

Recording Uncertainty in Measured Values

Making a single measurement

When reading instruments like a scale, a ruler, or a dial, we use common sense to determine the best estimate of the quantity in question: we examine the marks on the measurement scale and determine which mark the quantity lies closest to. For example, suppose we're measuring the volume of water in a 50-mL graduated cylinder with markings every 1 mL. We examine the meniscus and decide it is closest to 42 mL. What we really mean by this statement is that the meniscus is closer to 42 than it is to either 41 or 43, implying that the actual volume lies somewhere between 41.5 and 42.5 mL. A rule of thumb for reading marked scales is that the uncertainty in a single measurement is half of the quantity between markings (in the previous example the quantity between markings is 1 mL, so the uncertainty is 0.5 mL).

However, there are other factors we must consider when deciding what a reasonable range of uncertainty is. An extremely important factor to consider is human error. Suppose we are using a stopwatch to calibrate a pump flow rate. We fill a basin with water from the pump, and when the water level reaches a predetermined volume, we hit the button on the stopwatch. We see it reads 34.84 s. However, it would be inappropriate to report this value, as there is no way we could have stopped the stopwatch to within 0.005s of the actual value. Instead, a more reasonable estimate might be that we can stop the stopwatch to within 1 s of the actual time and we would round our answer up to 35 s. Another factor to consider when assessing error is the reliability of the instrument we're using. If the stopwatch is old and running slow, every measurement we take will be off by a small quantity due to improper instrument calibration. This is called a systematic error and will be discussed later.

As described above, to determine a quantity x , we make a measurement, report our best estimate, and report the range over which we are reasonably confident the actual value lies:

$$(\text{measured value of } x) = x_{\text{best}} \pm \delta x . \quad (1)$$

δx is called the uncertainty, error, or margin of error.

Significant Figures in Uncertainty

Uncertainty, δx , is by definition an estimate or approximation. Accordingly, it should not be reported with too much precision. In general, experimental uncertainty should almost always be rounded to one significant figure. If we perform a calculation of uncertainty of a velocity that yields $\delta u = 0.0246$, we would round to 0.02. The exception to this rule is if the leading digit of the uncertainty value is 1. If $\delta x = 0.14$, rounding down to 0.1 would ignore a large portion of the uncertainty range and it would be appropriate to keep two significant digits.

When reporting an estimate of a measurement and its uncertainty, the two numbers' significant digits must agree. That is, the uncertainty should reflect the level of precision in the estimate of the quantity. For example, if we measure a flow rate of 12.3 L/min and estimate an uncertainty of 1 L/min, we would not report $Q = 12.3 \pm 1$ L/min. This would be nonsensical and negates the significance of our reported value. Instead, we report $Q = 12 \pm 1$ L/min. The general rule that describes this is that the last significant figure in any stated answer should be of the same order of magnitude as the uncertainty. Indeed in the previous example, the last significant digit in the answer is 2 (in the ones place), which is of the same order of magnitude as the uncertainty. As further example, for a measured value of $y = 64.23$,

$$\begin{array}{ll} \text{if } \delta y = 0.2, & y = 64.2 \pm 0.2 \\ \delta y = 2, & y = 64 \pm 2 \\ \delta y = 20, & y = 60 \pm 20 \end{array}$$