is actually slightly more general. The only reason to state the assumption in terms of common priors is to stay closer to the model in this article.

21. Note that when $\theta = 0.5$, the expected project revenue would become independent from the decisions, so that both the allocation of residual income and how decisions interact in R (e.g., as complements or substitutes) would play no role for the total utility or for the allocation of control. Since these elements are at the heart of Propositions 2–6 and play a significant role in Propositions 1 and 8, it would thus be difficult to relate a model with $\theta = 0.5$ to the current article. Moreover, assuming $\theta = 0.5$ essentially eliminates any role for the project and thus eliminates an essential part of the model.

22. This assumption only affects the results for complements and substitutes.

Second, for the result that an increase of a player's private benefits may make it optimal to shift control away from that player, consider the model above and assume first that $\alpha_1(2\theta - 1) < B_1 < (2\theta - 1)$ and $\alpha_2(2\theta - 1) > B_2$ (and, for later purposes, $\alpha_1 \geq \alpha_2$). Since (only) player P_2 will always choose X and thus maximize project revenue (which is the utility-maximizing outcome in this setting where the decision must be made by one or the other player), it is strictly optimal to give control to P_2. If B_2 now increases so that $\alpha_2(2\theta - 1) < B_2 < B_1 < (2\theta - 1)$, then both players will choose their preferred action. The (unique) optimal allocation is now to give control to the player with the most private benefits at stake, which is P_1 in this case. It thus follows indeed that an increase in B_2 can make it strictly optimal to shift control away from P_2. This result and its intuition are again very different from the differing priors case.

Third, to see why it may be (weakly) optimal to distribute control rights and income rights, consider a setting with two decisions and $R = \frac{d_1+d_2}{2}$, so that the decisions do not interact. Assume that $B_{1,1} = B_{2,2} \in \left(\frac{(2\theta-1)}{4}, \frac{(2\theta-1)}{2}\right)$ and $B_{1,2} = B_{2,1} \in \left(\frac{(2\theta-1)}{8}, \frac{(2\theta-1)}{4}\right)$. In that case, total utility is maximized (though not uniquely) by allocating decision D_1 to player P_2 and D_2 to P_1 (i.e., each decision is allocated to a player with weak personal preferences over that particular decision) and by choosing $\alpha_1 = \alpha_2 = 0.5$ so that each player always chooses X_n. In this case, the allocation of income (given the allocation of control) is driven by the need to make players decide in a certain way. This is a very different intuition from the differing priors intuition for allocating residual income, which is driven by differences in valuation of the residual income.

For the result on complements, consider a setting where $R = d_1 d_2$, $\alpha_1 = \alpha_2 = 0.5$, and both players get the same private benefit B for their preferred choice on each decision. Assume that $\frac{1}{4}(2\theta - 1) < B < \frac{\theta}{2}(2\theta - 1)$. If the decisions are allocated to different players, then the two players will always choose X_n, which is the outcome that maximizes the joint utility. If both decisions are allocated to one player, then that player will choose X_n for both unless she privately prefers Y_n on both decisions, in which case she chooses Y_n for both. The joint utility is now higher under distributed control than under concentrated control, in contrast to the differing priors case. The gain from distributed control in this case is that the private cost of maximizing the common benefit (namely forgoing the privately preferred actions) is spread equally over both players, in proportion to the distribution of the common benefit.

For the result on substitutes, consider a setting where $R = d_1 + d_2 - d_1 d_2$, $\alpha_1 = \alpha_2 = 0.5$, and both players get the same level of private benefits B_1 (resp. B_2) for their preferred choice on decision D_1 (resp. D_2). Assume that $\frac{(1-\theta)}{2}(2\theta - 1) < B_1 < B_2 < \frac{\theta}{2}(2\theta - 1)$. If both decisions are allocated to one player, then that player will choose X_n for one of the decisions and follow her private preference for the other. In particular, if the player in control prefers Y_n for both decisions, she will always choose $D_1 = X_1$ and $D_2 = Y_2$ (since D_2 gives more private benefits than D_1). When both decisions are allocated to different players, the decisions are the same with one exception: when both players prefer Y_n for the decision under their control, then whoever is allowed to choose first will choose Y_n. It follows that expected utility will be lower when control is distributed since it will sometimes be the decision with the most private benefits on which the private benefits get sacrificed for the common benefit. In this case, concentrating control leads to better coordination with respect to which private preferences to follow in case of conflict.

These contrasting results for private benefits and differing priors are to a large extent due to two fundamental differences:

1. In the model with private benefits, players value the expected residual income identically (in contrast to the case with differing priors) so that the allocation of income is not driven by differences in valuation but instead by the need to give players the right incentives to make particular decisions.
2. In the model with private benefits, players agree on what is optimal from the project's perspective but these revenue-maximizing decisions may conflict with their own private benefits. The players may then prefer to maximize their private benefits at the expense of the project, and residual income may counter-balance this by aligning different players' objectives. In the differing priors model, on the contrary, players disagree on what is optimal for the project itself. This leads to very different interactions between residual income and control.

7. Conclusion

The allocation of control only matters when different people would do different things. An important source of such differences is that people may openly and knowingly disagree on the optimal course of action. This article derived a number of results for the allocation of control when people may openly disagree in the sense of differing priors.

The article first noted that people value control rights in this context because they believe they make better decisions than others. It then showed that it is optimal to concentrate all income and control rights of a project in one hand: as a person gets more control rights, she values income rights higher, so it is optimal to give her more income rights; as a person gets more income rights, she values control rights higher, making it optimal to give her more control rights. Different projects, however, will sometimes be optimally "owned" by different people. The article further showed that—when the allocation of residual income is exogenously given—complementary decisions should be more co-located, whereas substitute decisions should be more distributed. Confident people with a lot at stake should—in a wide range of settings—get more control.

From an empirical perspective, a useful feature of the model is that all player benefits are derived from the project's income stream, which is more easily measurable than, say, players' private benefits or private costs. However, just like private information or private benefits, a player's beliefs or confidence are typically difficult to measure directly. As an alternative, the empirical literature has suggested some useful proxies for the degree of disagreement, such as the development stage of the industry or firm, the divergence among analyst expectations, or the volatility of stock prices (which may be caused in part by diverging views on the effects of new information). Another alternative is obviously to rely on comparative statics along other dimensions.

This theory also has implications for the theory of the firm. In particular, Van den Steen (2007b) builds a theory of the firm where a firm's role is to give a manager authority over employees through centralized asset ownership and low-powered incentives. The results of the current article play an important role in that context. In particular, the co-location of income and control makes it optimal that the asset owner (who endogenously has control) is also the residual claimant. At the same time, though, different projects will sometimes optimally be owned by different firms, thus avoiding the result that "all production [is] carried on by one big firm" (Coase 1937).

The article also suggested some areas for further research that would translate the results in more direct empirical predictions. The formation of alliances under capital constraints and VC financing of entrepreneurs seem to be two promising areas for further research with this framework.

Appendix A. Calculations for Joint Utility

This appendix contains the general calculations (for $\rho \in [0, 1)$) for the players' joint utility. I will, in particular, do the following:

1. Show that it is a dominant strategy for each player P_i to choose for each decision the course that he or she believes is most likely to be correct (as long as $\alpha_i > 0$).
2. Calculate the joint utility of the players (with more detailed calculations for $n = 2$).
3. Show, as part of that, that all control rights will be allocated.
4. Calculate the change in joint utility from a change in allocation of control for $n = 2$.

For notational purposes, let $\boldsymbol{d} = (d_1, \ldots, d_N)$ denote a vector of d_n, let $\mathcal{D}$ denote the set of all such vectors, let $\boldsymbol{d}_{-\mathrm{k}}$ denote the vector $\boldsymbol{d}$ excluding the kth element and $\boldsymbol{d} = (\boldsymbol{d}_{-k}, d_k)$ again the full vector. Let $\boldsymbol{N}$ denote the set $\{1, 2, \ldots, N\}$ and $\boldsymbol{N}_{-k} = \{1, \ldots, k-1, k+1, \ldots, N\}$. Let $Z_{i,k} \in \{X_k, Y_k\}$ denote the course of action for decision D_k that player P_i considers most likely to be correct.

To see that it is indeed a dominant strategy for each player P_i to choose $Z_{i,k}$ (as long as $\alpha_i > 0$), consider a player P_i who—potentially among other decisions—controls D_k. Let $\tilde{\nu}_{i,k} = \max(\mu_{i,k}, 1 - \mu_{i,k})$ denote player i's realized confidence about decision D_k. Conditional on any particular realization $\tilde{\boldsymbol{d}}_{-k}$ for $\boldsymbol{d}_{-\mathrm{k}}$, this player's expected utility if he chooses $Z_{i,k}$ is $\alpha_i[\tilde{\nu}_{i,k} R(\tilde{\boldsymbol{d}}_{-k}, 1) + (1 - \tilde{\nu}_{i,k}) R(\tilde{\boldsymbol{d}}_{-k}, 0)]$ while it is $\alpha_i[\tilde{\nu}_{i,k} R(\tilde{\boldsymbol{d}}_{-k}, 0) + (1 - \tilde{\nu}_{i,k}) R(\tilde{\boldsymbol{d}}_{-k}, 1)]$ otherwise. Since $\alpha_i(2\tilde{\nu}_{i,k} - 1)(R(\tilde{\boldsymbol{d}}_{-k}, 1) - R(\tilde{\boldsymbol{d}}_{-k}, 0)) > 0$ when

$\alpha_i > 0$, it follows that it is a dominant strategy to choose $Z_{i,k}$. Note that with $\alpha_i = 0$, P_i is indifferent so that she chooses $Z_{i,k}$ by assumption. Overall, each player P_i will always choose $Z_{i,k}$ when in control of D_k.

Consider now the calculation of the joint utility at the end of period 1. Consider first the case—which occurs with probability ρ—that the players always agree. I will first calculate for any particular realization of $\boldsymbol{d}$, how likely that realization is according to P_i. Since, for any player P_i, the state realizations are independent and since both players choose $Z_{i,k}$, it follows that—according to P_i—the probability that $d_n = 1$ equals $\nu_{i,n}$ while the probability that $d_n = 0$ equals $(1 - \nu_{i,n})$, independent of who makes decision D_n (since they make the same decision) and independent of $\boldsymbol{d}_{-\mathrm{k}}$ (since the state realizations are independent). The probability that a particular vector $\boldsymbol{d}$ realizes is then, according to P_i,

$$\prod_n (\nu_{i,n} d_n + (1 - d_n)(1 - \nu_{i,n})) = \prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{i,n} - \frac{1}{2}) \right).$$

Consider next the case—which obtains with the complementary probability $(1 - \rho)$—that the players' beliefs are independent. From player P_i's perspective, the probability that $d_n = 1$ then equals

$$\beta_{i,n}\nu_{i,n} + (1 - \beta_{i,n})\frac{1}{2} = \frac{1}{2} + \beta_{i,n}(\nu_{i,n} - \frac{1}{2}),$$

while the probability that $d_n = 0$ equals

$$\beta_{i,n}(1 - \nu_{i,n}) + (1 - \beta_{i,n})\frac{1}{2} = \frac{1}{2} - \beta_{i,n}(\nu_{i,n} - \frac{1}{2}).$$

Since the state realizations are independent, the probability that a particular vector $\boldsymbol{d}$ obtains is then, according to P_i,

$$\prod_n \left(\frac{1}{2} + (2d_n - 1)\beta_{i,n}(\nu_{i,n} - \frac{1}{2}) \right).$$

Combining these two cases implies that joint utility equals

$$\begin{aligned} U = {} & \rho\alpha \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ & + \rho(1-\alpha) \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{2,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ & + (1-\rho)\alpha \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\beta_{1,n}(\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ & + (1-\rho)(1-\alpha) \sum_{\boldsymbol{d}\in\mathcal{D}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\beta_{2,n}(\nu_{2,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}). \end{aligned}$$

The next step is now to show that this joint utility is increasing in each of the $\beta_{i,n}$ (and strictly so in at least either $\beta_{1,n}$ or $\beta_{2,n}$) since this will imply that in equilibrium $\beta_{1,n} + \beta_{2,n} = 1$, which will allow me to further simplify the formulas.

The analysis here and elsewhere is often simpler if the $\beta_{i,n}$ were allowed to vary continuously on [0, 1] rather than being integer restricted to {0, 1}. To that purpose, I will introduce $\tilde{\beta}_{i,n} \in [0, 1]$ and derive the result by looking at $\beta_{i,n}$ as the integer-restricted version of $\tilde{\beta}_{i,n}$.

To show that U indeed increases in each $\beta_{i,n}$, note that the partial derivative for, say, $\tilde{\beta}_{1,k}$ is

$$\frac{\partial U}{\partial \tilde{\beta}_{1,k}} = (1-\rho)\alpha \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_{n\in \boldsymbol{N}_{-k}} \left(\frac{1}{2} + (2d_n - 1)\tilde{\beta}_{1,n}(\nu_{1,n} - \frac{1}{2}) \right) \right] (2d_k - 1) \\ \times (\nu_{1,k} - \frac{1}{2}) R(\boldsymbol{d}).$$

It then suffices to show that conditional on any particular $\tilde{\boldsymbol{d}}_{-k}$,

$$\sum_{d_k\in\{0,1\}} (2d_k - 1) R(\tilde{\boldsymbol{d}}_{-k}, d_k) > 0$$

or $R(\tilde{\boldsymbol{d}}_{-k}, 1) - R(\tilde{\boldsymbol{d}}_{-k}, 0) > 0$ which follows from $\frac{\partial Q}{\partial d_n} > 0$. Since at least either $\alpha > 0$ or $(1-\alpha) > 0$, U is strictly increasing in at least either $\beta_{1,k}$ or $\beta_{2,k}$. As mentioned above, this implies that all control rights will be fully allocated to P_1 and P_2, so that $\beta_{2,n} = 1 - \beta_{1,n}$. For simplicity, I will now denote $\beta_n = \beta_{1,n}$ so that $\beta_{2,n} = 1 - \beta_n$. The general expression then becomes

$$U = \rho\alpha \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ + \rho(1-\alpha) \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(\nu_{2,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ + (1-\rho)\alpha \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)\beta_n(\nu_{1,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}) \\ + (1-\rho)(1-\alpha) \sum_{\boldsymbol{d}\in\boldsymbol{\mathcal{D}}} \left[\prod_n \left(\frac{1}{2} + (2d_n - 1)(1-\beta_n)(\nu_{2,n} - \frac{1}{2}) \right) \right] R(\boldsymbol{d}).$$

I now consider the more specific case with 2 decisions. In that case, joint utility can be written

$$U = \rho\alpha[\nu_{1,1}\nu_{1,2}]R(1,1) + \rho(1-\alpha)[\nu_{2,1}\nu_{2,2}]R(1,1) \\ + (1-\rho)\alpha\left[\left(\frac{1}{2} + \beta_1(\nu_{1,1} - \frac{1}{2}) \right) \left(\frac{1}{2} + \beta_2(\nu_{1,2} - \frac{1}{2}) \right) \right] R(1,1) + (1-\rho) \\ \times (1-\alpha)\left[\left(\frac{1}{2} + (1-\beta_1)(\nu_{2,1} - \frac{1}{2}) \right) \left(\frac{1}{2} + (1-\beta_2)(\nu_{2,2} - \frac{1}{2}) \right) \right] R(1,1) \\ + \rho\alpha[\nu_{1,1}(1-\nu_{1,2})]R(1,0) + \rho(1-\alpha)[\nu_{2,1}(1-\nu_{2,2})]R(1,0)$$

$$
\begin{aligned}
&+ (1-\rho)\alpha\left[\left(\frac{1}{2}+\beta_1(\nu_{1,1}-\frac{1}{2})\right)\left(\frac{1}{2}-\beta_2(\nu_{1,2}-\frac{1}{2})\right)\right]R(1,0) + (1-\rho)\\
&\times (1-\alpha)\left[\left(\frac{1}{2}+(1-\beta_1)(\nu_{2,1}-\frac{1}{2})\right)\left(\frac{1}{2}-(1-\beta_2)(\nu_{2,2}-\frac{1}{2})\right)\right]R(1,0)\\
&+ \rho\alpha\left[(1-\nu_{1,1})\nu_{1,2}\right]R(0,1) + \rho(1-\alpha)\left[(1-\nu_{2,1})\nu_{2,2}\right]R(0,1) + (1-\rho)\\
&\times \alpha\left[\left(\frac{1}{2}-\beta_1(\nu_{1,1}-\frac{1}{2})\right)\left(\frac{1}{2}+\beta_2(\nu_{1,2}-\frac{1}{2})\right)\right]R(0,1) + (1-\rho)(1-\alpha)\\
&\times \left[\left(\frac{1}{2}-(1-\beta_1)(\nu_{2,1}-\frac{1}{2})\right)\left(\frac{1}{2}+(1-\beta_2)(\nu_{2,2}-\frac{1}{2})\right)\right]R(0,1)\\
&+ \rho\alpha\left[(1-\nu_{1,1})(1-\nu_{1,2})\right]R(0,0) + \rho(1-\alpha)\left[(1-\nu_{2,1})(1-\nu_{2,2})\right]R(0,0)\\
&+ (1-\rho)\alpha\left[\left(\frac{1}{2}-\beta_1(\nu_{1,1}-\frac{1}{2})\right)\left(\frac{1}{2}-\beta_2(\nu_{1,2}-\frac{1}{2})\right)\right]R(0,0)\\
&+ (1-\rho)(1-\alpha)\left[\left(\frac{1}{2}-(1-\beta_1)(\nu_{2,1}-\frac{1}{2})\right)\left(\frac{1}{2}-(1-\beta_2)(\nu_{2,2}-\frac{1}{2})\right)\right]\\
&\times R(0,0),
\end{aligned}
$$

or with $\underline{R} = R(0, 0)$

$$
\begin{aligned}
U = {} & \rho\alpha\left[\underline{R}+\nu_{1,1}\delta_1+\nu_{1,2}\delta_2+\nu_{1,1}\nu_{1,2}\Delta\right]\\
&+ \rho(1-\alpha)\left[\underline{R}+\nu_{2,1}\delta_1+\nu_{2,2}\delta_2+\nu_{2,1}\nu_{2,2}\Delta\right]\\
&+ (1-\rho)\alpha\frac{1}{4}\left[(1+\beta_1(2\nu_{1,1}-1))(1+\beta_2(2\nu_{1,2}-1))\right]R(1,1)\\
&+ (1-\rho)\alpha\frac{1}{4}\left[(1+\beta_1(2\nu_{1,1}-1))(1-\beta_2(2\nu_{1,2}-1))\right]R(1,0)\\
&+ (1-\rho)\alpha\frac{1}{4}\left[(1-\beta_1(2\nu_{1,1}-1))(1+\beta_2(2\nu_{1,2}-1))\right]R(0,1)\\
&+ (1-\rho)\alpha\frac{1}{4}\left[(1-\beta_1(2\nu_{1,1}-1))(1-\beta_2(2\nu_{1,2}-1))\right]R(0,0)\\
&+ (1-\rho)(1-\alpha)\frac{1}{4}\left[(1+(1-\beta_1)(2\nu_{2,1}-1))(1+(1-\beta_2)(2\nu_{2,2}-1))\right]\\
&\times R(1,1) + (1-\rho)(1-\alpha)\frac{1}{4}\left[(1+(1-\beta_1)(2\nu_{2,1}-1))(1-(1-\beta_2)\right.\\
&\left.\times(2\nu_{2,2}-1))\right]R(1,0) + (1-\rho)(1-\alpha)\frac{1}{4}\left[(1-(1-\beta_1)(2\nu_{2,1}-1))\right.\\
&\left.\times(1+(1-\beta_2)(2\nu_{2,2}-1))\right]R(0,1) + (1-\rho)(1-\alpha)\frac{1}{4}\left[(1-(1-\beta_1)\right.\\
&\left.\times(2\nu_{2,1}-1))(1-(1-\beta_2)(2\nu_{2,2}-1))\right]R(0,0).
\end{aligned}
$$

Using the fact that $R(1,1)+R(1,0)+R(0,1)+R(0,0) = 4R+2(\delta_1+\delta_2)+\Delta$, $R(1,1)+R(1,0)-R(0,1)-R(0,0) = 2\delta_1+\Delta$, and $R(1,1)-R(1,0)+R(0,1) - R(0,0) = 2\delta_2+\Delta$, this becomes

$$U = \underline{R} + \rho\alpha\left[v_{1,1}\delta_1 + v_{1,2}\delta_2 + v_{1,1}v_{1,2}\Delta\right] + \rho(1-\alpha)$$
$$\times\left[v_{2,1}\delta_1 + v_{2,2}\delta_2 + v_{2,1}v_{2,2}\Delta\right] + (1-\rho)\alpha\frac{1}{4}[2(\delta_1+\delta_2)+\Delta]$$
$$+ (1-\rho)\alpha\frac{1}{4}\beta_1(2v_{1,1}-1)[2\delta_1+\Delta] + (1-\rho)\alpha\frac{1}{4}\beta_2(2v_{1,2}-1)[2\delta_2+\Delta]$$
$$+ (1-\rho)\alpha\frac{1}{4}\beta_1\beta_2(2v_{1,1}-1)(2v_{1,2}-1)\Delta + (1-\rho)(1-\alpha)\frac{1}{4}[2(\delta_1+\delta_2)$$
$$+\Delta] + (1-\rho)(1-\alpha)\frac{1}{4}(1-\beta_1)(2v_{2,1}-1)[2\delta_1+\Delta] + (1-\rho)$$
$$\times(1-\alpha)\frac{1}{4}(1-\beta_2)(2v_{2,2}-1)[2\delta_2+\Delta] + (1-\rho)(1-\alpha)\frac{1}{4}(1-\beta_1)$$
$$\times(1-\beta_2)(2v_{2,1}-1)(2v_{2,2}-1)\Delta.$$

Let $\hat{U}$ and $\check{U}$ be the joint utilities for $(\hat{\beta}_1, \hat{\beta}_2)$ and $(\check{\beta}_1, \check{\beta}_2)$ respectively. The difference in joint utilities then equals

$$\hat{U} - \check{U} = (1-\rho)\alpha\frac{1}{4}(\hat{\beta}_1 - \check{\beta}_1)(2v_{1,1}-1)[2\delta_1+\Delta]$$
$$+ (1-\rho)\alpha\frac{1}{4}(\hat{\beta}_2 - \check{\beta}_2)(2v_{1,2}-1)[2\delta_2+\Delta]$$
$$+ (1-\rho)\alpha\frac{1}{4}(\hat{\beta}_1\hat{\beta}_2 - \check{\beta}_1\check{\beta}_2)(2v_{1,1}-1)(2v_{1,2}-1)\Delta$$
$$+ (1-\rho)(1-\alpha)\frac{1}{4}(\check{\beta}_1 - \hat{\beta}_1)(2v_{2,1}-1)[2\delta_1+\Delta]$$
$$+ (1-\rho)(1-\alpha)\frac{1}{4}(\check{\beta}_2 - \hat{\beta}_2)(2v_{2,2}-1)[2\delta_2+\Delta]$$
$$+ (1-\rho)(1-\alpha)\frac{1}{4}[(1-\hat{\beta}_1)(1-\hat{\beta}_2) - (1-\check{\beta}_1)$$
$$\times(1-\check{\beta}_2)](2v_{2,1}-1)(2v_{2,2}-1)\Delta.$$

Appendix B. Competitive Allocation of Control

The analysis in this article assumed that income and control rights were allocated through Nash bargaining. Very often, however, control is determined by a power struggle or by a competitive mechanism rather than by some orderly and efficient negotiation process. In this appendix, I will study such competitive allocation for one very simple case and show that the outcome coincides with the efficient allocation that obtains (in this transferable utility context) through bargaining.

To be more concrete, assume that each control right gets allocated through an ascending-price auction (while the income rights are exogenously given). In some cases, however, the auction may be complicated by the fact that a person's valuation may in principle depend on who gets the decision right in equilibrium. To ensure a unique equilibrium for such cases, assume the following auction process. The price, p, is measured on a clock which starts at zero and continuously rises. The process starts, at $p = 0$, with all players "in." At any point, any player can stop the clock and quit, after which the clock continues. When two or more players stop the clock at exactly the same time, then only one of these players can quit, with each player being equally likely.[23] This continues until exactly one player is left. This player gets the control right at the price at which the one-before-last player quit (and cannot refuse this transaction).

For this competitive allocation, I will limit my analysis to the basic model and to the case where the decisions do not interact, that is, $R = \sum_n \kappa_n d_n$, where the constants $\kappa_n \geq 0$ attach weights to the different decisions and thus satisfy $\sum_n \kappa_n = 1$. The reason for considering only this case without interaction is that interactions among the decisions cause interactions among the bids for different control rights, and may thus require combination bids (for sets of control rights). Although this complicates the analysis considerably, the basic points and insights can be made in the much simpler case. Let $\tilde{\boldsymbol{B}}$ denote the allocation of control under the auction and $\hat{\boldsymbol{B}}$ the allocation under Nash bargaining (which is efficient). The key result is then that with differing priors, these allocations coincide.

Proposition 9. The allocation of control under the ascending-price auction is the same as under Nash bargaining: $\tilde{\boldsymbol{B}} = \hat{\boldsymbol{B}}$.

Proof. Since the revenue function is totally separable in decisions, it suffices to show this for one decision. Consider therefore the special case that $R = d_1$, and drop all indices that refer to the decision.

Consider first the allocation that obtains under Nash bargaining. Total utility in this case equals

$$U = \sum_i \alpha_i \left(\frac{1}{2} + \beta_i (\nu_i - \frac{1}{2}) \right),$$

which is maximized by allocating all control to one player with the highest $\alpha_i(\nu_i - \frac{1}{2}) = W_i$.

Consider next the allocation under the auction. A player's expected utility when he does not control the decision equals $\frac{\alpha_i}{2}$. Player i's gain from control is then $W_i = \alpha_i(\nu_i - \frac{1}{2})$. Since this valuation is well defined and independent of any other factors, the game is now equivalent to a private-benefits ascending-price auction with complete information.

I now claim that in any equilibrium of the ascending-price auction specified above, the control right will be allocated to a player with the highest W_i, which

23. Of course, the other player can immediately stop the clock again and quit. When, however, the two of them are the only ones left, then this is how it gets determined who gets the control right and at what price.

I will denote as player $P_{\tilde{i}}$. Assume, by contradiction, that that were not the case. Then there is some player P_j with $W_j < W_{\tilde{i}}$ who gets control. It follows that all but P_j must drop out of the auction before some price $\tilde{p} \leq W_j$, and thus get zero (since otherwise player j has negative expected utility and would prefer to drop out at, e.g., $p = 0$). But then $P_{\tilde{i}}$ can improve his utility by not dropping out before $\tilde{p}$ but, instead, stay in until $p = W_{\tilde{i}} > W_j \geq \tilde{p}$ and then stop the clock and quit. Now any player P_i with $W_i < W_{\tilde{i}}$ will want to drop out at some price $p < W_{\tilde{i}}$, which leads to a contradiction.

To show that an equilibrium actually exists, it is straightforward to check that the following is an equilibrium: each player i quits when the price reaches W_i. In that case, the player with the highest W_i gets indeed control, at a price equal to the second-highest W_j. It also follows that $\tilde{\boldsymbol{B}} = \hat{\boldsymbol{B}}$. This completes the proposition. ■

References

Acemoglu, Daron, Victor Chernozhukov, and Muhamet Yildiz. 2006. "Learning and Disagreement in an Uncertain World," MIT Department of Economics Working Paper No. 6–28.

Aghion, Philippe and Jean Tirole. 1997. "Formal and Real authority in Organizations," 105(1) *Journal of Political Economy* 1–29.

Aghion, Philippe, and Patrick Bolton. 1992. "An Incomplete Contracts Approach to Financial Contracting," 59 *Review of Economic Studies* 473–94.

Aghion, Philippe, Mathias Dewatripont, and Patrick Rey. 2004. "Transferable Control," 2(1) *Journal of the European Economic Association* 115–38.

Alonso, Ricardo, and Niko Matouschek. 2007. "Relational Delegation," 38 *Rand Journal of Economics* 1070–89.

Arrow, Ken J. 1964. "The Role of Securities in the Optimal Allocation of Risk-bearing," 31 *Review of Economic Studies* 91–6.

Aumann, Robert J. 1976. "Agreeing to Disagree," 4 *Annals of Statistics* 1236–9.

Baker, George, Robert Gibbons, and Kevin J. Murphy. 2004. "Strategic Alliances: Bridges Between 'Islands of Conscious Power'," Working Paper.

———. 2006. "Contracting for Control," Working Paper.

Boot, Arnoud W.A., Radhakrishnan Gopalan, and Anjan V. Thakor. 2006. "The Entrepreneur's Choice Between Private and Public Ownership," 61 *Journal of Finance* 803–36.

Brunnermeier, Markus K., and Jonathan A. Parker. 2005. "Optimal Expectations," 95 *American Economic Review* 1092–118.

Chen, Joseph Si, Harrison Hong, and Jeremy C. Stein. 2002. "Breadth of Ownership and Stock Returns," 66 *Journal of Financial Economics* 171–205.

Coase, R.H. 1937. "The Nature of the Firm," 4 *Economica* 386–405.

Cooper, Arnold C., William C. Dunkelberg, and Carolyn Yauyan Woo. 1988. "Entrepreneurs' Perceived Chances for Success," 3 *Journal of Business Venturing* 97–108.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. "Investor Psychology and Security Market Under- and Overreactions," 53 *Journal of Finance* 1839–85.

Dessein, Wouter. 2002. "Authority and Communication in Organizations," 69 *Review of Economic Studies* 811–38.

———. 2005. "Information and Control in Ventures and Alliances," 60 *Journal of Finance* 2513–50.

Diaconis, Persi, and David Freedman. 1986. "On the Consistency of Bayes Estimates," 14(1) *The Annals of Statistics* 1–26.

Donaldson, Gordon, and Jay W. Lorsch. 1983. *Decision Making at the Top: The Shaping of Strategic Direction.* New York: Basic Books.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales. 2006. "Does Culture Affect Economic Outcomes?," 20(2) *Journal of Economic Perspectives* 23–48.

Harris, Milt, and Artur Raviv. 1993. "Differences of Opinion Make a Horse Race," 6 *Review of Financial Studies* 473–506.

Harrison, Michael, and David M. Kreps. 1978. "Speculative Investor Behavior in a Stock Market with Heterogenous Expectations," 92 *Quarterly Journal of Economics* 323–36.

Hart, Oliver. 1995. *Firms, Contracts, and Financial Structure.* Oxford: Clarendon Press.

Hart, Oliver, and Bengt Holmstrom. 2002. "A Theory of Firm Scope," Working Paper.

Hart, Oliver, and John Moore. 1990. "Property Rights and the Nature of the Firm," 98 *Journal of Political Economy* 1119–58.

———. 2005. "On the Design of Hierarchies: Coordination Versus Specialization," 113 *Journal of Political Economy* 675–702.

Hong, Harrison, and Jeremy C. Stein. 2007. "Disagreement and the Stock Market," 21 *Journal of Economic Perspectives* 109–128.

Kremer, Michael. 1993. "The O-Ring Theory of Economic Development," 108 *Quarterly Journal of Economics* 551–75.

Landier, Augustin, and David Thesmar. 2007. "Financial Contracting with Optimistic Entrepreneurs," *Review of Financial Studies.*

Leland, Hayne E. 1980. "Who Should Buy Portfolio Insurance?," 35 *Journal of Finance* 581–94.

Lerner, Josh, and Robert P. Merges. 1998. "The Control of Technology Alliances: An Empirical Analysis of the Biotechnology Industry," 46 *Journal of Industrial Economics* 125–56.

Lord, C., L. Ross, and M. Lepper. 1979. "Biased Assimilation and Attitude Polarization: The Effects of Prior Theories on Subsequently Considered Evidence," 37 *Journal of Personality and Social Psychology* 2098–109.

Marino, Anthony M., and John G. Matsusaka. 2005. "Decision Processes, Agency Problems, and Information: An Economic Analysis of Capital Budgeting Procedures," 18 *Review of Financial Studies* 301–25.

Marschak, Jacob, and Roy Radner. 1972. *Economic Theory of Teams.* New Haven, CT: Yale University Press.

McHoskey, J.W. 1995. "Case Closed? On the John F. Kennedy Assassination: Biased Assimilation of Evidence and Attitude Polarization," 17 *Basic and Applied Social Psychology* 395–409.

Milgrom, Paul, and John Roberts. 1994. "Comparing Equilibria," 84 *American Economic Review* 441–59.

Morris, Stephen. 1994. "Trade with Heterogeneous Prior Beliefs and Asymmetric Information," 62 *Econometrica* 1327–47.

———. 1995. "The Common Prior Assumption in Economic Theory," 11 *Economics and Philosophy* 227–53.

———. 1997. "Risk, Uncertainty and Hidden Information," 42 *Theory and Decision* 235–69.

Plous, S. 1991. "Biases in the Assimiliation of Technological Breakdowns: Do Accidents Make us Safer?," 21 *Journal of Applied Social Psychology* 1058–82.

Prendergast, Canice. 2002. "The Tenuous Trade-off between Risk and Incentives," 110 *Journal of Political Economy* 1071–102.

Rosen, Sherwin. 1982. "Authority, Control, and the Distribution of Earnings," 13 *Bell Journal of Economics* 311–23.

Schein, Edgar H. 1985. *Organizational Culture and Leadership.* San Francisco, CA: Jossey-Bass Publishers.

Scheinkman, Jose A., and Wei Xiong. 2003. "Overconfidence and Speculative Bubbles," 111 *Journal of Political Economy* 1183–219.

Van den Steen, Eric J. 2004. "Rational Overoptimism (and Other Biases)," 94 *American Economic Review* 1141–51.

———. 2006a. "Disagreement and the Allocation of Control," MIT Sloan Research Paper No. 4610−06.

———. 2006b. "The Limits of Authority: Motivation versus Coordination," MIT Sloan Research Paper No. 4626−06.

———. 2007a. "The Costs of Incentives Under Disagreement: Too Motivated?" MIT Sloan mimeo.

———. 2007b. "Interpersonal Authority in a Theory of the Firm," MIT Sloan Research Paper No. 4667–07.
Varian, Hal R. 1989. "Differences of Opinion in Financial Markets," in Courtenay S. Stone, ed., *Financial Risk: Theory, Evidence, and Implications*, 3–37. New York: Springer.
Whinston, Michael. 2003. "On the Transaction Cost Determinants of Vertical Integration," 19(1) *Journal of Law, Economics, and Organization* 1–23.
Williamson, Oliver. 1985. *The Economic Institutions of Capitalism.* New York: Free Press.
Wilson, Robert. 1968. "The Theory of Syndicates," 36(1) *Econometrica* 119–32.
Yildiz, Muhamet. 2003. "Bargaining Without a Common Prior—An Immediate Agreement Theorem," 71 *Econometrica* 793–811.
———. 2004. "Waiting to Persuade," 119 *Quarterly Journal of Economics* 223–48.
Záboník Ján. 2002. "Centralized and Decentralized Decision-Making in Organizations," 20 *Journal of Labour Economics* 1–22.