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Contact Shift & Dipolar ShiftGeneralHigh fieldIsotropic g
$$\cdot H_{CON} = A < S > \cdot I$$
 $A < S_2 > I_2$  $A < S_2 > I_2$  $(I = I)$  $A < S_2 > I_2$  $(I = I)$  $\cdot H_{PC} = \{<\mu > /g_{\theta}\} \cdot D \cdot I$  $\{(<\mu > /g_{\theta}) \cdot D\}_2 / I_2$  $D < S_2 > I_2$  $(I = I)$  $I = I$  $(<\mu > /g_{\theta}) \cdot D\}_2 / I_2$  $I = I < I$  $- Let's$  obtain  $<\mu > \& < S >$  first.

**Magnetic Moment under Averaging 2 Case 2: A more general case**   $<\mathbf{S} = Tr\{\mathbf{S}\exp(-H/kT)\}/Tr\{\exp(-H/kT)\} \quad \exp(-A) \sim 1-A$   $\sim Tr\{\mathbf{S}(1 - \frac{\mu_B \mathbf{S} \cdot \mathbf{g} \cdot \mathbf{B}_0}{kT} \}/Tr\{1 - \frac{\mu_B \mathbf{S} \cdot \mathbf{g} \cdot \mathbf{B}_0}{kT} \}$   $= \sum_{\varsigma} < \zeta \mid (\sum_{jkl} \mathbf{e}_j S_j)(S_k g_{kl} B_{0l}) \mid \zeta > \mu_B/(kT) Tr\{1\}$ where  $\mathbf{e}_i$  is an unit vector along the axis j (j = x, y, z) and  $|\zeta|$  denotes a basis ket.  $<\mathbf{S} = \sum_{\varsigma} \sum_{jkl} \mathbf{e}_j g_{kl} B_{0l} < \zeta \mid S_j S_k \mid \zeta > \mu_B/(kT) Tr\{1\}$   $= \sum_{jkl} \mathbf{e}_j g_{kl} B_{0l} \{\delta_{jk} S(S+1)/3\} \mu_B/(kT)$   $= \mathbf{g} \cdot \mathbf{B}_0 \{S(S+1)/3\} \mu_B/(kT)$ In the high field approximation for the parameterized g tensor,  $<\mu>$  is given by  $<\boldsymbol{\mu} = \mu_B \mathbf{g} \cdot <\mathbf{S} = \mathbf{g} \cdot \mathbf{g} \cdot \mathbf{g} \cdot \mathbf{B}_0 \{S(S+1)\} \mu_B^2/(3kT) [2.18]$ 



Calculation of Thermally Averaged Contact Shift & Dipolar Shift Case 1: g-anitoropy neglected  $\delta_{CON} = A < S_Z = Ag_e B_0 \{S(S+1)\} \mu_B / (3kT)$  [2.22] Isotropic shift  $\rightarrow$  NOT Removable by MAS  $\delta_{PC} = D(\theta) < S_Z = D(\theta, R)g_e B_0 \{S(S+1)\} \mu_B / (3kT)$ Anisotropic shift  $D(\theta, R) = (1-3\cos^2\theta)/R^3$  $\Rightarrow$  Removable by MAS



Thermally Averaged Hyperfine Shifts Case 2: g-anitoropy NOT neglected  $\delta_{\text{CON}} = A < S_z > I_z$  $= \{A(\boldsymbol{g} \cdot \boldsymbol{B}_0)_Z C / (\mu_B T)\} I_Z \quad \text{This is actually anisotropic}$  $=\frac{AB_0C}{\mu_0T}\{g_{xx}\sin^2\beta+g_{yy}\cos^2\beta\sin^2\alpha+g_{zz}\cos^2\beta\cos^2\alpha\}$ [2.24]  $\delta_{PC} = (\langle \mu \rangle / g_e \cdot D)_z / z_z$  $(\alpha,\beta,\gamma)$  denote Euler angles that define the g-tensor orientation with  $= (C/g_eT)(\boldsymbol{B}_0 \cdot \boldsymbol{g} \cdot \boldsymbol{g} \cdot \boldsymbol{D})_Z I_Z$ [2.25] The tensor  $(\boldsymbol{g} \cdot \boldsymbol{g} \cdot \boldsymbol{D})$  is NOT traceless  $(g^2 \boldsymbol{D}$  is traceless).  $\rightarrow$  This term also includes both anisotropic and isotropic shifts Bertini et al. "Solution NMR of Paramagnetic Molecules" 26 Yesnowski et al JCP 89, 4600 (1988)























































