JANUARY 2000
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wednesday, January 19, 2000 37-212 9:00 a.m.-1:00 p.m.

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less).

You must answer at least two (2) questions from Column A.
Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your name appears **on the cover of each of your blue books that you turn in to be graded**.

Oral examinations will be held on Tuesday, January 25, 2000. Pick up your schedule on Monday, January 24, 2000 after 4:00 p.m. from Nancy Masley in E10-110.

Results will be available on Wednesday, January 26, 1999 after 3:00 p.m.
Please contact your advisor.

Column A	Column B
Mathematics	Avionics
Physics	Fluids
Dynamics	Humans and Automation
	Instrumentation, Control and Estimation
	Propulsion
	Structures
	Systems
	Thermodynamics

Written Doctoral Qualifying Exam

Math

a) A particular symmetric $N \times N$ matrix $\mathbf{A}$ has distinct eigenvalues λ_i $(i = 1 \ldots N)$, and corresponding eigenvectors $\mathbf{e}_i$:

$$\mathbf{A}\mathbf{e}_i = \lambda_i \mathbf{e}_i$$

If F is an analytical function (expressible as a Taylor series), what are the eigenvalues and eigenvectors of $F(\mathbf{A})$?

b) Given the eigenvalues μ_i and the orthogonal eigenvectors $\mathbf{f}_i$ of some square matrix, how can the matrix itself be calculated?

c) Using the above results, and with

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

calculate $\mathbf{A}^2$, $\mathbf{A}^{10}$, and $\exp(\mathbf{A})$. Verify directly for the case of $\mathbf{A}^2$.

Physics
Written Exam Question

Comment on six of the following, giving explanations of your comments in terms of basic physics. Wherever possible, include calculations to complement the qualitative description (your calculations can include rough estimates of physical quantities).

(a) The light from all distant galaxies appears to be red shifted. The amount of this red shift is nearly the same for all distant galaxies.

(b) It is not possible to use suction to draw water from a well where the water table is more than 35 ft (10.7m) below the surface.

Negative absolute pressure $\rho g h$

greatest relative pressure

(c) Most commercial airliners deploy "spoilers" shortly after landing which reduces the lift generated by the wings.

(d) Imagine one oberves (from above) a surfer waiting for just the right wave. As a wave passes, the position of the surfer (who is not paddling in any way) will oscillate normal to the wave front but will finally return to the same position as before the wave arrived.

transverse + longitudinal

(e) Military aircraft often deploy "chaff" as a countermeasure against missiles thjat employ active radar to locate their targets. "Chaff" is a cloud of thin strips of Aluminum cut to lengths which are determined by the frequencies of the missile's radar.

(f) The direction of current flow in the windings of DC permanent magnet motors is designed to reverse twice for each rotation of the armature shaft (this reversal of current is commonly referred to as "commutation").

(g) When one looks into the front of a rectangular aquarium constructed of transparent glass on all five sides, one can see the fish inside and one can often also see a reflection of the fish from the sides of the aquarium as if the sides were mirrored.

(i) During a complete solar eclipse (in which the sun is eclipsed by the moon), the diameter of the sun appears somewhat greater than just a few minutes before and after the eclipse.

(j) If one pulls slowly and steadily on a piece of toilet paper, the roll will rotate and pay out more paper. If one applies force suddenly to the same bit of toilet paper, the paper will break at the perforations.

Written Doctoral Qualifying Exam
Dynamics

Two bar pendulums, each of total mass m and length ℓ, hang under gravity, and are interconnected at their midpoints by a spring k.

Determine the natural frequencies ω_i of the bars if:

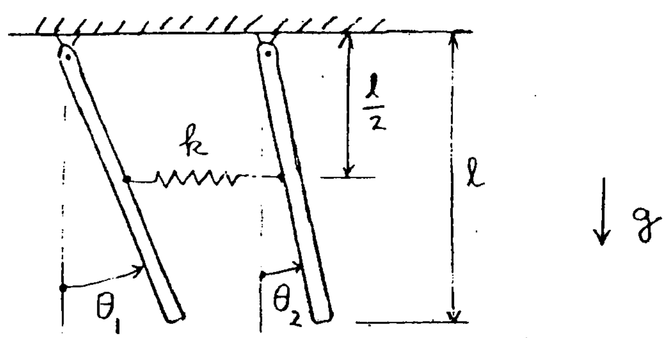
1a) There is no spring.

1b) The spring is replaced by a rigid link.

1c) The spring is present as drawn. Describe how the frequencies change as k varies.

2) If bar 1 is held fixed at $\theta_1 = 0$, and gravity is reversed by inverting the apparatus, what is the requirement on k to keep the other free bar stable at $\theta_2 = 0$?

$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$



Doctoral Qualifying Examination
Department of Aeronautics and Astronautics
Avionics Question

In this question we are interested in the dynamics of interacting transport aircraft. We therefore consider the generic situation of two aircraft encountering each other, as shown in the figure. The nomenclature is as follows:

v_1, v_2: aircraft velocities.
$d(t)$: distance between aircraft.
ϕ_1, ϕ_2: aircraft heading (relative to horizontal axis).
θ_1, θ_2: aircraft azimuths (relative to the line of sight between them).

1. Assuming the two aircraft follow straight and level paths, and assuming they are flying at the same altitude, how do the relative azimuths θ_1 and θ_2 evolve as a function of time? How do the azimuth histories evolve as a function of the trajectories followed by those airplanes? Sketch typical relative azimuth histories as a function of time.

2. Assume that only a few measurements of the relative azimuth of the intruder are available to the ownship. This situation often arises during operations under visual flight rules. How many measurements does the ownship need to predict the time of minimum miss distance between the two aircraft? Are those measurements sufficient to compute the minimum miss distance itself?

   Note: The minimum miss distance is the distance between two aircraft when they are closest to each other.

3. Assume the two aircraft are on a collision course. The velocities satisfy $v_1 = v_2$ and remain constant. Assume the aircraft attempt to avoid each other by using the following control laws:

$$\frac{d}{dt}\phi_1 = \frac{k}{1+\tau s}(\theta_1 - (\pi - \theta_2))$$
$$\frac{d}{dt}\phi_2 = \frac{k}{1+\tau s}(\theta_2 - \pi + \theta_1),$$

   where k and τ are positive constants; τ is a variable that represents decision-making delay. Are these control laws satisfactory for conflict avoidance? Discuss the case $\tau = 0$ first, then the case $\tau > 0$.

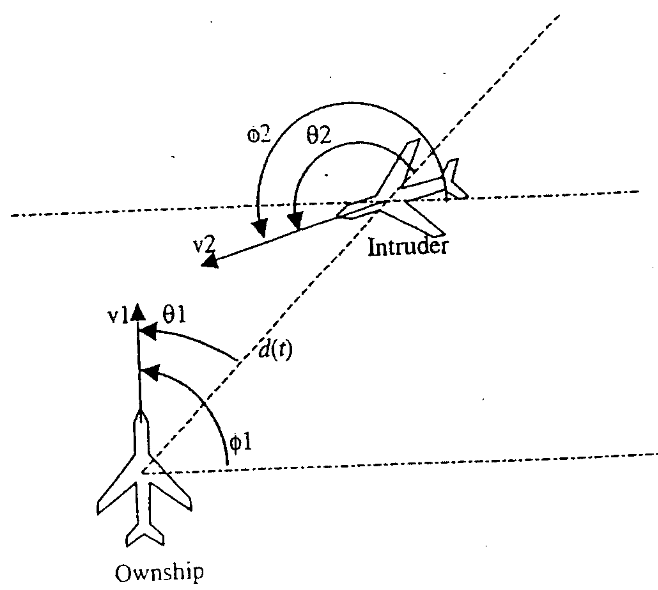


Figure: Aircraft encounter geometry

Written Doctoral Qualifying Exam

Fluids

An ideal gas with specific heat ratio $c_p/c_v = \gamma = 1.4$ flows through a choked nozzle which has a shock. The flow conditions measured at the two ends away from the nozzle and shock are:

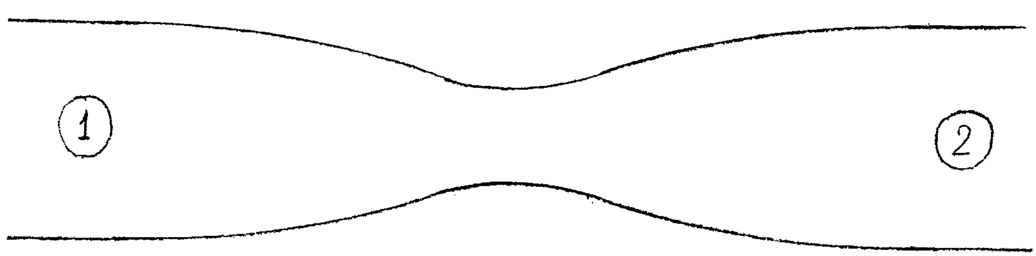
static pressure:	$p_1 = 1.0 \times 10^5 \text{Pa}$	$p_2 = 0.9 \times 10^5 \text{Pa}$
static enthalpy:	$h_1 = 2.8 \times 10^5 \text{m}^2/\text{s}^2$	$h_2 = 2.7 \times 10^5 \text{m}^2/\text{s}^2$
total enthalpy:	$h_{t_1} = 3.0 \times 10^5 \text{m}^2/\text{s}^2$	$h_{t_2} = 3.0 \times 10^5 \text{m}^2/\text{s}^2$

a) Is the flow subsonic or supersonic at stations 1 and 2?

b) Which way must the gas be flowing (left or right)?

c) What is the ratio of channel areas A_1/A_2 ?

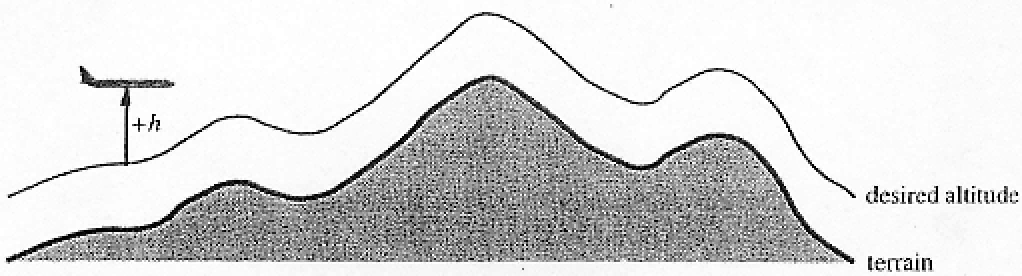
d) What is the ratio of channel areas A_{throat}/A_2 ?

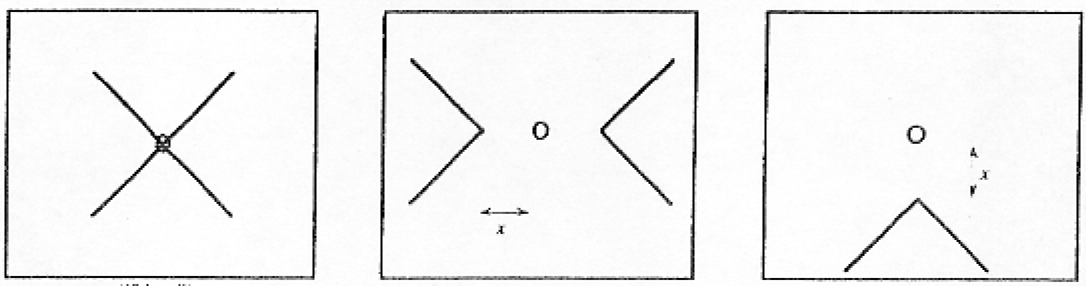


2000 DOCTORAL QUALIFYING EXAMINATION

HUMANS AND AUTOMATION

A new cockpit display is needed to aid pilots in flying high-speed low-level terrain following missions in poor visibility. Three candidate displays (A, B, C) have been proposed:





(if $h < 0$)
A B C

The three displays work by monitoring the current altitude of the aircraft relative to a desired altitude above the terrain. This relative altitude is denoted h as shown.

All three displays have a fixed circle icon in the center. Display A overlays an X icon whenever h is negative, and shows nothing when h is positive. Display B shows two chevron symbols whose lateral displacement x from the center of the display is directly proportional to h. Display C shows a triangle symbol which moves vertically, with a displacement x from the center of the display also proportional to h.

1. To compare the displays, some measure of 'performance' is required. Propose and justify a set of performance metrics that would enable an engineer to select one of the displays for use in an aircraft.

2. Design and describe an experiment in which you would be able to measure the performance metrics you identified in #1. Be sure to discuss issues such as experimental protocol, subject selection, test conditions, and data collection methods.

3. Discuss the advantages and disadvantages that you would expect from each of the three displays. Specifically, would you expect Display A to perform better than Display B? Why or why not? Would you expect Display B to perform better than Display C? Why or why not?

4. Propose and justify additional modifications to the displays that should further improve performance based on human factors principles.

Qualifying Exam Question in Controls

Because of its deta wing configuration and the large elevons used to control its longitudinal motion, the Space Shuttle has non-minimum phase dynamics during its final approach to landing. For perturbations about a nominal descent trajectory, the transfer function relating altitude rate (ft/sec) to elevon deflection (deg) can be modeled approximately as:

$$\frac{\delta \dot{h}}{\delta e} = \frac{(-s+5)}{s(s+1)}$$

We wish to create a feedback system to control altitude rate perturbations on final approach.

a. Sketch the root locus for a system which only feeds back altitude rate perturbations through a pure gain.

b. Sketch the Nyquist diagram for this system and explain how the two analyses give consistent results in terms of system stability.

c. Determine an appropriate feedback control system so that the bandwidth is about 1 rad/sec with a phase margin of about 45 degrees.

Written Doctoral Qualifying Exam

Propulsion

Tip-mounted monopropellant rockets drive a hovering rotor. The liquid monopropellant flows along the shaft, along each blade, and into each catalytic decomposition chamber. Pressurization of the liquid of density ρ_ℓ is entirely via centrifugation. Assumptions:

- The rockets' specific impulse is $I_{sp} = c_F c^*/g$, where c^* is the characteristic speed given by $\dot{m}/2 = p_c A_c/c^*$, and c_F is some given nozzle thrust coefficient.
- The injector pressure drop is a given fraction δ of the chamber pressure: $p_1 - p_c = \delta\, p_c$
- The chamber pressure greatly exceeds the atmospheric pressure, $p_a \ll p_c$. The liquid's pressure at the rotor center is negligible: $p_0 \ll p_1$
- Simple actuator disk theory gives the lift of the rotor: $L = \pi R^2\, \frac{1}{2}\rho_a V_{\text{wake}}^2$. The shaft power is then $P = Q\Omega = L\, V_{\text{wake}}/2$, where Q is the rockets' driving torque.

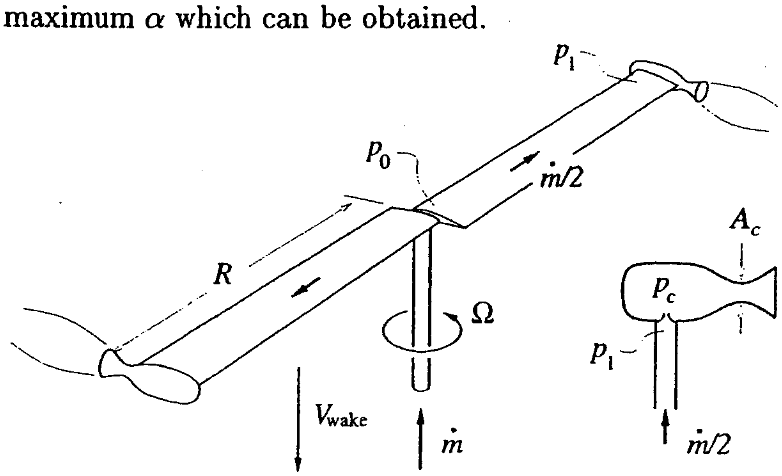
a) Construct a model that will allow calculation of $\dot{m}$, Ω, and Q which are necessary to obtain a required lift L from a rotor of given geometry.

b) The effectiveness of this system can be defined via the "augmentation ratio" α, which is the ratio of the rotor lift to the lift which the rockets could provide if directed downward.

$$\alpha = \frac{L}{\dot{m} g I_{sp}}$$

How must A_c be changed to improve α? What is the necessary corresponding tip speed ΩR to maintain L?

c) For a particular application the tip Mach number must be limited to $M_T = \Omega R/c_a \leq 0.6$ because of noise constraints. The disk loading must also be fixed at $L/(\pi R^2 p_a) = 0.1$. Determine the maximum α which can be obtained.

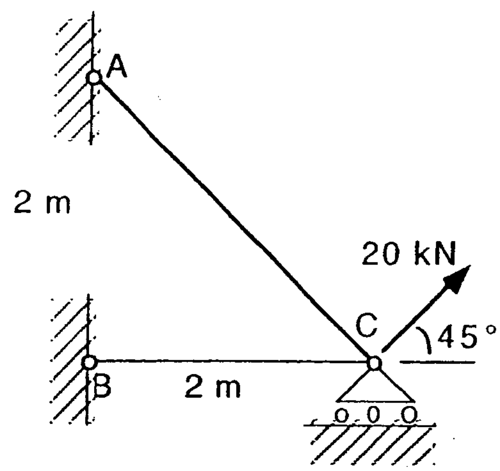


Materials and Structures
Written Exam Question 1999

Note. Show all your working. The process you follow to solve a problem is often of more importance than the solution itself.

A two bar truss is shown below. The bars are made of aluminum tubes with an internal diameter = 16 mm and an external diameter of 32 mm . The modulus of the aluminum alloy is 70×10^9 N/m^2 . The yield strength is 300×10^6 N/m^2 and the coefficient of thermal expansion is 24×10^{-6} K^{-1} .

a) Determine the reactions at A, B and C under the 20 kN loading shown.



b) Determine the reactions if there is a 100 K temperature rise from the temperature at which the truss was fabricated and the 20 kN load is still applied.

c) State, and where possible verify, all the modeling assumptions that you have made in order to analyze this structure.

Systems Qualifying Exam - Candidate Written Question

Systems are most completely defined using three means. These include:

1) Architecture (*what the system is*),
2) Behavior (*what the system does*), and
3) Context (*what environments the system operates in and what interfaces with the system*).

Consider the following system:

An orbiting space-tug will be used to transition satellites and various spacecraft between orbits of different heights above the Earth. It can be expected that this autonomous tug will 1) extract payloads from the Space Shuttle payload bay and deliver them to orbits that are beyond the Space Shuttle's capabilities, 2) retrieve space vehicles from higher orbits to rendezvous with the Space Shuttle for either repairs or return to Earth, 3) enhance the orbits of existing spacecraft to extend their useful life, and 4) adjust spacecraft orbits to improve overall utility/effectiveness.

• Using schematic diagrams in the time available, characterize a candidate or baseline architecture for the space-tug system. In doing this, ensure that interfaces between the major subsystems or components are annotated.

• Develop a behavioral diagram showing a typical scenario for the space-tug. In this diagram ensure that functions performed sequentially and in parallel are identified.

• Finally, develop a set of context diagrams in which the space-tug's environments and external interfaces are represented.

<u>Note:</u> In performing these tasks, creating all three diagram sets concurrently may be more effective then building them sequentially

Written Doctoral Qualifying Exam

Thermodynamics

1. A two-phase Carnot cycle is operated as shown in the figure below. The letters A, B, C, D, E, and F, refer to different states that occur during the cycle. The pressures and temperatures are such that the vapor behaves as a perfect gas. The <u>specific volume</u> at C is twice that at B. The specific volume of the liquid is much less than the specific volume of the vapor.

   a) What is the ratio of the pressure at B to that at A?
   b) What is the ratio of the pressure at C to that at A?
   c) Is the specific volume at E greater than that at F?
   d) Is the specific volume at D greater than that at E?
   e) What is the difference in enthalpy between states B and A? (give the answer in terms of T and S)
   f) What is the ratio of the <u>overall</u> heat received to the heat received in the regime in which the system is in a two phase condition?
   g) What is the thermal efficiency of the Carnot cycle?