

**MODELING, VALUATION AND RISK MANAGEMENT
OF ASSETS AND DERIVATIVES
IN ENERGY AND SHIPPING**

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October 2011

ABSTRACT: Derivatives are financial contingent claims designed for the pricing, transfer and management of risk embedded in underlying securities in the fixed income, equity and foreign exchange markets. Their rapid growth spurred their introduction to the energy commodity and shipping markets where the underlying assets are real commodities, crude oil, refined products, natural gas, electricity and shipping tonnage. Risk neutral pricing and stochastic models developed for financial derivatives have been extended to energy derivatives for the modeling of correlated commodity and shipping forward curves and for the pricing of their contingent claims. This has enabled the valuation and risk management of a wide range of assets and derivatives in the energy and shipping markets. They include storage for natural gas, floating storage of crude oil, products and Liquefied Natural Gas in tankers, refineries, power plants and utility scale wind farms, shipping structured securities, cargo vessels and shipping derivatives portfolios.

KEY WORDS: Derivatives, risk neutral valuation, risk management, forward curves, futures, options, energy commodities, shipping, Forward Freight Agreements, freight options, Principal Components Analysis, Canonical Correlation Analysis, hedging, stochastic optimal control, natural gas storage, crude oil floating storage, structured securities, wind energy.

Investments in energy and shipping assets are exposed to interest rate, commodity price and freight rate risks. The management of these risks has led to the introduction and widespread use of derivatives which have experienced explosive growth over the past several decades. In the

fixed income market interest rate futures and futures options emerged in the 1980's in response to the need to hedge interest rate swap risk. This led to the development of financial models for the arbitrage free evolution of the term structure of interest rates and the pricing of a wide range of fixed income derivatives, laying the foundation for the development of analogous models for the arbitrage free evolution of the forward curves of physical commodities including crude oil, its refined products, natural gas and recently shipping freight rates.

Commodity futures settle physically against the price of a spot commodity that must be delivered at the contract expiration or in cash against a spot commodity index. The latter is the case in shipping where Forward Freight Agreements (FFAs) and freight rate futures settle against shipping indices composed of a basket of freight rates. The first generation of commodity futures models was based on the development of stochastic models for the spot index and the use of the principles of risk neutral pricing for the valuation of derivatives written on the spot. Recent models are based on the insight that in the absence of arbitrage futures prices with daily credits and debits into a margin account are martingales. This has enabled the modeling of the evolution of futures prices of any tenor as lognormal diffusions with zero drift in a Gaussian setting. The primary unknown in this model of futures prices is the volatility term structure which may be estimated from market prices of liquid futures and futures options. The arbitrage free price process for the spot commodity or underlying index follows from this martingale representation of the entire commodity forward curve in the limit of small tenors.

The martingale representation of the commodity forward curve lends itself to parsimonious modeling using the powerful statistical techniques of principal components analysis in the case

of a single futures curve and of canonical correlation analysis in the case of multiple correlated forward curves. In both cases a small number of statistical factors may be derived which are shown to follow mean reverting log-Ornstein-Uhlenbeck diffusions. This financial modeling and statistical inference framework of cross-correlated energy commodity and shipping forward curves allows the pricing of a wide range of vanilla, spread and exotic derivatives written on futures contracts. Moreover the model of the forward curve in terms of a small number of statistical factors enables the explicit valuation and hedging of wide range of energy and shipping assets with cash flow exposures that may be replicated by the prices of traded futures contracts.

This chapter reviews the fundamental developments that led to the introduction of financial and commodity derivatives their stochastic modeling and risk neutral pricing. Pricing in Gaussian and non-Gaussian settings is addressed and shown in most cases to lead to closed form results even when the underlying is represented by an advanced stochastic model. The martingale modeling of forward curves and their estimation by a principal components analysis is discussed for the crude oil market, its refined products, natural gas and the shipping market.

The valuation of real assets and financial claims of energy and shipping entities is discussed. The parsimonious form of the arbitrage free factor models of the pertinent forward curves enable the pricing of these assets and securities by risk neutral pricing. Their risk management is also addressed drawing upon the explicit form of the underlying factor model and the techniques of stochastic optimal control which have found widespread use for the management of portfolios of financial securities.

ENERGY COMMODITY PRICE MODELS

The past several decades have witnessed the emergence and rapid development of the fields of financial engineering and derivatives. Grown out of Paul Samuelson's foundational insights on the relationship between informationally efficient markets and the random walk and his introduction of the lognormal diffusion model of security prices, a wide range of stochastic models of security prices and arbitrage free valuation methods were developed for the pricing of derivatives written on financial securities, real assets and other variables [see Samuelson (1965)]. The use of these models and pricing methods in the fixed income, equity, foreign exchange and credit markets is growing as is the complexity of the mathematical, econometric and filtering methods necessary for their implementation. More recently, these methods have been adapted to the energy and shipping sectors in order to control the high volatility of energy prices and freight rates and spur new investment.

Spot Price Models

Energy commodity prices are characterized by idiosyncrasies not encountered in the financial markets. The volatility of the price of oil, natural gas and especially electricity is a lot larger than that of currencies, interest rates and equities. Energy prices often exhibit mean reversion, seasonality, sharp and asymmetric spikes which require the development of advanced price models and derivative valuation methods, extensions of the Black-Scholes-Merton stock option pricing formula. Moreover, a complex interaction often exists between the attributes of the spot physical commodity and its forward contracts and other derivatives, not present for financial

securities and their derivatives which settle electronically and do not require the delivery of a physical asset. This requires the use of the extended risk-neutral valuation method of derivatives written on real assets and other variables that are not tradable [Ross (1976)].

Standard reduced form stochastic models for the spot price of crude oil and natural gas are diffusions which account for mean reversion, seasonality and depend on hidden economic factors. They include a stochastic convenience yield – the implied dividend received by the owner of the commodity held in storage – and a long-term stochastic equilibrium price to which the spot price mean reverts. The two-factor spot price models of Gibson and Schwartz (1990), Schwartz (1997) and Smith and Schwartz (2000) model the spot price and its factors as diffusions and permit the explicit valuation of futures and forward contracts and their options written on the spot commodity using the extended risk-neutral valuation of derivatives written on real assets [Hull (2003)]. More general spot price models that may include stochastic volatility and jumps are discussed in Clewlow and Strickland (2000) and London (2007). In the study of Cortazar and Naranjo (2006) the entire oil futures curve and its volatility term structure are shown to be very well modeled by a four-factor spot price Gaussian model which was estimated by Kalman filtering.

Stochastic models for the evolution of the electricity prices must account for sharp and asymmetric spikes, strong mean reversion, jumps and a dependence on structural factors affecting the electricity market. Reduced form stochastic models of electricity prices are usually jump-diffusions and Levy processes. An example is the jump-diffusion model of Kou (2002) which permits the independent parametric adjustment of the tail thicknesses of its probability

distribution and allows the explicit pricing of electricity derivatives. Other models are discussed in Eydeland and Wolyniec (2003), London (2007) and Bengt, Bengt and Koekebakker (2008). Analogous models apply to the modeling of the spot price process of shipping freight rates.

Forward Curve Models

Crude oil and natural gas have liquid futures contracts trading on the New York Mercantile Exchange (Nymex) and the Intercontinental Exchange (ICE) with tenors of several years. A large market also exists of swaps, forward and options contracts written on energy commodities that trade Over The Counter (OTC). The relation between the prices of forward and futures contracts is discussed by Cox, Ingersoll and Ross (1981). Arbitrage-free forward curve models for energy commodities have been developed by Miltersen and Schwartz (1998) which accept as input the market prices of liquid futures and lead to the pricing of a number of other derivatives. The arbitrage-free evolution of the spot price follows from futures contracts of small tenors.

The modeling of the oil and natural gas futures curve is based on the Heath-Jarrow-Morton (HJM) framework developed for the arbitrage-free modeling of the term structure of interest rates. A principal task of the HJM framework is the parameterization of the volatility and correlation structure of the futures curve by a small number of independent factors using a Principal Components Analysis. This was carried out by Scлавounos and Ellefsen (2009) where it was shown that three Principal Components capture most of the fluctuations of the forward curve. In the same paper the arbitrage free evolution of the spot price was derived as implied in equilibrium by the forward curve and was shown to be driven by three independent factors that follow mean reverting logarithmic Ornstein-Uhlenbeck (log-OU) processes with stochastic drifts.

Calls, puts, swaps, caps and their options written on futures contracts may then be valued explicitly as in the interest rate markets for use in energy risk management applications. [Hull (2003), Musiela and Rutkowski (2008)].

Energy Derivatives

In addition to the standard derivatives discussed above, more complex derivatives have been introduced in the energy markets reflecting the economics of energy assets. In particular, power plants are exposed to the spot/futures price difference of two energy commodities, e.g. natural gas/electricity, coal/electricity, refineries are exposed to the price differentials of two fuels – crude oil/gasoline, crude oil/jet fuel -- and oil and natural gas pipelines and electricity transmission lines are exposed to the price differentials of the same spot commodity at two different geographical locations.

A partial list of exotic derivatives used for the valuation, hedging and risk management of energy assets include options on the spread between two futures contracts with different expirations written on the same commodity, options on the price difference of two futures contracts with the same expiration written on two separate commodities, options to exchange two spot commodities or their futures, average-price and average-strike Asian options, Barrier options which are exercised when the commodity price crosses a threshold and American swing options for the delivery of an uncertain amount of the commodity. A discussion of these and other exotic energy derivatives is presented in Clewlow and Strickland (2000), Eydeland and Wolyniec (2003) and Geman (2005).

Exotic energy derivatives are complex to price and hedge for advanced commodity price models. Furthermore, spread derivatives depend not only on the volatility but also on the correlation between various spot/futures contracts which may be challenging to model and calibrate to market prices. Consequently, the development of accurate stochastic price models and pricing methods for exotic derivatives and spread options may be particularly helpful for the valuation and hedging of energy assets. Accurate analytical approximations of spread options prices and their hedge ratios are derived by Li, Deng and Zhou (2008) for two assets that follow correlated log-OU diffusions. Extensions to multi-asset spread option pricing and hedging are presented in Li, Zhou and Deng (2010).

Shipping Derivatives

The success and rapid growth of derivatives in the energy commodity markets has spurred their introduction in the shipping markets. Shipping derivatives – Forward Freight Agreements (FFAs) and Freight Futures – were introduced in 1985 and are widely used by the dry bulk and tanker shipping markets as discussed by Alizadeh and Nomikos (2009). Freight rate swaps were also recently introduced in the containership markets. The growth of shipping derivatives is also motivated by the correlation of the supply and demand for shipping ton-miles with that of the bulk commodities transported by cargo vessels -- crude oil, refined products, iron ore and coal. An example is the recent introduction of Over The Counter iron ore swaps following the initiation of quarterly pricing of that bulk commodity. Therefore the need arises for the robust statistical modeling of the correlated forward curves of shipping and commodity markets and the pricing of shipping derivatives for use in risk management.

VALUATION AND HEDGING OF DERIVATIVES

The pricing of derivatives written on a financial security, a spot commodity or another variable – the underlying -- may be carried out by using the fundamental principles of risk-neutral valuation. When the underlying is a non-tradable – e.g. temperature – an associated market price of risk process enters in the derivative price which must be estimated from the prices of traded instruments. Otherwise, the fundamental economic insight of risk-neutral pricing and the associated mathematical techniques apply over a wide range of assets and stochastic models used for the modeling of the underlying process.

Vanilla Derivatives for Jump-Diffusions

A standard derivative pricing method for the wide class of jump-diffusion processes is based on the derivation of a risk-neutral probability measure under which European derivative prices may be expressed as conditional expectations of a payoff at a specified horizon [Duffie (2001), Hull (2003), Shreve (2004), Musiela and Rutkowski (2008)]. Derivative prices expressed as conditional expectations may be evaluated explicitly in the form of Fourier integrals of the complex characteristic function of the jump-diffusion by using the methods developed by Heston (1993), Carr and Madan (1998), Duffie, Pan and Singleton (2000) and Lewis (2005). The use of this derivative pricing method in practice for the modeling of the equity implied volatility surface and the calibration of a wide range of jump-diffusion models are discussed in Gatheral (2006).

Derivative prices expressed in the form of Fourier integrals allow the explicit evaluation of the derivative sensitivities known as the Greeks, they permit the analytical derivation of the stochastic process followed by the derivative price itself by using the Ito-Doebelin formula and often allow the explicit pricing of European derivatives with more general payoffs. The evaluation of Fourier integrals may be carried out efficiently by complex contour integration, numerical integration or Fast Fourier Transform techniques.

The valuation of American options for jump-diffusions and the optimal stopping problems that arise when early exercise is permitted is discussed in Oksendal and Sulem (2005). When the use of analytical techniques is not possible for the evaluation of American options and the determination of the early exercise boundary, the approximate method of Longstaff and Scwhartz (2001), the quasi-analytical method described in Albanese and Campolieti (2006) and Monte Carlo simulation methods described in Glasserman (2004) may be used.

Exotic Derivatives for Jump-Diffusions

The valuation of a number of exotic derivatives is considerably more complex than their vanilla counterparts because their price depends on the path of the underlying process. Typical examples are Barrier and Asian options. Therefore, the price of exotic derivatives is more sensitive on the structure of the underlying stochastic process than is the price of vanilla calls and puts. Consequently, the choice of the underlying process and the subsequent pricing and hedging of exotic derivatives may be a task of considerable complexity, a topic discussed for equities by Gatheral (2006).

For the geometric Brownian motion with constant drift and volatility explicit prices of a number of exotic derivatives are derived in Shreve (2004). When the underlying process follows a jump-diffusion, the pricing of exotic derivatives by Fourier methods leads to Wiener-Hopf problems in the complex plane the factorization of which is often possible analytically. This is the case for the jump-diffusion model of Kou (2002) which leads to the explicit valuation of Barrier options. These analytical results are developed in Cont and Tankov (2004) where the class of Levy stochastic processes is also studied.

The extension of these Fourier methods to the valuation of options on spread contracts and other complex energy derivatives is discussed in London (2007). In the same reference the derivation of the characteristic functions of a number of jump diffusion models of energy prices is presented along with the valuation of weather derivatives.

Statistical Inference of Asset Price Models

Asset price models usually contain a number of parameters that need to be estimated upon calibration of the model against market prices. This may be carried out by using the econometric techniques presented in Cambell, Lo and MacKinley (1997), Greene (2000), Singleton (2006) and Tsay (2011).

Stochastic models of commodity prices often contain hidden factors -- stochastic trends, volatilities and the convenience yield -- which are usually modeled as diffusions. The estimation of the models may be carried out by casting the time series obtained upon discretization in state space form and using the Kalman filtering methods presented by Durbin and Koopman (2001).

The simultaneous inference of the model parameters and hidden factors may then be carried out by using dual Kalman filters and the Expectations Maximization algorithm presented in Haykin (2001). These statistical inference techniques may also be used for the estimation of nonlinear structural form models of power prices and shipping freight rates which are known to depend on nonlinearities in the supply and demand schedules of the underlying markets.

Model parameters are usually estimated using standard econometric methods for example maximum likelihood estimation. However in practice parameters are uncertain and this must be taken into account in model selection and use. Moreover in a multi-dimensional setting Stein (1956) showed that maximum likelihood estimators of the mean are inadmissible, namely more accurate estimators of means exist. An extensive literature ensued that showed that Stein's observation is justified in an empirical or hierarchical Bayes setting where model parameters are uncertain. This has led to the derivation of powerful Bayes-Stein estimators of drifts in multi-dimensional problems which are discussed in more detail below in the context of the energy and shipping markets. With the advent of fast computers Bayes inference evolved into Markov Chain Monte Carlo methods which are currently widely used in several disciplines [Gelman, Carlin, Stern and Rubin (2004)].

Stochastic Optimal Control Methods

The availability of analytical models governing the evolution of spot commodity prices and their derivatives, allow the formulation and solution of a wide range of valuation and hedging problems involving energy assets and their derivatives. The resulting stochastic dynamic programming problems are often possible to treat analytically by using the stochastic optimal

control methods presented in Yong and Zhou (1999) for diffusions with time dependent deterministic coefficients. These results follow from the solution of the Hamilton-Jacobi-Belman (HJB) partial differential equation or the Pontryagin Stochastic Maximum Principle and its connection to backwards stochastic differential equations. Extensions of these stochastic optimal control methods for underlying processes that follow diffusions with stochastic coefficients are discussed in Lewis (2005). Stochastic control methods for jump-diffusions and the treatment of the associated integro-differential equations are discussed in Oksendal and Sulem (2005).

APPLICATIONS

The stochastic price models, derivative valuation methods and stochastic optimal control algorithms presented above have found widespread use in the securities markets. A number of applications drawn from the energy and shipping sectors are discussed below.

Valuation of Natural Gas and Oil Storage

Storage facilities for natural gas and oil are assets that enable the transfer of power generation capacity between two time periods in response to supply and demand fluctuations. Such fluctuations are affected by the different seasonal variations of the natural gas and electricity prices, the former usually being higher and more volatile during the winter and the latter often being a lot higher during the summer.

The availability of inexpensive gas storage facilities, and the need to invest in new capacity, allows the low cost shifting of cheap summer production and storage of gas into the winter season. Moreover, the availability of gas storage facilities allows the quick delivery of natural gas when demand peaks, circumventing the need for expensive new production. These economic drivers call for the valuation and optimal operation of storage facilities for natural gas and other fuels, in the face of stochastic gas prices.

The storage valuation problem may be cast in a stochastic dynamic programming framework that relies on the analytical modeling of the commodity spot prices, futures curve and their derivatives as outlined above. In its generality, this valuation problem reduces to the

determination of optimal storage in/out-flows given the commodity seasonal price dynamics. The analytical framework for this valuation problem is presented in Eydeland and Wolyniec (2003) and discussed below in the context of the valuation of crude oil floating storage using a Principal Components factor model for the forward curve.

Natural gas may be transported over large distances using above ground and subsea pipelines, Liquefied Natural Gas (LNG) carriers and Compressed Natural Gas (CNG) carriers. These real assets, and LNG and CNG vessels in particular, may also be used and valued as storage facilities for natural gas traded in the spot or forward markets. LNG and CNG shipping is an emerging sector that is likely to receive increased attention in the future and is discussed in more detail below.

Valuation of Flexible Hydrocarbon Reservoirs

The optimal dynamic management of proven but undeveloped hydrocarbon reservoirs and flexible oil fields leads to a sequence of decisions analogous to those described above for above-ground storage facilities. When significant irreversible investments with option like value are necessary for the development of flexible hydrocarbon fields, the extended valuation framework of Real Options is needed. Its development is presented in Dixit and Pindyck (1994) and a number of applications are discussed in Brennan and Trigeorgis (2000) and Copeland and Antikarov (2001). Given a HJM model for the oil and natural gas futures curve and its derivatives, the operation of flexible hydrocarbon fields may be reduced to a stochastic dynamic programming problem leading to the determination of optimal investment and hydrocarbon extraction flows. A number of real projects where these valuation methods are applicable are

presented in Ronn (2002).

Hedging of Fuel Costs

The risk management of fuel costs in the transportation and energy sectors entails the hedging of commitments to purchase or deliver energy commodities – crude oil, natural gas, aviation jet fuel, gasoline, heating oil and shipping bunker fuels by various entities – refineries, utilities, airlines and shipping companies. An objective of such hedging programs is the minimization of the variance of the commodity price exposures over a given horizon. Variance minimizing quadratic hedges of complex derivative exposures using simpler securities is common in the financial markets and may be reduced to the solution of a stochastic dynamic programming problem [Yong and Zhou (1999) and Jouini, Cvitanic and Musiela (2001)].

A fuel cost hedging program may be implemented by using a combination of physical storage and the futures market. Such a hedging task faces a number of challenges, including commodity price and volume uncertainty, a decreasing liquidity of futures contracts of increasing tenor, an increasing volatility of futures contracts of decreasing tenor that need to be rolled over and exposure to basis risk when liquid futures contracts for the fuel of interest do not exist. The solution of the resulting dynamic optimization problem may be carried out by taking advantage of the analytical modeling, pricing and optimal control techniques outlined above. The complexity of such hedging programs is considerable as is highlighted by the collapse of the stacked hedges of Metallgesellschaft studied in Culp and Miller (1999).

Valuation of Seaborne Energy Cargoes

Crude oil and other liquid energy cargoes transported in tanker fleets may be traded while the cargo is in transit. This is akin to the optimal financial management of energy commodities in movable storage. Here, the location and speed of the tankers enter as controls in a stochastic dynamic programming framework which may be treated with the analytical techniques described above. The timing, sales price and port of delivery of the energy cargo are variables that may be selected in a value maximizing manner while the commodity is in transit. These decisions must take into consideration the shape of the oil futures curve which may be trading in contango, backwardation or in a composite formation, as well as the tanker freight rate forward curve. Moreover, since a large portion of the above-ground crude oil is in transit, the aggregate tonnage and average speed of crude oil tanker fleets may have a material impact upon the crude oil convenience yield, the shape of its futures term structure and its impact on the valuation of seaborne oil.

The principal components model of the crude oil forward curve developed by Sclavounos and Ellefsen (2009) was applied by Ellefsen (2010) to the valuation of crude oil floating storage. The value of a crude oil cargo carried by a Very Large Crude Carrier (VLCC) is shown to be that of an American option with an imbedded early exercise premium. The valuation of this option is carried out in a semi-analytical form by virtue of the explicit form of the Ornstein-Uhlenbeck diffusions and their transition densities that govern the independent factors that drive the crude oil forward curve using the method presented in Albanese and Campolieti (2006). It is shown that the value of the early exercise premium can be significant particularly in volatile markets and even if the forward curve is not trading in extreme contango. The returns of crude oil

floating storage investments are also studied and shown to be significant. Their hedging using crude oil futures is also addressed.

The valuation methodology developed for crude oil floating storage extends with minor modifications to land based storage of crude oil, products, bunker fuels, natural gas and other commodities. The necessary analytical machinery lies in the development of the principal component analysis of the forward curve of the commodity under consideration and the analytical derivation of the diffusions governing a small number of independent factors that drive the evolution of the respective forward curves.

LNG and CNG Shipping

Analogous considerations apply to the transportation of Liquefied Natural Gas in LNG and Compressed Natural Gas in (CNG) carriers. The LNG market is not as liquid or global as the oil market, yet it is likely to grow in the future in light of the growing demand for natural gas for the generation electricity [Lloyd's Shipping Economist, August 2011]. LNG carriers have grown in size over the past two decades and as of 2009 the largest vessel had a volumetric capacity of 267,000 cubic meters which translates to 113,000 metric tons of liquefied natural gas which is equivalent to about 6.2 billion standard cubic feet of natural gas under normal atmospheric conditions.

The CNG shipping market is likely to draw increased attention in the future as an inexpensive and flexible means for the seaborne transportation and storage of natural gas. A survey of CNG carrier technologies is presented by Ruppin, Noetzold, Pentschew and Kaeding (2011). The

technology of compressing natural gas up to 250-300 atmospheres in containers is mature. A volume unit of liquid natural gas yields 600 volume units of natural gas under normal atmospheric conditions, therefore compressing natural gas at 300 atmospheres in a CNG carrier leads on an energy equivalence basis to a capacity half that of a LNG carrier of the same volume. CNG shipping circumvents investment in expensive liquefaction and gasification facilities and above ground or subsea pipelines, also mitigating geopolitical issues. The natural gas may be loaded on CNG vessels equipped with pressurized containers, it may be compressed in cylinders stacked in standard TEU or FEU containers carried by a conventional containership or it may be loaded on barges carrying pressurized containers for routes with mild weather conditions. Loading may take place at the site of natural gas production onshore or at an offshore platform and delivered directly to the market with the ships acting as floating pipelines. Discounted cash flow valuation of investments in LNG vs CNG supply chains and carriers strongly favors CNG shipping for transportation distances less than 2,000-3,000 km [Wang and Economides (2009)]. A CNG vessel may also be used as a floating natural gas storage facility which may be valued using the methods discussed above. CNG shipping due to its low cost and flexibility is also likely to motivate the development of stranded natural gas reservoirs worldwide which either because of their size, expense of construction of local liquefaction plants or geopolitical reasons are yet undeveloped.

CNG vessels may also be configured to carry compressed hydrogen. CNGH vessels would remove the logistical bottleneck of transporting large quantities of hydrogen, produced by steam methane reforming, onshore or offshore wind energy and solar energy, to distant markets. Mixing hydrogen with methane in the right proportions (about 20-25% hydrogen and 80-75%

methane by volume) produces the fuel Hythane[®] which is known to have excellent combustion properties and also lead to a significant reduction in greenhouse gas emissions when mixed with gasoline in internal combustion engines. Consequently CNGH shipping stands to emerge as a new and key shipping sector that would enable the seaborne trading of large quantities of natural gas, hydrogen and Hythane[®]. This would spur their more rapid introduction into the transportation sector and the development of greener and more efficient dual fuel engines in the automobile and shipping industries.

Fuel Efficient Navigation and Optimal Chartering of Shipping Fleets

The shipping industry consumes approximately 5% of the world crude oil production in the form of bunker fuels. Assuming a daily world oil production of 87 million barrels and a price of oil of \$100 per barrel, the daily bunker fuel costs for the shipping industry are estimated at \$400 million dollars. The long term daily average freight rate revenue is harder to estimate and is assumed over twice the daily bunker fuel costs.

The selection of the optimal speed and route of cargo vessels exposed to stochastic freight rates and subject to the constraints imposed by the charter contract, cargo loading schedules, port and other fees, leads to a stochastic dynamic programming problem. The ship resistance and propulsion characteristics may be supplied by the shipowner, estimated from models or inferred from real-time measurements of the ship speed, propeller revolutions, engine performance and the weather using the inference methods described in Haykin (2001). Using a reduced form or structural stochastic price model for the shipping freight rate forward curve, optimal routing and chartering strategies may be derived analytically aiming to minimize the fuel consumption and

maximize freight rate revenue over single or consecutive voyages. A cumulative 5% reduction in bunker fuel costs and increase in freight rate revenue would translate into a \$50 million increase in the daily net income of the shipping industry. The promise of these advanced dynamic optimization algorithms is underscored by their adoption by the aviation industry for the optimal routing of commercial jets.

Several strategies are currently being implemented by the shipping industry to reduce fuel consumption and greenhouse gas emissions. Slow steaming is common when allowed by the charterparty agreement. Fuel consumption decreases like the cube of the vessel speed. A reduction of the speed of a VLCC from 14 to 10 knots leads to a reduction of fuel consumption by 65% assuming that the propeller and engine efficiencies are unchanged. Another strategy leading to fuel savings of 20% is “virtual arrival”. When delays are anticipated at the destination port where the cargo is to be unloaded it is advantageous for the captain to remain in constant communication with the charterer and continuously adjust the vessel speed in order to ensure arrival of the vessel at the time the port berth would be available. The alternative of sailing at a constant high speed and waiting to unload the cargo at a congested port leads to high fuel consumption and may lead to demurrage charges to the charterer. The savings in fuel consumption from the implementation of “virtual arrival” is estimated by BP to be \$5bn per year for the tanker sector [Lloyds List, July 6, 2011]. This is an example of the value of optionalities embedded in the optimal navigation and chartering strategies discussed above.

Shipping is in the process of implementing environmental regulations aiming to achieve a reduction in greenhouse gas emissions. The reduction of fuel consumption is one measure,

installing emissions scrubbing systems is another. Dual fuel engines that burn distilled fuels mixed with natural gas are also being developed by marine engine manufacturers. The introduction of natural gas as a cleaner fuel for the propulsion of cargo vessels is a development that stands to benefit from LNG and CNG carriers that would transport natural gas and hydrogen to coastal or offshore bunkering facilities where they would be stored in liquefied or compressed form for use by the shipping industry as component propulsion fuels.

Valuation and Hedging of Power Plants and Refineries

The optimal economic dispatch of power plants presents a challenging problem that depends in part on the price differential of two energy commodities. The input commodity is usually a fuel – natural gas or oil – which may be traded in the spot and forward markets. The output commodity is electricity which cannot be stored, it trades into a spot cash market and may not have liquid forward contracts, as discussed by Joskow (2006).

In simple cases, the valuation of power generating units may be reduced to the pricing of a strip of options written on the price differentials of electricity and the input fuel, for example natural gas. Given analytical price models for the price of the input fuel and electricity, the power plant valuation and hedging problem may be based on the pricing of these spark-spread options which may be available explicitly. In more general settings where operational constraints apply, the valuation problem may be cast in a stochastic dynamic programming framework which may benefit from the use of the analytical modeling and hedging methods outlined above. A similar set of issues arise in the valuation and hedging of refineries which process crude oil, which has a well developed spot and futures market, into products – gasoline, heating oil, jet fuel, bunker fuel

-- which often do not have actively traded forward contracts. The use of this general valuation and hedging methodology in practice is presented in Eydeland and Wolyniec (2003).

Valuation of Wind Farms and Electricity Storage Facilities

Wind is an ample clean renewable energy source, yet its availability is variable. The electricity generated from a wind farm varies stochastically, is a function of the statistical properties of the wind speed and the volatility of the annual mean wind speed is about 10%. The development of onshore wind farms is growing at a rapid rate worldwide. Offshore wind energy is the next frontier with high expected growth rates over the next decades from the development of vast expanses of sea areas with high winds and capacity factors of 40-45% using innovative low cost floating wind turbine technologies that may be deployed in water depths ranging from 30 to several hundred meters [Sclavounos et. al (2009), Tsouroukdissian et. al. (2011)]. An offshore wind farm with a rated capacity of 1 GW and a lifespan of 25 years is on an energy equivalent basis comparable to a 100 million barrel oil reservoir. Moreover this energy resource is available just 100 meters above sea level as opposed to thousands of meters below it.

The valuation of a utility scale onshore or offshore wind farm as an energy asset may be carried out using the standard Weighted Average Cost of Capital (WACC) discounted cash flow method assuming a relatively constant leverage ratio. Alternatively the Adjusted Present Value (APV) method may be used for a varying leverage ratio and when tax shields and other incentives available to wind farm investors must be valued separately [Myers (1974), Miles and Ezzell (1980)]. Wind turbines are high value capital assets that generate steady cash flows with an annualized volatility of about 10%. Utility scale wind farm investments may therefore be

structured using non-recourse project finance with a leverage that may reach 70-80%. The risk imbedded in debt and equity securities issued to finance utility scale wind farms depends on technical, environmental and market factors. Their rational modeling permits the pricing of debt and equity claims at various levels of leverage. Moreover, the availability of statistical records of wind speeds allows the development and calibration of diffusion models of wind speeds and power output [London (2007), Bengt, Bengt and Koekebakker (2008)]. These models lead to the rational pricing of adjustable rate amortizing bank loans and long term debt financed by issuing bonds with fixed coupons and a bullet payment at maturity. Such bonds must be valued so that they are of investment grade in order to attract interest from institutional investors and pension funds. The pricing of these liabilities may be carried out using models analogous to those used by rating agencies for the pricing of credit risk. Statistical models of the power produced by a single or a portfolio of wind farms also allow the pricing of structured securities like convertible debt and other derivatives that may be used to design an optimal capital structure, hedge financial exposures of wind farms as energy assets and determine the optimal mix of fixed Purchasing Power Agreements and fluctuating market price contracts for the delivery of electricity.

Investments in storage facilities for electricity generated by wind farms may be economically attractive if they permit the storage of a large amount of kilowatt-hours when electricity prices are low and wind speeds are high and their sale when electricity prices are attractive. Such storage facilities include pumped water storage in elevated reservoirs and compressed air stored either in above ground containers analogous to those used in CNG vessels or inside the floaters of offshore wind farms which are empty, paid for and may be used as pressurized containers.

Energy may also be stored in the form of compressed hydrogen produced by the electrolysis of seawater using the power generated by onshore or offshore wind farms. Hydrogen may be produced at a coastal or offshore facility using megawatt scale seawater electrolyzers and stored in compressed or liquefied form. The valuation and optimal operation of compressed air storage depends on the short term volatility and longer term fluctuations of wind speeds and electricity prices. In the case of hydrogen the value of storage depends on the cost of electrolyzers, the prevailing market value of hydrogen [Padro and Putsche (1999), Ivy (2004)], the cost of transporting the hydrogen to the markets using CNGH vessels and the cost of fuel cells for the generation of electricity. Alternatively hydrogen produced and stored at an offshore facility may be used as a green fuel for the shipping market. The availability of a stochastic price model for the spot and forward electricity prices allows the explicit valuation of such storage facilities using the methods presented above. This analysis would suggest the merits, size and optimal management of utility scale electricity storage facilities and would guide investments in these assets.

Canonical Correlation of Commodity and Shipping Forward Curves

The principal components analysis of the term structure of interest rates and of the forward curves in the commodities markets is a powerful method for the representation of the evolution of a large number of correlated spot and forward securities in the respective markets in terms of a small number of factors. Examples of this statistical modeling method were discussed above.

The forward curves of energy commodities, e.g. crude oil, gasoline, gasoil bunker fuels, are often correlated. The same applies to the FFA forward curves of distinct routes in the dry bulk and

tanker shipping markets. Therefore the development of parsimonious statistical models of the correlation structure of two or more forward curves is often necessary for the valuation of assets exposed to multiple commodities. This may be accomplished by carrying out a canonical correlation analysis of the block covariance matrix of the commodity forward curves of interest. The diagonal blocks are the intra-commodity covariance matrices which may be treated by the principal component analysis discussed above. The off-diagonal blocks are the inter-commodity covariance matrices which may be reduced by the canonical correlation analysis described in Basilevsky (1994) and Anderson (2003).

In a principal components analysis a small number of dominant factors is derived for each commodity forward curve, linear combinations of the traded futures contracts of varying tenors. In a canonical correlation analysis, for example of two commodity forward curves, portfolios of futures trading on each forward curve may be derived that are maximally correlated. The maximum correlation coefficient between the two curves is a summary metric that is independent of the tenor of the futures contracts used to derive each portfolio. The extension of this method to multiple commodity forward curves is straightforward. A canonical correlation analysis allows an in depth study of the cross-correlation structure of multiple commodity and shipping markets and may be used for the development of cross hedging strategies, the valuation of assets and for risk management.

The canonical correlation of the forward curves of distinct routes in the dry bulk and tanker shipping markets was carried out by Hadjiyiannis (2010). This study revealed various degrees of maximal correlations between shipping routes and a surprisingly high maximal correlation

between the dry bulk and tanker markets. This suggests that there exist portfolios of FFAs trading on major routes in the dry bulk market that are highly correlated with FFA portfolios in the tanker market. The composition of these portfolios follows from the canonical correlation analysis. The implication is that a small number liquid forward curves in shipping may be used for the hedging of exposures in routes with less liquid derivatives. The shipping forward curve principal components and canonical correlation analysis described above may be extended to include the cross-correlation of shipping freight rates with the forward curves of bunker fuels and the liquid energy and bulk commodity cargoes carried by tankers.

The derivation of a small number of dominant factors governing the dynamics of energy and shipping markets may be combined with Bayes-Stein shrinkage estimators of drifts to estimate the returns of portfolios of commodity and shipping securities with uncertain parameters. These methods have been applied to the selection of optimal portfolios of financial securities under parameter uncertainty by Jorion (1986), (1991) and Kan and Zhou (2007). The drifts and covariance matrix of the returns of securities in a portfolio are only known with statistical error. Taking this into account using a Bayes-Stein estimator for portfolio selection lead to a significant improvement in returns for example by investing in the risk free asset, the tangency portfolio and the minimum variance portfolio. Further improvements in returns are possible and depend on the number of securities in the portfolio, the time history used to estimate the drifts and covariance matrix and the type of Bayes-Stein estimator used. Returns are assumed to be multi-variate normal distributed but Bayes-Stein estimators have also been found to be robust under fat-tailed Student-t multi-variate distributions.

The Bayes-Stein shrinkage estimators of drifts allow subjective beliefs of trends to be incorporated into forecasts. This enables the combination of public information provided by the market prices of traded securities with private information about investment strategies. This is a particularly useful attribute in shipping where shipowners make investment decisions based on experience and their personal assessment of the shipping markets. Such private information may be combined with publicly available values of ships and prices of shipping securities and their derivatives and factored into investment decisions.

Pricing of Shipping Options

The arbitrage free pricing of shipping options is carried out along lines similar to those in the energy markets. A technical complexity of shipping derivatives is that shipping options settle against the arithmetic average of the underlying spot index. Shipping options may be priced either by modeling the evolution of the underlying index, or by modeling the evolution of the underlying futures contract. The first method is prevalent to date and is discussed in Alizadeh and Nomikos (2009). Yet, the second method has a number of advantages. By modeling the underlying futures or FFA contract as a lognormal diffusion, the pricing of calls and puts may be carried out readily by using the Black formula. Moreover, the underlying futures or FFA contracts may be used for delta, gamma and vega hedging of options exposures.

This approach of pricing shipping options has been adopted in the multi-factor principal components model of the forward curve developed by Sclavounos and Ellefsen (2009). It leads to explicit expressions of the option prices and their Greeks and also allows for a volatility term structure which is the result of the mean reversion of the factors driving the shipping forward

curves. The explicit form of the Ornstein-Uhlenbeck diffusions governing the evolution of the factors leads to explicit algebraic expressions for the options and their Greeks discussed in Ellefsen (2010).

Pricing of Credit Risk and Structured Securities in Shipping

Shipping fleets are primarily financed by debt issued by banks and other lending institutions, followed by equity raised by shipping firms in private placements or in public markets. The underlying assets financed by this capital are cargo vessels which have observable prices quoted by shipping brokers and firms specializing in vessel valuation e.g. VesselsValue.com. Credit derivatives and other structured securities analogous to those in widespread use in other asset markets are not yet as widely traded in the shipping sector.

The pricing of credit risk is based on the fundamental structural form firm value method of Merton (1974) and the reduced form hazard rate method of Duffie and Singleton (2003). These valuation methods have enabled the pricing of derivatives written on individual credits – e.g. Credit Default Swaps – as well as derivatives written on baskets of credits. The values of the underlying entities in a basket and their default probabilities are correlated and this dependency structure may be modeled in its generality by using multivariate Gaussian statistics and in non-Gaussian settings copulae functions [Li (2000)]. This financial technology has enabled the design and pricing of an array of structured financial securities discussed in Duffie and Singleton (2003), Lando (2004) and London (2007).

Shipping credit risk may be modeled and priced using a hybrid model which blends the structural

and reduced form valuation methods discussed in Ammann (2001). The price of the assets of a shipping firm – the cargo vessels – is stochastic but observable, therefore recovery at default is known. The price of equity of public shipping firms is also observable and may be used to model the hazard rate, the probability of default and hence the pricing of shipping debt by calibrating a hybrid credit risk model as described by Overhaus et. al. (2007). Cargo ship prices within and across shipping sectors are correlated and this dependency may be modeled by identifying common underlying factors via a principal components and canonical correlation analysis. The above attributes of the shipping sector may be introduced to price loans, convertible bonds, equity and credit linked notes and other structured securities which may be used to better manage shipping risk, reduce bank regulatory risk capital and make available new sources of financing to shipowners.

Key Points

- Producers of energy commodities and owners of shipping tonnage may take short positions in futures and Freight Forward Agreements (FFAs) in order to hedge their forward delivery commitments against a decrease of prices.

- Consumers of energy commodities and shippers who charter cargo vessels may take long positions in futures and FFAs in order to hedge their forward commodity and freight rate exposures against rising prices.

- Power plants and refineries that transform an input commodity into an output commodity, e.g. natural gas-to-electricity, crude oil-to-gasoline, may go long the futures of the input commodity and short the futures of the output commodity in order to protect their profit margins against adverse moves of the input/output commodity price spread.

- Liquid energy commodity forward curves convey information about the stochastic evolution of the spot price of the commodity.

- The stochastic dynamics of individual energy commodity and shipping forward curves may be modeled by a small number of independent statistical factors using a Principal Components Analysis (PCA). The factors are

portfolios of traded futures contracts and their stochastic dynamics is governed by diffusions that may be derived in explicit form.

- The joint stochastic dynamics of cross-correlated commodity and shipping forward curves may be modeled by a small number of statistical factors using an intra-commodity PCA curve and an inter-commodity Canonical Correlation Analysis (CCA).
- The parsimonious statistical factor modeling of the commodity and shipping forward curves may be used for the valuation and risk management of energy assets, structured securities and portfolios of commodity and shipping derivatives.

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