## Spin Diffusion NMR For Distance Determination

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## Spin Diffusion Methods in SSNMR

- ${ }^{1} \mathrm{H}$-driven X-spin isotropic spin diffusion:
- no ${ }^{1} \mathrm{H}$ decoupling (PDSD)
- with ${ }^{1} \mathrm{H}$ decoupling, $\omega_{1}=\omega_{\mathrm{r}}$ (DARR/RAD)

- ${ }^{1} \mathrm{H}$-driven X -spin anisotropic spin diffusion: CODEX

-> distances between chemically equivalent but orientationally inequivalent spins.
- Direct ${ }^{1} \mathrm{H}$ spin diffusion:
- With ${ }^{1} \mathrm{H}$ evolution and X -spin detection

- With X-spin evolution and $X / Y$ detection (XHHY)
-> lipid-protein distances ~<20 Å.



## Oligomeric Structure From Anisotropic Spin Diffusion



Goal: determine the intermolecular packing and distances of oligomeric protein assemblies.


- The sequence detects reorientations due to either slow motion or spin diffusion. Can distinguish the two by:
- varying temperature to affect motion, or
- varying ${ }^{1} \mathrm{H}$ decoupling during $\mathrm{t}_{\mathrm{m}}$ to affect spin diffusion.
- Mechanism of spin diffusion: dipolar coupling -> distance determination.

As $\mathrm{t}_{\mathrm{m}} \rightarrow 0, \mathrm{~S} / \mathrm{S}_{0} \rightarrow 1 / n$, where $n$ is the number of orientationally inequivalent sites.
$\rightarrow$ spin counting

short $t_{m}$
$n=3$


## Spin Counting: ${ }^{13} \mathrm{C}$ CODEX



shortest C-C distance: 4.22 A


C-C distances: $4.17 \AA, 5.23 \AA$

Buffy et al, JACS, 2004, 127, 4477 (2005).

## ${ }^{19}$ F Spin Diffusion: Faster than ${ }^{13} \mathrm{C}$

F-F coupling is 14 -fold stronger than C-C coupling for the same distance.


CODEX Decay Trajectory: Rate Matrix Approach

- For spin diffusion among $n \times$ spins, the time-evolution of the $n$-dimensional vector of the $z$ magnetization, $M(t)$, is given by the differential equation:

$$
\begin{aligned}
& \frac{d \vec{M}(t)}{d t}=-K \vec{M}(t) \quad\binom{d M_{1}(t) / d t}{d M_{n}(t) / d t}=\underbrace{\left(\begin{array}{ccc}
k_{11} & \ldots & k_{1 n} \\
\ldots & \ldots & \ldots \\
k_{n 1} & \ldots & k_{n n}
\end{array}\right)}_{K}\binom{M_{1}(t)}{\left(M_{n}(t)\right.} \\
& \text {-M(t)=M}(t)-M(0) .
\end{aligned}
$$

- $\mathrm{K}: n$-D exchange matrix of rate constants $\mathrm{k}_{\mathrm{ij}}$.
- $T_{1}$ relaxation not included since it's removed by

$$
\mathrm{k}_{\mathrm{ij}}=0.5 \pi \cdot \omega_{\mathrm{ij}}^{2} \cdot \mathrm{~F}_{\mathrm{ij}}(0)
$$ the CODEX control $\mathrm{S}_{0}$.

- Detailed balance of equilibrium M requires:
- the sum of each column of the K matrix is zero, $\mathrm{k}_{\mathrm{ii}}=-\sum_{\mathrm{j} \neq \mathrm{i}} \mathrm{k}_{\mathrm{ji}}$

$$
\begin{aligned}
& \frac{d M_{1}}{d t}+\ldots+\frac{d M_{n}}{d t}=0 \rightarrow\left(k_{11} M_{1}+\ldots+k_{1 n} M_{n}\right)+(\ldots)+\left(k_{n 1} M_{1}+\ldots+k_{n n} M_{n}\right)=0 \rightarrow \\
& \left(k_{11}+k_{21} \ldots+k_{n 1}\right) M_{1}+(\ldots)+\left(k_{n 1}+k_{n 2} \ldots+k_{n n}\right) M_{n} \equiv 0 \\
& \Rightarrow k_{11}+k_{21 \ldots} \ldots k_{n 1}=0, \quad \ldots k_{n 1}+k_{n 2} \ldots+k_{n n}=0 \rightarrow-\sum_{j \neq i} k_{j i}=k_{i i}
\end{aligned}
$$

$$
\mathbf{K}=\left(\begin{array}{ccc}
k_{11} & \ldots & k_{1 n} \\
\ldots & \ldots & \ldots \\
k_{n 1} & \ldots & k_{n n}
\end{array}\right) \quad \mathrm{k}_{\mathrm{ij}}=0.5 \pi \cdot \omega_{\mathrm{ij}}^{2} \cdot \mathrm{~F}_{\mathrm{ij}}(0)
$$

- $\mathrm{k}_{\mathrm{ij}}=\mathrm{k}_{\mathrm{ji}}$ for equal populations of equilibrium M .
- Thus sum of each row is also zero.
- e.g. 4-spin K matrix:

$$
\mathbf{K}=\left(\begin{array}{cccc}
k_{A B}+k_{A C}+k_{A D} & -k_{B A} & -k_{C A} & -k_{D A} \\
-k_{A B} & k_{B A}+k_{B C}+k_{B D} & -k_{C B} & -k_{D B} \\
-k_{A C} & -k_{B C} & k_{C A}+k_{C B}+k_{C D} & -k_{D C} \\
-k_{A D} & -k_{B D} & -k_{C D} & k_{D A}+k_{D B}+k_{D C}
\end{array}\right)
$$

- The rate matrix includes both direct and relayed transfer effects. e.g. magn. transfer from $A$ to $C$ : $-\mathrm{k}_{\mathrm{AC}},-\mathrm{k}_{\mathrm{AB}}$ and $-\mathrm{k}_{\mathrm{BC}}$.
- CODEX is a natural method to measure distances in inherently multi-spin environments, among spins of the same identity but in different molecules $\rightarrow$ intermolecular distance constraints in oligomeric assemblies.


## CODEX Decay to Equilibrium Value

- The solution to the differential equation of $M(t)$ is:

$$
\overrightarrow{\mathrm{M}}(\mathrm{t})=\mathrm{e}^{-\mathrm{Kt}} \overrightarrow{\mathrm{M}}(0)
$$

- The exponential operator can be treated by diagonalization of $\mathbf{K}$ or calculated in a matrix-based software. Expressed in terms of the diagonalized exchange matrix $\Lambda=\mathbf{U K} \mathbf{U}^{-1}\left(\mathbf{K}=\mathbf{U}^{-1} \Lambda \mathbf{U}\right)$ where $\mathbf{U}$ is the eigenvector matrix of $\mathbf{K}$,
$\vec{M}(t)=e^{-K t} \cdot \vec{M}(0)=e^{-\left(U \Lambda U^{-1}\right) t} \vec{M}(0)=\left(U e^{-\Lambda t} U^{-1}\right) \vec{M}(0)=U\left(\begin{array}{ccc}e^{-\lambda_{1} t} & 0 & 0 \\ 0 & \cdots & 0 \\ 0 & 0 & e^{-\lambda_{n} t}\end{array}\right) U^{-1} \cdot \vec{M}(0)$
- For an $n$-D matrix (for $n$ spins) with zero-sum columns, one eigenvalue is always zero with the eigenvector of $\left(\begin{array}{lll}1 / \sqrt{n} & \ldots & 1 / \sqrt{n}\end{array}\right)^{\top}$, while all other eigenvalues are positive.

Proof:

$$
\mathbf{K} \cdot\left(\begin{array}{l}
1 / \sqrt{n} \\
\cdots \\
1 / \sqrt{n}
\end{array}\right)=\sum_{n} K_{m n} \cdot \frac{1}{\sqrt{n}}=\frac{1}{\sqrt{n}} \underbrace{\sum_{n} \mathrm{~K}_{m n}}_{\text {sum over row }} \xrightarrow{\substack{\sum_{m n} \mathrm{~K}_{m n}=0}}=\frac{1}{\sqrt{n}} \cdot 0=0 \cdot\left(\begin{array}{c}
1 / \sqrt{n} \\
\cdots \\
1 / \sqrt{n}
\end{array}\right)
$$

- Thus, at long mixing times $\mathrm{t}_{\mathrm{m}}$,

$$
\left.\begin{array}{l}
\vec{M}(t)=\left(U^{-\Lambda t} U^{-1}\right) \cdot \vec{M}(0) \quad \Rightarrow \\
\vec{M}\left(t \gg \frac{1}{\lambda_{i}}\right)=\sum_{i=1}^{n} \vec{M}(0) \cdot\left(\vec{u}_{i} \cdot e^{-\lambda_{i} t} \cdot \vec{u}_{i}^{-1}\right)=\sum_{i=1}^{n-1} \vec{M}(0) \cdot\left(\vec{u}_{i} \cdot e^{-\infty} \cdot \vec{u}_{i}^{-1}\right.
\end{array}\right)+\vec{M}(0) \cdot\left(\begin{array}{c}
1 / \sqrt{n} \\
\ldots \\
1 / \sqrt{n}
\end{array}\right) e^{-0 \cdot t\left(\begin{array}{c}
1 / \sqrt{n} \\
\cdots \\
1 / \sqrt{n}
\end{array}\right)} \begin{aligned}
& =0+\left(\begin{array}{llll}
0 & \ldots & 1 & \ldots 0
\end{array}\right)\left(\begin{array}{l}
1 / \sqrt{n} \\
\cdots \\
1 / \sqrt{n}
\end{array}\right) \cdot 1 \cdot\left(\begin{array}{l}
1 / \sqrt{n} \\
\cdots \\
1 / \sqrt{n}
\end{array}\right)=1 / \sqrt{n} \cdot\binom{1 / \sqrt{n}}{1 / \sqrt{n}}=\left(\begin{array}{c}
1 / n \\
\cdots \\
1 / n
\end{array}\right)
\end{aligned}
$$

$\mathrm{M}\left(\mathrm{t} \gg \frac{1}{\lambda_{\mathrm{i}}}\right)=(1 / n, 1 / n, \ldots 1 / n) \quad$ Complete equilibration of CODEX magnetization.

## Rate Constant and Overlap Integral

- In the rate constant expression: $\mathrm{k}_{\mathrm{ij}}=0.5 \pi \cdot \omega_{\mathrm{ij}}^{2} \cdot \mathrm{~F}_{\mathrm{ij}}(0) \quad \omega_{\mathrm{ij}}=\frac{\mu_{0}}{4 \pi} \frac{\gamma^{2} \hbar}{r_{\mathrm{ij}}^{3}} \frac{\left(1-3 \cos ^{2} \theta_{\mathrm{ij}}\right)}{2}$
- The angular term, $\left(1-3 \cos ^{2} \theta_{\mathrm{ij}}\right)$ depends on the powder angles of the molecules in the $B_{0}$ field. The square of $\omega_{\mathrm{ij}}$ can be simplified by its powder-averaged value, 0.8.

Main adjustable parameter in the $\omega_{\mathrm{ij}}$ extraction: $\mathrm{F}_{\mathrm{ij}}(0)$

- Overlap integral: probability that SQ transitions occur at the same frequency for spins $i$ and $j$ :

$$
F_{i j}(0)=\int_{-\infty}^{+\infty} f_{i}\left(\omega-\omega_{i}\right) f_{j}\left(\omega-\omega_{j}\right) d \omega
$$

- $\mathrm{f}_{\mathrm{i}}\left(\omega-\omega_{\mathrm{i}}\right)$ : normalized SQ lineshape of spin i without ${ }^{1} \mathrm{H}$ decoupling.
- $\omega_{i}$ : center of the lineshape.
- $\mathrm{F}_{\mathrm{ij}}(0)$ : reflects the overlap area of two ${ }^{1} \mathrm{H}$ undecoupled SQ lines, and is related to the normalized $Z Q$ lineshape at 0 frequency.
- The larger the $F_{i j}(0)$, the faster the decay, the larger the spin diffusion rate $\mathrm{k}_{\mathrm{ij}}$, and the smaller the decay constant $\tau_{\mathrm{SD}}$.
- $\mathrm{F}_{\mathrm{ij}}(0)$ has the unit of time ( s ).


## Overlap Integral

$$
F_{i j}(0)=\int_{-\infty}^{+\infty} f_{i}\left(\omega-\omega_{i}\right) f_{j}\left(\omega-\omega_{j}\right) d \omega
$$

- $F_{i j}(0)$ depends on the
- isotropic shift difference
- anisotropic chemical shift
- $\mathrm{X}-{ }^{-1} \mathrm{H}$ dipolar coupling
- ${ }^{1} \mathrm{H}-{ }^{1} \mathrm{H}$ dipolar coupling
- Spinning speed
- For singly labeled systems, can approximate $\mathrm{F}_{\mathrm{ij}}(0)$ as the same for all intermolecular spin pairs ij .
- The rate constant $\mathrm{k}_{\mathrm{ij}}=0.5 \pi \cdot \omega_{\mathrm{ij}}^{2} \cdot \mathrm{~F}_{\mathrm{ij}}(0)$ was developed for ${ }^{1} \mathrm{H}$-driven X -spin diffusion. But it has been used to analyze direct ${ }^{1} \mathrm{H}$ spin diffusion as well, and on small molecule compounds it gives good agreement with the


Zero-quantum spectrum


Single-quantum spectrum
 crystal-structure distances.

## Determining $\mathrm{F}(0)$ from Model Compounds

For small-molecule compounds, need to consider distances over a number of unit cells.

$$
\omega_{i j}^{2} \longrightarrow \sum_{m, n} \omega_{i_{m} \cdot j_{n}}^{2} \begin{aligned}
& \text { second moment } \\
& \text { coupling }
\end{aligned}
$$

shortest distance between spin pair dipolar coupling square $\omega_{u}{ }^{2}$



Smallest RMSD?
$F(0)$ value is determined


## $F(0)$ of ${ }^{13} \mathrm{C}$ CODEX

13C'-Leu


Spinning speed dependence of $F(0)$



- At 5 kHz MAS, $F(0) \approx 80 \mu \mathrm{~s}$.
- Faster spinning reduces $F(0)$-> slower spin diffusion.
- $F(0) \sim\left(1 / v_{r}\right)^{0.5-1}$.


## $F(0)$ of ${ }^{19} F$ CODEX

## 5-19F-Tryptophan


nearest neighbor: $1.1 \mathrm{kHz}(4.6 \AA$ ) second moment: $1.6 \mathrm{kHz}(4.0 \AA$ )



4-19F-2'-nitroacetanilide

nearest neighbor: $70 \mathrm{~Hz}(11.5 \AA$ ) second moment: 470 Hz ( $6.1 \AA$ Å)



Consensus ${ }^{19} \mathrm{FF}(0)$ at 8 kHz MAS is $37 \mu \mathrm{~s}$.
$\mathrm{k}_{\mathrm{ij}}=0.5 \pi \cdot \omega_{\mathrm{ij}}^{2} \cdot \mathrm{~F}_{\mathrm{ij}}(0) \propto \mathrm{F}_{\mathrm{ij}}(0) / r^{6} \Rightarrow \mathrm{k}$ is much less sensitive to $\mathrm{F}(0)$ than r .

## M2-TMP: a Tetrameric $\mathrm{H}^{+}$Channel in the Membrane



Ala30 -> [4-19F] Phe30, $P: L=1: 15, D M P C$ bilayers, 240 K, 8 kHz MAS







Luo \& Hong, JACS, 128, 7242 (2006)

## Other Practical Aspects of CODEX for Oligomeric Structure Determination

- Symmetric oligomers: only one unknown distance in the $\mathbf{K}$ matrix.

$$
\begin{aligned}
& \text { e.g. } k_{A B}=k_{A D}=0.5 \pi F(0) \cdot \omega(r)^{2} \\
& k_{A C}=0.5 \pi F(0) \cdot \omega(\sqrt{2 r})^{2}=\frac{1}{2^{3 / 2}} 0.5 \pi F(0) \cdot \omega(r)^{2}
\end{aligned}
$$



- Asymmetric oligomers: multiple distances unknown. Unclear whether the CODEX curve can yield multiple distances. The rigorous approach: measure multiple distances to avoid under-determining the problem.
- With ${ }^{19} \mathrm{~F}-{ }^{19} \mathrm{~F}$ dipolar coupling, the maximum distance detected in model compounds is $\sim 15 \AA$.
- Phenylene ring $4-{ }^{19}$ F position insensitive to ring flip: good for distance expts.
- $\mathrm{CF}_{3}$ labels not recommended: fast ${ }^{19} \mathrm{~F} \mathrm{~T}_{1}$ relaxation during $\mathrm{t}_{\mathrm{m}}$.
- Other aromatic ${ }^{19} \mathrm{~F}$-labels for proteins: $5-{ }^{19} \mathrm{~F}$-Trp, 6-19F-Trp.
- Large ${ }^{19}$ F CSA is sensitive to small-angle differences between two molecules.
E.g. $\delta \approx 55 \mathrm{ppm}$ for $4-{ }^{19}$ F-Phe; at $9.4 \mathrm{~T}, \delta \approx 20 \mathrm{kHz}$. With $\mathrm{Nt}_{\mathrm{r}}=250 \mu \mathrm{~s}, 2 \pi \delta \mathrm{Nt}_{\mathrm{r}} \approx$ $10 \pi$, sensitive to $10^{\circ}$ orientation differences between molecules.
- Need to ensure no slow motion is present at the desired temperature.


## ${ }^{1} \mathrm{H}$ Spin Diffusion of Membrane Proteins



Purposes:

- Protein distance to the membrane center.
- Protein distance to the membrane surface.
- Main features:
- Undecoupled ${ }^{1} \mathrm{H} \mathrm{T}_{2}$ filter before $\mathrm{t}_{1}$ selects mobile components.
- ${ }^{1} \mathrm{H}$ undecoupled $\mathrm{t}_{1}$ evolution further suppresses rigid components.
- direct ${ }^{1} \mathrm{H}$ spin diffusion, mobile $->$ rigid transfer.
- $X$ spin detection can be ${ }^{13} \mathrm{C},{ }^{15} \mathrm{~N},{ }^{31} \mathrm{P}$, etc.
- Application modes:
- ambient temp. (LC phase): lipid (L) -> protein (P) transfer,
- $2 \tau \sim 2 \mathrm{~ms}$
- $\mathrm{t}_{\mathrm{m}} \sim[10 \mathrm{~ms}, 10 \mathrm{~s}]$
- mainly 2D (can also be 1D), to resolve multiple mobile ${ }^{1} \mathrm{H}$ signals.
- low temp. (gel phase): water (W) -> protein transfer,
- $2 \tau \sim 0.2 \mathrm{~ms}$
- $\mathrm{t}_{\mathrm{m}} \sim[0.1 \mathrm{~ms}, 25 \mathrm{~ms}]$
- 1D, no ${ }^{1} \mathrm{H}$ evolution needed (only water remains).


## LC Phase ${ }^{1} \mathrm{H}$ Spin Diffusion: Intensity Buildup Reflects Minimum L-P Distance



- ${ }^{1} \mathrm{H}$ spectrally resolved mobile components in a membrane sample:
- $\mathrm{H}_{2} \mathrm{O}: \mathrm{S} \sim 0.03$
- lipid $\mathrm{CH}_{3}: \mathrm{S} \sim 0.02-0.04$
- lipid $\left(\mathrm{CH}_{2}\right)_{\mathrm{n}}$ : S ~ 0.08-0.20
- lipid $\mathrm{H} \gamma$ : S very small
- If the protein is mostly immobile, $S>\sim 0.7$, then spin diffusion is slow within the soft lipid matrix and water, and rapid within the protein.
- A rate-limiting step in the L/W $->P$ transfer is transfer across the intermolecular interface due to translational and rotational diffusion of L/W.
- Once intermolecular transfer occurs, ${ }^{1} \mathrm{H}$ magnetization equilibrates in the protein in $\leq 1 \mathrm{~ms}$ ( $\sim \mathrm{CHHC}$ ), obliterating distance resolution for typical $\mathrm{t}_{\mathrm{m}}$ values of $\sim 100 \mathrm{~ms}$ and higher.
- Buildup curve (Intensity vs $\sqrt{\mathrm{t}_{\mathrm{m}}}$ ) reflects the shortest distance from the source spin to the protein $\rightarrow$ qualitative information of protein topology.
- It doesn't matter where the ${ }^{13} \mathrm{C} /{ }^{15} \mathrm{~N}$ label is in the protein.


## 2D Data and Buildup Curves

Colicin la channel domain in POPC/POPG membrane


Transmembrane model


Huster et al, JACS, 124, 874 (2002)

Surface model


## Distances from Linear-Chain Spin Diffusion Calculation

- General 1D diffusion equation (Fick's $2^{\text {nd }}$ Law): $\frac{\partial \mathrm{M}}{\partial \mathrm{t}}=\mathrm{D} \cdot \frac{\partial^{2} \mathrm{M}}{\partial \mathrm{x}^{2}}$ (Nature abhors a wrinkle.)
- On a discrete 1D lattice (along the bilayer normal):

$$
\begin{aligned}
\frac{\Delta \mathrm{M}_{\mathrm{i}}}{\Delta \mathrm{t}} & =\mathrm{D} \cdot \frac{1}{a^{2}}\left[\left(\mathrm{M}_{\mathrm{i}+1}-\mathrm{M}_{\mathrm{i}}\right)-\left(\mathrm{M}_{\mathrm{i}}-\mathrm{M}_{\mathrm{i}-1}\right)\right] \\
& =\Omega\left(-2 \mathrm{M}_{\mathrm{i}}+\mathrm{M}_{\mathrm{i}+1}+\mathrm{M}_{\mathrm{i}-1}\right)
\end{aligned}
$$

D: diffusion coefficient ( $\mathrm{nm}^{2} / \mathrm{ms}$ )
$\Omega$ : transfer rate $=\mathrm{D} / a^{2}$
$a$ : lattice spacing, 2 Å or 1 Å


- $\Omega$ or $D$ is related to the ${ }^{1} \mathrm{H}-{ }^{1} \mathrm{H}$ dipolar couplings. In rigid polymers, $D \approx 0.8$ $\mathrm{nm}^{2} / \mathrm{ms}$ has been measured, equivalent to $\Omega \approx 20 \mathrm{kHz}$ for two protons $2.0 \AA$ apart.
- Two lipid vicinal protons are $\sim 2.4 \AA$ apart (rigid-limit $\delta=8.8 \mathrm{kHz}$ ).
- Using a $S \approx 0.04$ for protons close to the acyl chain termini, the motionally averaged ${ }^{1} \mathrm{H}-{ }^{1} \mathrm{H}$ coupling is $\Omega \approx 350 \mathrm{~Hz}$.
- With a spacing $a$ of $2 \AA$, the resulting $\mathrm{D}_{\mathrm{L}}=\Omega a^{2}$ is $\sim 0.014 \mathrm{~nm}^{2} / \mathrm{ms}$.
- For proteins with $\mathrm{S} \approx 0.7$, the ${ }^{1} \mathrm{H}-{ }^{1} \mathrm{H}$ coupling is $\Omega(2.4 \AA) \sim 6.0 \mathrm{kHz},=>D_{P} \approx$ $0.25 \mathrm{~nm}^{2} / \mathrm{ms}(a=2 \AA$ Å).
- For interfacial transfer, typical $D_{\text {int }} \sim 0.002 \mathrm{~nm}^{2} / \mathrm{ms}$ (order of magnitude).

| Sample simulation: | D $\left(n m^{2} / m s\right)$ | $r(\AA)$ |
| :--- | :--- | :--- |
| Source - lipid $\mathrm{CH}_{3}:$ | 0.012 | $4 \AA$ |
| Source $-\mathrm{H}_{2} \mathrm{O}:$ | 0.03 | $2 \AA$ |
| Gap: | 0.012 | $x \AA$ |
| Interface: | 0.00125 | $2 \AA$ |
| Sink -peptide | 0.3 | $30 \AA$ |



## Effects of $\mathrm{D}_{\text {int }}$ and Distance on the Buildup Curves

$\mathrm{D}_{\text {int }}$ : the adjustable parameter in the SD simulation. Estimate by reproducing the slope of the experimental buildup curve.
$D_{L}=0.0125 \mathrm{~nm}^{2} / \mathrm{ms}$
$D_{p}=0.3 \mathrm{~nm}^{2} / \mathrm{ms}$
$\mathrm{D}_{\text {int }}\left(\mathrm{nm}^{2 / m s}\right)$

- 0.0125
- 0.0025
- 0.00125
- 0.00025
- 0.000125


- $\mathrm{D}_{\text {int }}$ mainly changes the slope of the buildup curve.
- $r$, lipid-protein distance, mainly changes the initial lag of the buildup curve.
- Empirically, phospholipid-protein mixtures have $\mathrm{D}_{\text {int }} \sim 0.0025 \mathrm{~nm}^{2} / \mathrm{ms}$, lipidDNA have $\mathrm{D}_{\text {int }} \sim 0.00025 \mathrm{~nm}{ }^{2} / \mathrm{ms}$ (low ${ }^{1} \mathrm{H}$ density in DNA), and cholesterolcontaining membranes also give $D_{\text {int }} \sim 0.00025 \mathrm{~nm}^{2} / \mathrm{ms}$.


## Origin of the $t^{1 / 2}$ Dependence of Intensity Buildup

- For a point source at $x_{0}, M(x, 0)=\delta\left(x-x_{0}\right)$, the solution of the diffusion equation $\partial M / \partial t=D \cdot \partial^{2} M / \partial x^{2}$ is a Gaussian function of $x, M(x, t)=e^{-\left(x-x_{0}\right)^{2} / 4 D t} / \sqrt{\pi D t}$.
- A domain source $M_{\text {dom }}(x, t)$ is a superposition of many point sources:

$$
M_{d o m}(x, 0)=\int_{-\infty}^{0} \delta\left(x-x_{0}\right) d x_{0}
$$

- $M_{\text {dom }}(x, t)$ evolves as an error function centered at the source-sink interface:
$M_{d o m}(x, t)=\int_{-\infty}^{0} \frac{e^{-\left(x-x_{0}\right)^{2} / 4 D t}}{\sqrt{\pi D t}} d x_{0} \xrightarrow{x^{\prime}=\frac{x-x_{0}}{4 D t}}\left(\begin{array}{l}x_{0}=-\infty, x^{\prime}=+\infty \\ x_{0}=0, x^{\prime}=x / \sqrt{4 D t} \\ d x^{\prime}=-d x_{0} / \sqrt{4 D t}\end{array}\right)$
$=\frac{-\sqrt{4 \mathrm{Dt}}}{\sqrt{\pi \mathrm{Dt}}} \cdot \int_{+\infty}^{\mathrm{x} / \sqrt{4 \mathrm{Dt}}} \mathrm{e}^{-\mathrm{x}^{\prime 2}} \mathrm{dx}^{\prime}=\frac{2}{\sqrt{\pi}} \cdot \int_{\mathrm{x} / \sqrt{4 \mathrm{Dt}}}^{+\infty} \mathrm{e}^{-\mathrm{x}^{\prime 2}} \mathrm{dx}^{\prime}=\operatorname{erfc}\left(\frac{\mathrm{x}}{\sqrt{4 \mathrm{Dt}}}\right)$
- The total magn $I_{\text {sink }}(t)$ of the sink increases as $t^{1 / 2}$ :

$$
\begin{aligned}
& I_{\text {sink }}(t) \propto \int_{0}^{+\infty} M_{\text {dom }}(x, t) d x=\int_{0}^{+\infty} \operatorname{erfg}\left(\frac{x}{\sqrt{4 D t}}\right) d x \xrightarrow{x^{\prime \prime}=\frac{x}{\sqrt{4 D t}}} \\
& \left(\begin{array}{l}
x=0, x^{\prime \prime}=0 \\
x=+\infty, x^{\prime \prime}=+\infty \\
d x^{\prime \prime}=d x / \sqrt{4 D t}
\end{array}\right)=\sqrt{4 D D} \int_{0}^{+\infty} \operatorname{erfd}\left(x^{\prime \prime}\right) d x^{\prime \prime}=\sqrt{4 D t} \frac{1}{\sqrt{\pi}}=\sqrt{\frac{4 D}{\pi}} \cdot \sqrt{\sqrt{t}}
\end{aligned}
$$



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## Buildup Curves Plotted with Time ${ }^{1 / 2}$ vs Time

- Thus, for domain spin diffusion, the I( $\mathrm{t}^{1 / 2}$ ) plot is linear.
- For point-source spin diffusion, there is a latency period $(M \approx 0)$ whose duration depends on the distance from the point source.
- Plotting $I\left({ }^{1 / 2}\right)$ stretches out the initial period compared to $I(t)$, thus better distinguishing different distances.




## Buildup Curves of Non-Transmembrane Systems

- In membrane systems, spin diffusion is usually from point sources, giving a lag period in the $\mathrm{I}_{\text {sink }}\left(\mathrm{t}^{1 / 2}\right)$ plot. This is especially clear in non-TM macromolecules.

DNA - cationic membrane

POPC/cholesterol membrane with PG-1


## Higher-Sensitivity LC-Phase ${ }^{1} \mathrm{H}$ Spin Diffusion



- 2D HHC:
- Indirect ${ }^{1} \mathrm{H}$ dimension of lipids and water, high resolution, require long $t_{1}$.
- Direct ${ }^{13} \mathrm{C}$ dimension of protein, lower resolution and sensitivity.
- Buildup curves require multiple 2D, long expt time, need careful monitoring of CP stability, sample hydration etc, to obtain reliable curves.

$\tau_{1} \sim 10 \mathrm{~ms}, \tau_{2} \sim 5 \mathrm{~ms}$.
Detection sensitivity gain: $\left(\gamma_{H} / \gamma_{C}\right)^{3 / 2}=8$
- Two obstacles of 1D CHH:
- Suppressing large equilibrium ${ }^{1} \mathrm{H}$ magnetization of lipids \& water.
- Sensitivity gain limited by the fraction of labeled ${ }^{13} \mathrm{C}$ sites versus natural ${ }_{26}$ abundance lipid ${ }^{13} \mathrm{C}$.


## 1D CHH Protein-Lipid Spin Diffusion

TEASE U- ${ }^{13} \mathrm{C},{ }^{15} \mathrm{~N}$-labeled colicin la channel domain in POPC/POPG membrane. $\mathrm{P} / \mathrm{L}=1: 100, \sim 50 \%{ }^{13} \mathrm{C}$ labeling.


Time-saving: 180-350 fold.


1D reproduces the 2D buildup curves.

## Sensitivity of the CHH Spin Diffusion Experiment

- All detected ${ }^{1} \mathrm{H}$ magn originates from the labeled ${ }^{13} \mathrm{C}$ sites $\left(\mathrm{C}_{\mathrm{p}}\right)$ in the protein. So the sensitivity mainly depends on the ${ }^{13} \mathrm{C}$ labeling level.
- Sensitivity also depends on the \% of mobile protons $\left(\mathrm{H}_{\mathrm{L}}+\mathrm{H}_{\mathrm{W}}\right)$ in the sample.
- Assuming complete equilibrium ( $\mathrm{CP}+\mathrm{SD}$ ), the number of detected protons is:

$$
H_{C H H}=C_{P} \times \frac{H_{P}}{H_{P}+C_{P}} \times \frac{H_{L}+H_{W}}{H_{P}+H_{L}+H_{W}}
$$

- The \% detected ${ }^{1} \mathrm{H}$ 's among the total lipid and water protons is $\mathrm{H}_{\mathrm{CHH}} /\left(\mathrm{H}_{\mathrm{L}}+\mathrm{H}_{\mathrm{W}}\right)$
- For a membrane protein sample with mass ratio $P: L: W \approx 1: 3: 2$ and a ${ }^{13} \mathrm{C}$ labeling level of $\sim 50 \%$, the calculated fraction of detected protons is $\sim 2.5 \%$. This gave reproducible and correct CHH buildup curves.
- The experiment needs to suppress $\sim 98 \%$ undesired ${ }^{1} \mathrm{H}$ signals. This is achieved by the $T_{2}$ filter, phase cycling, and a $90^{\circ}$ purge pulse. Suppression of the rigid ${ }^{1} \mathrm{H}$ magn is easy, but of the mobile ${ }^{1} \mathrm{H}$ magn. of the natural abundance lipid ${ }^{13} \mathrm{C}$ is more difficult.
- Empirically, $<0.8 \%$ detected protons causes systematic errors in the buildup curves. Thus, ${ }^{13} \mathrm{C}$ labeling level needs to be $>\sim 15 \%$ for CHH to work.


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## ${ }^{19}$ F Spin Diffusion for Determining Intermolecular Distances in Oligomeric Membrane Proteins



Mei Hong, Iowa State University
Membrane protein structural features:

- Orientation.
- Depth of Insertion.
- Sidechain conformation.
- Assembly of polypeptide chains: quaternary structure.


Oligomeric structure of membrane proteins:

- Oligomeric number
- Intermolecular distance constraints.


## M2 Protein: a Proton Channel of Influenza A Virus



- Forms tetrameric bundles in micelles.
- Oligomeric state in the lipid bilayer unknown. Only one short interhelical distance reported (Cross et al.).


## F-F Distance Confirms the Tetramer Model



Most probable rotamer:
$r=7.5 \AA, F(0)=28 \mu s$


$$
r=18.5 \AA,
$$

$$
F(0)=2000 \mu \mathrm{~s}
$$



Least probable rotamer:
ring clashes with backbone

The interhelical distance of 7.9-9.5 A for Phe30 agrees well with the M2 tetramer model obtained from ${ }^{15} \mathrm{~N}$ orientation data (Cross et al).

## F-F Distance Confirms Existing Tetramer Model




- $+60^{\circ}$ rotamer: forbidden by steric clash with the backbone
- Only the trans rotamer is possible.
helix $i+1$
helix i+2

The inter-helical distance of $7.9-9.5 \AA$ for $F 30$ agrees well with the M2 tetramer model obtained from 15N orientational data.

## Distance Restraint for Helix Orientation



- Functional model may have un-optimized rotation angles.

