6.434J/16.391J | May 9
Statistics for Engineers and Scientists | Handout #18
MIT, Spring 2006 | Due: Never

Problem Set 8

Problem 1: Let X_1, X_2,...,X_n be a random sample from a uniform distribution on $(0, \theta)$, and let $Y_1 \leq Y_2 \leq ... \leq Y_n$ be the associated order statistics. For "a" fixed, show that $[Y_n, Y_n a^{-1/n}]$ is a confidence interval for θ and find its confidence coefficient and expected length $\mathbb{E}\{U - L\}$.

Problem 2: Consider a multi-variable function (for example, it could be the joint density) of three variables,

$$f(x_1, x_2, x_3) = x_1 x_2 + x_1 x_3$$

such that each $x_i \in \Omega$, where $|\Omega| = N$. That is, $i = 1, 2, 3$ and each x_i takes N values. We need to compute marginal on x_1, i.e.,

$$g(x_1) \triangleq \sum_{\forall x_2, x_3} f(x_1, x_2, x_3)$$

a) Show steps to have a direct calculation of $g(x_1)$. How many computations does it require? (We treat additions and multiplications equally, and count each as one unit of computation.)

b) Recall the *distributive law of multiplication over addition* and re-write $f(x_1, x_2, x_3)$ in a form suitable for computing the desired marginal $g(x_1)$. Show its steps and give a computational count.

Problem 3: Consider the following function of three variables,

$$f(x_1, x_2, x_3) = a x_1 x_2 + b x_2 x_3 + c x_3 x_1$$

where for $i = 1, 2, 3$, $x_i \in \Omega$, and $|\Omega| = N$. a, b, c are known constants. We are interested in the marginal on x_2. Draw a factor graph representation for the given function. Show how would you compute the required marginal efficiently. Compare the computations.