JANUARY 200~~X~~5

DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., January 19, 2005 Room 37-212 9:00 AM – 1:00 PM

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less)

You must answer at least two (2) questions from Column A, (one (1) Math and one (1) Physics), and three (3) questions from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Be sure that your NUMBER (last four digits of your MIT ID) appears on the cover of each of your blue books that you turn in to be graded.

Oral examinations will be held on Tuesday, January 25th. Please pick up your schedule on Monday, January 24, 2003 after 3:00 PM from the Aero Astro Student Services Office (33-208).

Results will be available from your advisor on Wednesday, January 26 after 3:00 PM.

Column A

Mathematics (Discrete OR Continuous)
Physics (Dynamics OR Fields)

Column B

Autonomy
Communication and Networks
Control
Fluid Mechanics
Human Factors Engineering
Propulsion and Thermodynamics
Software Engineering
Structures and Materials
Vehicle Design and Performance

Continuous Math

1a) Find a normal vector $\vec{n}_1$ perpendicular to plane 1, and a normal vector $\vec{n}_2$ perpendicular to plane 2.

$$\begin{array}{lrcl} \text{plane 1:} & y - 2z & = & 0 \\ \text{plane 2:} & x - 2y + 8z & = & 1 \end{array}$$

1b) Find a vector $\vec{p}$ parallel to the intersection line of the two planes.

1c) Determine three functions $x(t)$, $y(t)$, $z(t)$, which parametrically define the intersection line of the two planes.

2) A function is defined by

$$F(x) = \int_0^x \ln(x^2 + y^2)\, dy$$

Evaluate dF/dx at $x = 1$.

3a) For the two functions

$$f(x, y) = y \qquad\qquad g(x, y) = -x$$

evaluate the following line integral taken along the perimeter of the unit circle in the counter-clockwise direction.

$$\oint [f(x, y)dx + g(x, y)dy]$$

3b) Now assume that f and g are partial derivatives of another function $\phi(x, y)$, e.g.

$$f(x, y) = \frac{\partial \phi}{\partial x} \qquad\qquad g(x, y) = \frac{\partial \phi}{\partial y}$$

Show that for any closed contour S of arbitrary shape,

$$\oint_S [f(x, y)dx + g(x, y)dy] = 0$$

3c) Is it possible that the vector function $y\hat{\imath} - x\hat{\jmath}$ is the gradient of some scalar function $\phi(x, y)$? Explain.

Discrete Math

Consider a two-person game, played over some number of stages. A stage consists of the following (also summarized in the table):

- A coin toss is made.
- If heads show, then Player A wins the stage.
- If tails show, then Player B wins the stage.
- Winning a stage is worth one point.

The coin is imperfect, in such a way that heads show with probability $P \neq 0.5$, and tails show with probability $1 - P$. The outcome of each stage is independent from the outcome of previous stages.

	A's score	B's score	Probability
Heads	1	0	P
Tails	0	1	$1 - P$

The game is won by the first player to score n points, with n specified before the start.

1) What is the expected number of points and the variance of the number of points that Player A will have a higher score after the first four stages of the game?

2) Write an expression for the probability that Player A will win the game.

3) Write an expression for the probability that the game will end (with one of the two players winning) after exactly m trials (for $m \geq n$).

Fields

A conductive substance (plasma), occupies the half-space $x > 0$. The other half-space $x < 0$ is vacuum. A plane electromagnetic incident wave is sent normal to the $x = 0$ boundary. The electric fields of the three waves which are involved in this interaction are

$$\begin{aligned}
\vec{E}_i &= \hat{y}\ \exp(ik_0 x - i\omega t) && \text{incident wave} \\
\vec{E}_r &= \hat{y}\ \exp(-ik_0 x - i\omega t) && \text{reflected wave} \\
\vec{E}_t &= \hat{y}\ \exp(ik_1 x - i\omega t) && \text{transmitted wave}
\end{aligned}$$

The associated magnetic fields for the incident and reflected waves are

$$\begin{aligned}
\vec{H}_i &= \hat{z}\ \exp(ik_0 x - i\omega t)\ k_0/\omega\mu_0 && \text{incident wave} \\
\vec{H}_r &= -\hat{z}\ \exp(-ik_0 x - i\omega t)\ k_0/\omega\mu_0 && \text{reflected wave}
\end{aligned}$$

1) Using Maxwell's equations, determine $\vec{H}_t$ of the transmitted wave. Also determine the current density $\vec{j}$ which must be present in the plasma.

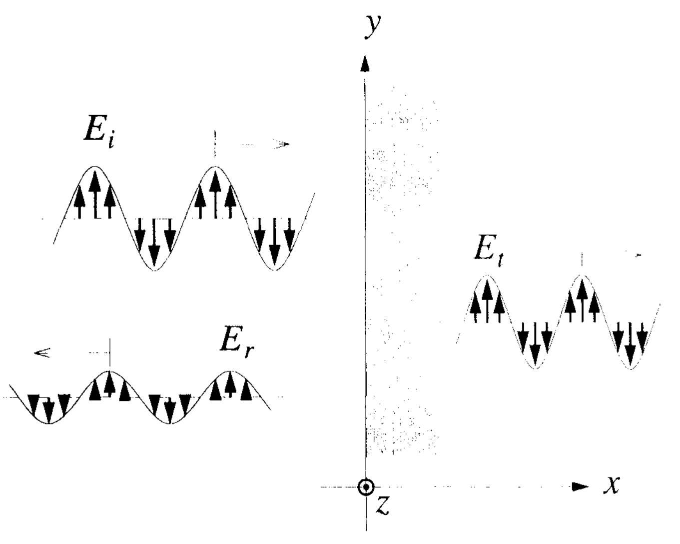
2) Explain why Maxwell's equations imply that the total $\vec{E}$ and $\vec{H}$ must each be continuous across the boundary. Determine the magnitudes E_r and E_t in terms of the wavenumbers k_0 and k_1 for the two media. Hint: Impose continuity of the total E and H at the two points $x = 0^-$ and $x = 0^+$.

3) Assuming the current is entirely due to electron motion, the current density is related to the electron charge, number density, and velocity, the latter also being related to $\vec{E}$ by the electron momentum equation.

$$\vec{j} = -e\, n_e\, \vec{v}_e \qquad\qquad m_e \frac{\partial \vec{v}_e}{\partial t} = -e\vec{E}$$

Determine the electric field amplitude E_t in the plasma.

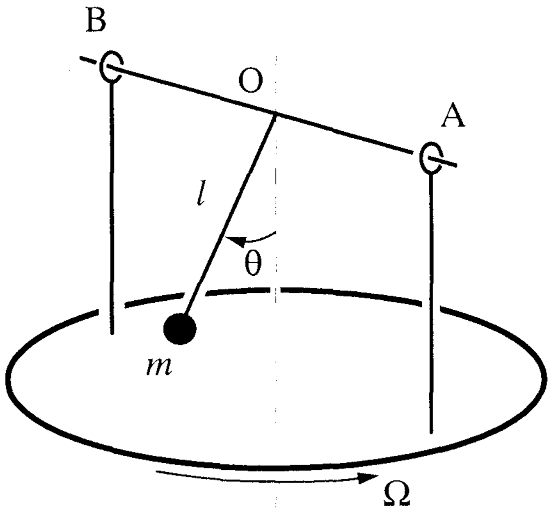
4) Comparing the results of E_t from 2) and 3), determine the wavenumber k_1 in the plasma. For some conditions, this k_1 value may turn out to be imaginary. What is the physical implication of this result?



$$\begin{aligned}
\nabla \times \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} \\
\nabla \times \vec{H} &= \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{j}
\end{aligned}$$

Dynamics

The apparatus below consists of a mass m suspended by a massless rod of length l which is welded to the massless horizontal rod at point O. The horizontal rod is free to rotate in the frictionless bearings at A and B. The whole assembly is mounted on a circular rotating table which turns about its center with an angular velocity Ω as shown. Gravity is acting downwards.



1) For $\Omega = 0$, determine the period of small oscillations of the mass about its equilibrium position $\theta = 0$.

2) For $\Omega > 0$, determine the period of small oscillations of the mass about $\theta = 0$. Indicate the range of values $\Omega = (0 \ldots \Omega_{\text{crit}})$ for which oscillations about $\theta = 0$ are possible.

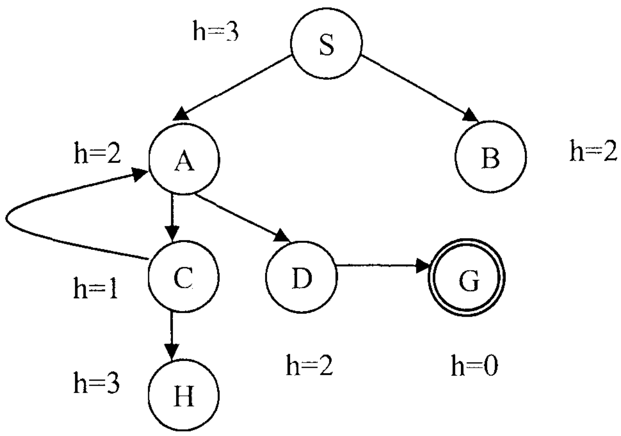
3) For values of Ω above the critical value Ω_{crit} found in 2, determine the θ value of stable equilibrium.

4) Determine the period of small oscillations about the equilibrium point found in 3.

Autonomy

Problem 1 Search (20 points)

You are trying to find a path from the state **S** to state **G** in the following directed graph using two search algorithms:



Assume the following:

- Edges are length 1.
- h denotes heuristic cost for each node.
- Given two or more equally good node's explore them in **alphabetical order**.
- The search stops as soon as the goal is expanded.

Write the sequence of nodes **expanded** by the specified search methods. A node is **expanded** when the method takes it off the queue, and attempts to create its children.

Part A Iterative Deepening (6 points)

Show the iterative-deepening expansion sequence:

S-

Part B A* Search (14 points)

Show the A* sequence (we have started it for you). **Write above each node** in your expansion sequence **the number used by** A* to pick the next node to expand.

0 1
S-A-

For A*, indicate the final path found…

S-

Is the heuristic H admissible? Yes, or No

Problem 2 Constraint Propagation (18 points)

Consider a constraint satisfaction problem consisting of four variables: A, B, C and D. Each variable has a domain consisting of two legal values; in particular, the variable domains for each variable are:

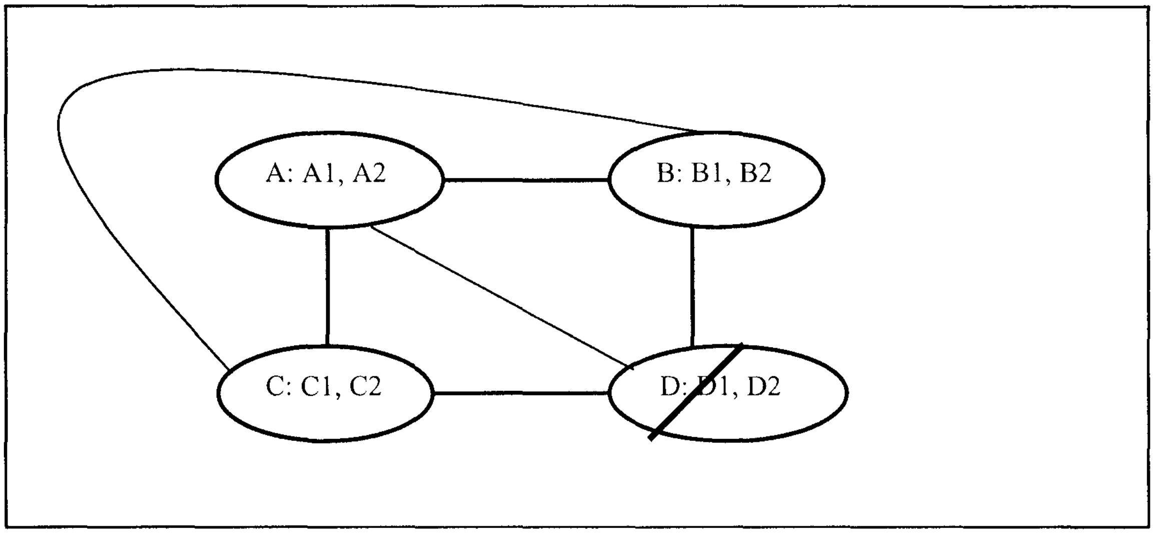
A: {A1, A2}
B: {B1, B2}
C: {C1, C2}
D: {D1, D2}

The **only** valid assignments to pairs of variables are described by the following constraints. For each variable pair, no other pairs of values are valid

- **A-B**: <A1, B1> or <A2,B2>
- **A-C**: <A1, C2> or <A2,C1>
- **B-D**: <B1, D1> or <B1,D2>
- **C-D**: <C2, D1> or <C2, D2>
- **B-C**: <B1, C2>
- **A-D**: <A2, D2>

Part A Pruned Domains (8 points)

First, consider what domain elements can be eliminated using constraint propagation. The constraints and variable domains are depicted on the following constraint graph. On the graph, cross out each domain element that is eliminated by constraint propagation. For example, to start you off, we have eliminated D1, which is eliminated using constraint **A-D**:



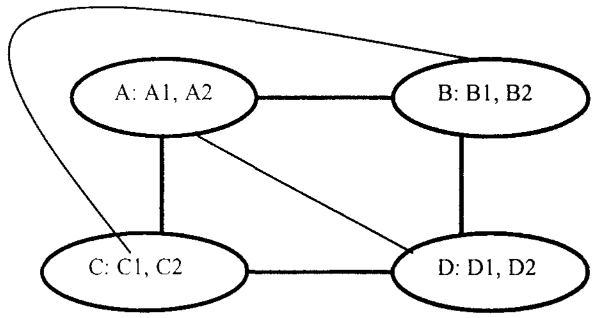
Part B Backtrack Search with Forward Checking (10 points)

In this part we **search** for **all** consistent solutions to the constraint problem described above. Variables are ordered alphabetically, and values are ordered numerically.

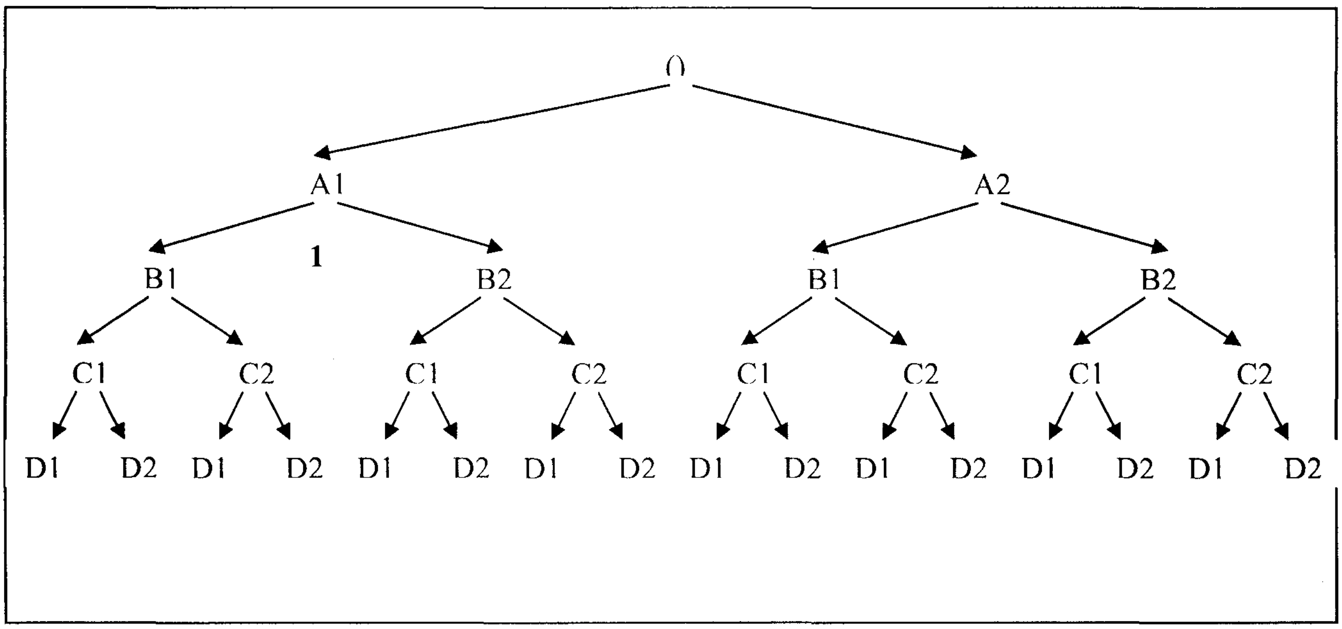
A copy of the complete search tree for this problem is given below. Each node (except the root) is labeled with the value involved in the assignment.

The constraint table and graph are repeated here for your convenience:

- **A-B**: <A1, B1> or <A2,B2>
- **A-C**: <A1, C2> or <A2,C1>
- **B-D**: <B1, D1> or <B1,D2>
- **C-D**: <C2, D1> or <C2, D2>
- **B-C**: <B1, C2>
- **A-D**: <A2, D2>



Indicate which assignments are made and the order of those assignments by writing a number under each assignment made on the copy of the search tree below. We have written **1** under the first assignment:



Problem 3 Linear Programming (12 points)

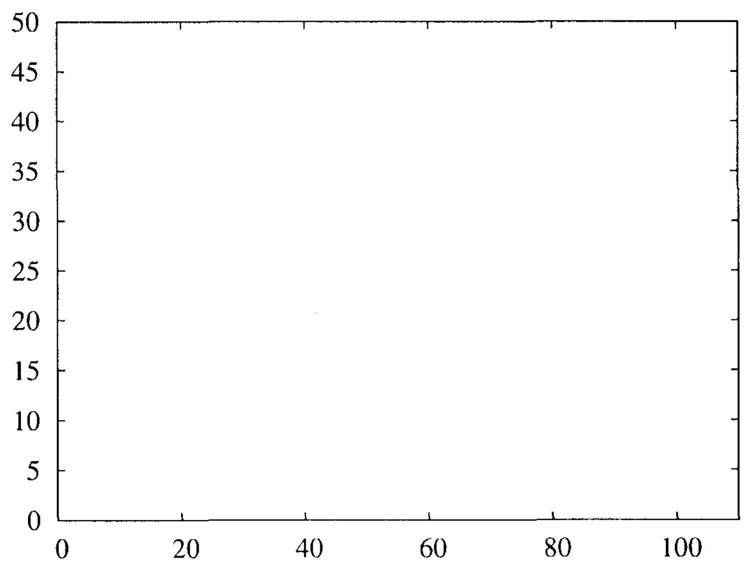
Part A Setting up Linear Programs (4 points)

Enron owns a coal-burning plant, which produces 2 Megawatts of power for every barrel of coal. Enron also owns an oil-burning plant, which produces 3 Megawatts of power for every barrel of oil. However, coal costs \$2/barrel, and oil costs \$10/barrel. Enron's creditors will only allow it to spend \$100/day. Also, Enron only has one truck, and it can only carry 40 barrels a day.

Enron needs to decide how much oil and how much coal to burn in order to maximize its power output and recover from bankruptcy. We can set this problem up as an LP. Please show how Enron's purchasing dilemma can be solved by writing in the following box a linear program that specifies the optimal choice of oil and coal to burn.

Part B The Simplex (8 points)

1. Graph the constraints and label the feasible region
2. Label the corners of the polytope by their co-ordinates
3. Label the order in which the simplex method searches the corners
4. Identify which corner represents the optimal purchase.



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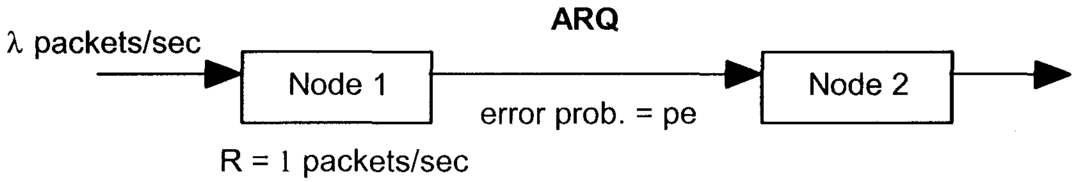
Human Factors

A human operator has the task of controlling a double integrator (acceleration control) system in a single axis, subject to a low frequency (0.1 rad/s) disturbance. A visual display and a joy stick are used.

1. What is the *form* of the operator's transfer function? Indicate reasonable values for the time delay, crossover frequency and remnant. (You are not expected to have memorized these.)
2. How would you simplify the task and help the operator to perform it?
3. Discuss two separate methods for measuring the operator's mental workload during the task.

Communications and Networks



A) Consider a Stop & Wait ARQ system, as shown in the figure. Packets arrive to node 1 according to a Poisson process of rate λ packets/second. Node 1 transmits the packets to node 2 at a fixed rate of 1 packet per second. The link has a packet error probability of Pe, and Stop and Wait is used for retransmission of packets containing errors. Let the amount of time for sending a packet and getting back an ACK be **exactly** S seconds (S includes transmission times) and assume no errors in the ACKs).

   i. What is the maximum throughput (in packets per second) achievable by the system?

   ii. What is the average queueing delay for a packet at node 1?

   iii. What is the average number of customers at node 1 (queue + server)?

B) Suppose that Go Back N is used instead of Stop and Wait. You should assume that Go back N with a window of S packets is used and a timeout occurs if an ACK is not received before the window expires.

   i. What is the maximum throughput (in packets per second) achievable by the system?

   ii. What is the average queueing delay for a packet at node 1?

You may find the following formulas useful:

$$\sum_{i=0}^{\infty} i\, q^i = \frac{q}{(1-q)^2}, \qquad \sum_{i=0}^{\infty} i^2 q^i = \frac{q+q^2}{(1-q)^3}.$$

The average queueing delay in an M/G/1 system is given by, $D = \frac{\lambda \overline{X^2}}{\quad}$

Control

The equation of motion for a mass-spring system shown in the left figure below is

$$m\ddot{x} = -k(x - u)$$

where x is the position of the mass to be controlled, and u is the control.

1. Derive the transfer function $h(s)$ from u to x.

2. Draw the locus of the closed-loop poles for the control diagram shown in the right figure, when the feedback $K(s)$ is simply a gain K_c that varies from $-\infty$ to ∞. For what values of K_c is the system i) Unstable? ii) Marginally stable? iii) Stable?

3. Consider now a proportional-plus-derivative controller $K(s) = K_c(s + s_0)$, where s_0 is a positive parameter of your own choosing. Choose K_c and s_0 such that the closed-loop poles are located at -3 rad/sec. For your choice of s_0, draw the root-locus of the system vs. K_c.

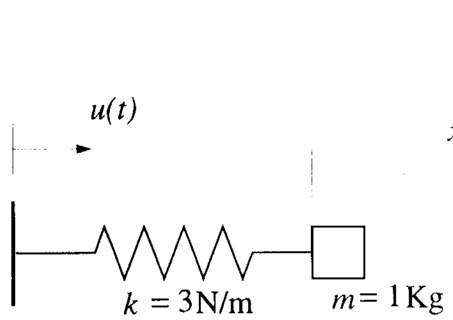
4. Now consider a refined model for the system. Due to motor dynamics, an additional pole is present in the open-loop transfer function, such that the "true" open-loop system is

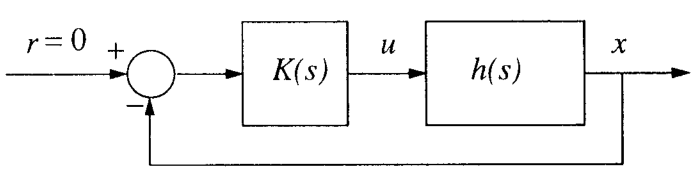
$$\begin{aligned} m\ddot{x} &= -k(x - u) \\ \dot{u} &= -10u + e \end{aligned}$$

   where the voltage e is the new input to this system.

   For this new system, we would like to use a proportional-plus-derivative controller of the form $K(s) = K_c(s + s_0)$, where s_0 is the same as that you have already determined in question 3. Sketch the new root-locus corresponding to this controller vs. K_c. Find the approximate position of the poles for the gain you have found in 3.

5. The $K(s)$ we have used so far has undesirable high gain characteristics at high frequencies. We therefore must modify it so as to keep its magnitude bounded at all frequencies. Based on your earlier design, design a controller whose effect on your system is approximately the same, but has bounded frequency-domain response. Sketch the corresponding root-locus.





Fluids

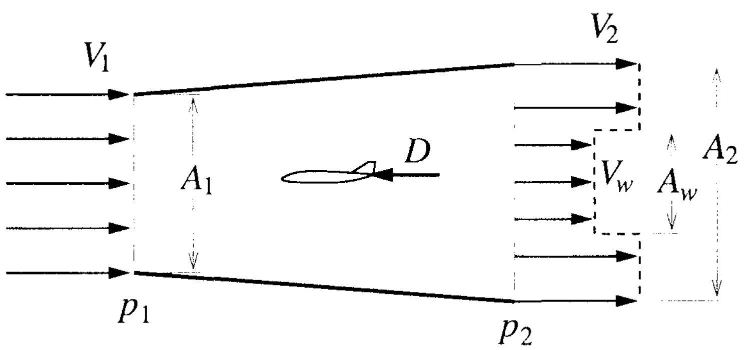
A variable-area wind tunnel is used to determine the drag force D on an aircraft model. The upstream flow station of area A_1 has uniform pressure p_1 and speed V_1. The downstream flow station of area A_2 has uniform pressure p_2, and a wake of area A_w and speed V_w surrounded by the otherwise uniform speed V_2. The air density ρ is effectively constant everywhere.

1) In the first test, the area A_2 is adjusted until $V_2 = V_1$, and $p_2 = p_1$, closely matching flight in the atmosphere. Determine the drag D in terms of the measured A_1, V_1, A_w, V_w, p_1, ρ.

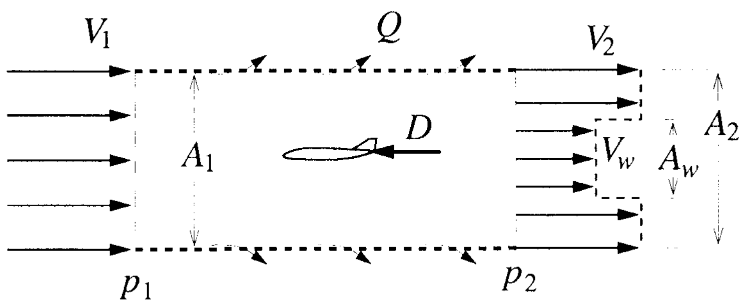
2) In a second test, the walls are set parallel, so that $A_2 = A_1$. Air is also sucked out through the porous walls at volume flow rate Q, adjusted so that $V_2 = V_1$, and $p_2 = p_1$. The removed air leaves the tunnel with a tangential speed component equal to V_1. Determine both Q and D in terms of A_1, V_1, A_w, V_w, p_1, ρ.

3) In a third test, the walls are still set parallel, $A_2 = A_1$, but the suction is turned off so that $Q = 0$. All the quantities are measured at stations 1 and 2. Determine both Q and D in terms of all these quantities, and also determine a constraint between them that expresses the conservation of mass.

1)



2), 3)



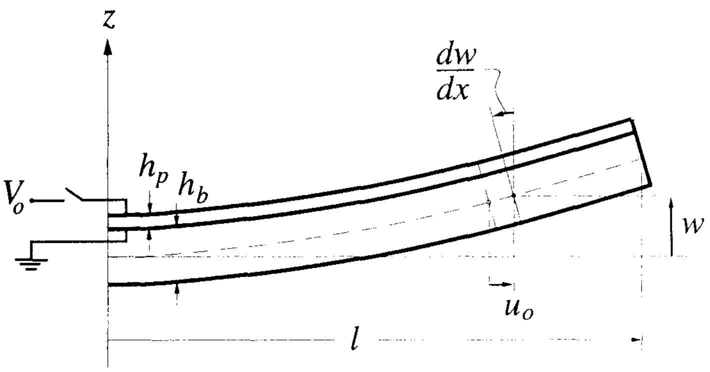
Materials & Structures

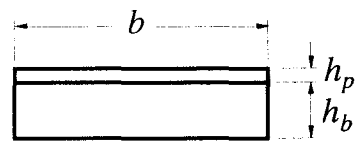
A cantilever beam has a piezoelectric element cemented to its top side, as shown in the figure. A voltage of $V_o = 100\,\mathrm{V}$ is applied across the piezoelectric element.

1) Derive the governing differential equations for stretching and bending of the cantilever structure. Explicitly state where you choose to place the $z = 0$ reference line.

2) Determine the tip deflection w_t.

3) Determine the normal stresses in the beam and the piezoelectric element.





cross–section

For piezoelectric		
E_p	=	$60\times10^9\ \mathrm{N/m^2}$
h_p	=	0.2×10^{-3} m
b	=	5×10^{-3} m
d_{13}	=	0.2×10^{-9} m/V

For beam		
E_b	=	$75\times10^9\ \mathrm{N/m^2}$
h_b	=	0.6×10^{-3} m
b	=	5×10^{-3} m
ℓ	=	0.05 m

$$\epsilon_x = \frac{\sigma_x}{E_p} + \frac{d_{13}}{h_p} V_o$$

Software Engineering

1) Differentiate between CMM and agile processes. Itemize and discuss the major similarities and differences.

2) Under what conditions would you use each?

Thermodynamics

Two thermodynamic cycles are shown below. The one on the left (Cycle A) is reversible.

a) What is the work done if a unit mass is taken round the reversible cycle A going from 0 to 1 to 2 back to 0?
[Suggestion: Before plunging into algebra, it might be useful to think about how the work of this cycle compares with the work of a Carnot cycle between temperatures T_0 and $2T_0$.]

b) What is the thermal efficiency of cycle A?

The cycle on the right (Cycle B) has two of the legs, 0 to 1 and 2 to 0, reversible. The leg from 1 to 2 (shown dashed) is *adiabatic but irreversible.*

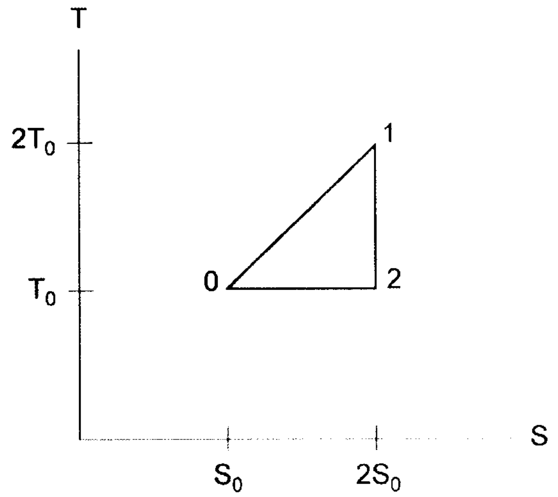
c) What is the ratio of the heat input for cycle B compared to the heat input for cycle A?

d) What is the ratio of the heat rejected for cycle B compared to the heat rejected by cycle A?

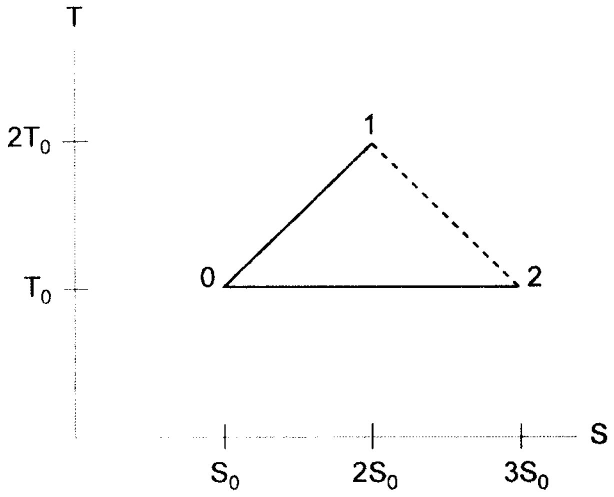
e) Is there work produced or absorbed by cycle B? What is its magnitude?

f) If cycle B *were* fully reversible, what would be the additional work that could be obtained compared to the situation with leg 1-2 irreversible?

g) *Could* one run cycle B in reverse? Why or why not? (A sentence or two explaining the answer is required.)



Cycle A



Cycle B

Vehicle Design and Performance

You may answer no more than one of the following two questions:

A. **Vehicle Design and Performance** (Aeronautics)

B. **Vehicle Design and Performance** (Astronautics)

Vehicle Design and Performance (Aeronautics)

An aircraft sizing study is performed, using the following independent design parameters.

m	gross takeoff mass
Σ	takeoff wing loading
AR	aspect ratio
$\bar{t}$	airfoil thickness/chord ratio
C_L	lift coefficient

From these, simple structural, aero, powerplant, and weight models are then used to derive numerous relevant dependent parameters. Some examples are

$b = \sqrt{mg\mathit{AR}/\Sigma}$	wing span
$v \simeq 0.5b^3/\mathit{AR}^2$	wing (fuel) volume
$V = \sqrt{2\Sigma/\rho C_L}$	flight speed at takeoff mass
$\mathit{Re} = VS/\nu b$	average-chord Reynolds number
$\mathit{PSFC} = \mathit{PSFC}(m_{\text{engine}}, \ldots)$	power-specific fuel consumption
$C_D = C_L^2/\pi\mathit{AR} + c_{d_p}(\mathit{Re}, C_L, \bar{t}) + \ldots$	drag coefficient
$m_{\text{fuel}} = m(1-f_{\text{pay}}) - m_{\text{structure}}(m, \Sigma, \mathit{AR}\ldots) - m_{\text{engine}} \cdots$	allowable fuel mass

A payload/gross mass fraction of $f_{\text{pay}} = 0.1$ is assumed. The assumed structural material properties correspond to common composites. The Breguet range is then finally examined, ultimately as a function of the independent parameters.

$$R(m, \Sigma, \mathit{AR}\ldots) \;=\; \frac{1}{g}\frac{\eta_p}{\mathit{PSFC}}\frac{C_L}{C_D}\ln\left(\frac{m}{m - m_{\text{fuel}}}\right)$$

In a sizing study, the gross mass is varied over a wide range $m = 1\ldots 100\,000$ kg (small UAV to large transport). All the other independent parameters are *optimized* so as to maximize the range R at each specified m. The resulting optimum R is shown in Figure 1, and the associated optimum AR and Σ are shown in Figures 2 and 3.

1) In Figure 1, what physical effects cause R to increase as m is increased from 1 kg to about 20000 kg? What causes the slight decrease as m increases beyond 20000 kg. Hint: examine the variables in the Breguet expression.

2) In Figure 2, why is the optimum AR largest for an intermediate mass? What causes is to decrease on the left of this maximum? On the right of this maximum?

3) In Figure 3, the optimum wing loading increases very nearly as $m^{1/3}$. What physical principle is responsible for this?

4) The assumed material properties are now upgraded to match exotic higher-strength and higher-stiffness composites, and the sizing study is repeated. Briefly describe how each of the curves in the three figures would change.

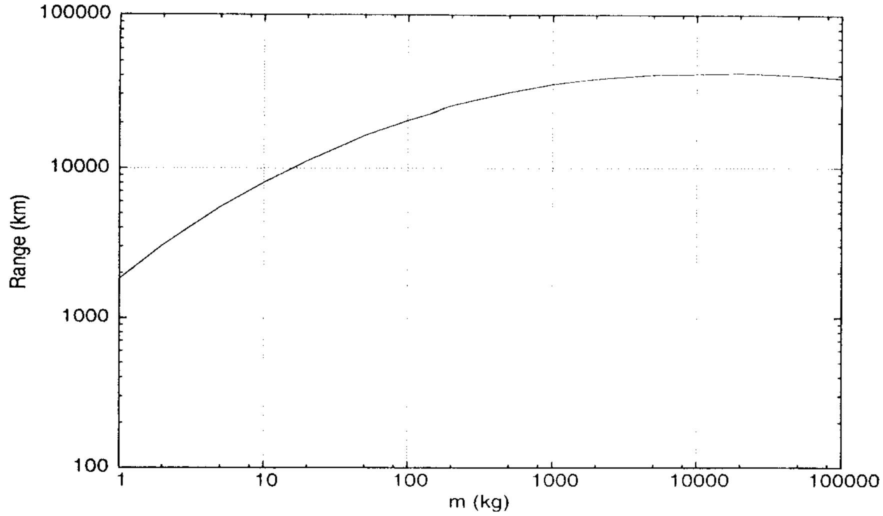


Figure 1: Range R versus mass m. All other parameters optimized for maximum range.

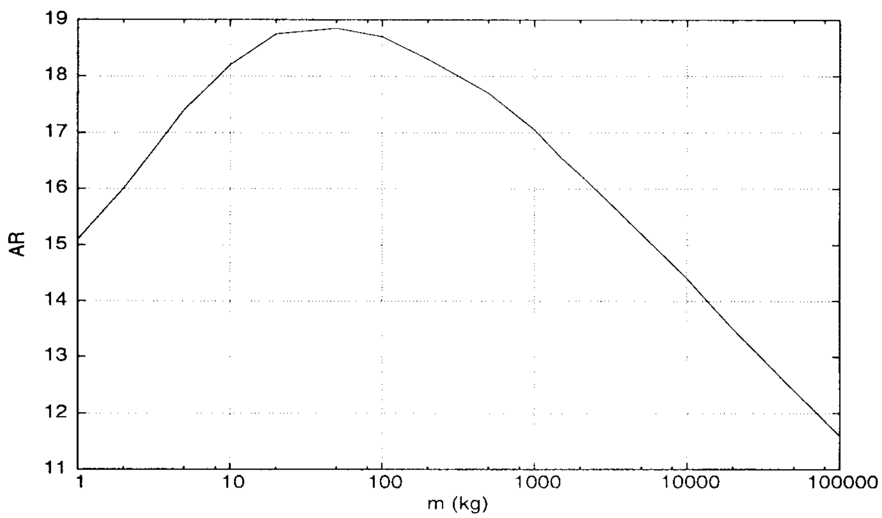


Figure 2: Aspect ratio AR optimized for maximum range, versus mass m.

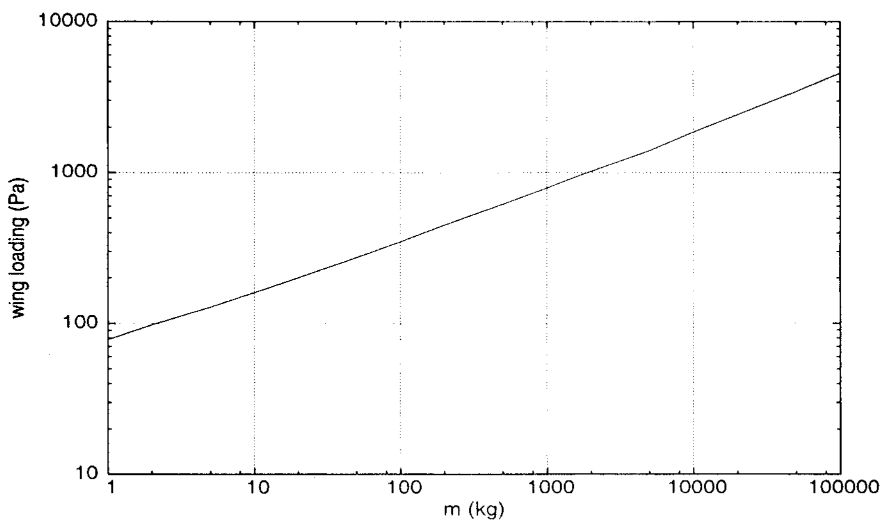


Figure 3: Wing loading Σ optimized for maximum range, versus mass m.

Vehicle Design and Performance (Astronautics)

Two competing architectures for lunar exploration must be compared in terms of their total departure mass in low Earth orbit (LEO), see Figure 1. Each architecture uses a slightly different set of modules and operational sequence.

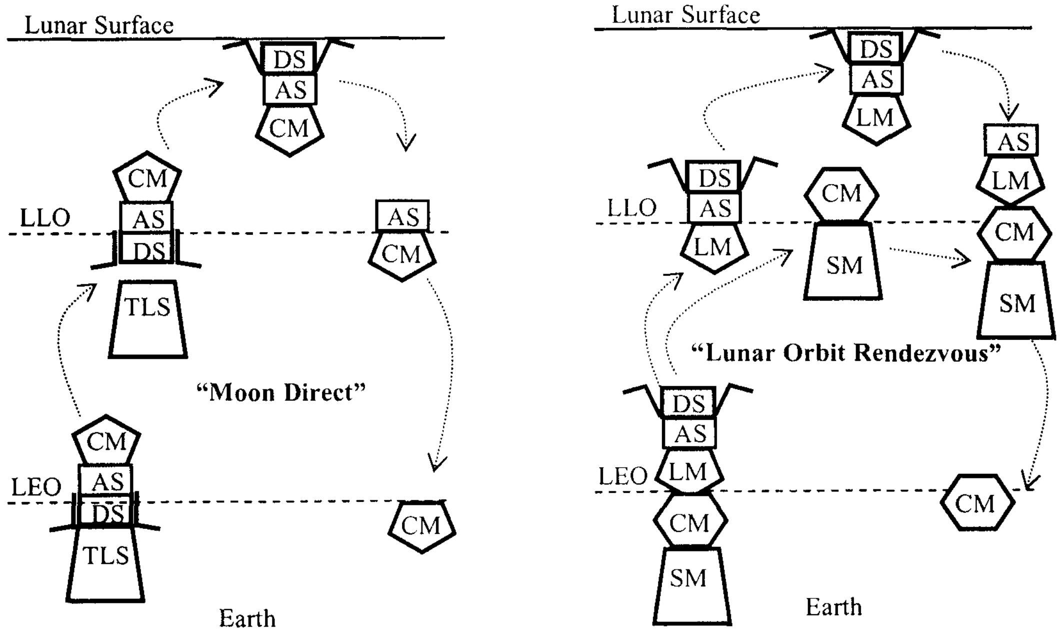


Figure 1: (left) "Moon Direct", (right) "Lunar Orbit Rendezvous"

Moon Direct: The Command Module (CM), Ascent Stage (AS), Descent Stage (DS) and Trans-Lunar Stage (TLS) leave together from low Earth orbit (LEO). The trans-lunar injection burn is provided by the TLS. Upon arrival in low lunar orbit (LLO) the TLS fires again to circularize the orbit around the Moon and is subsequently abandoned. The CM-AS-DS descend to the Moon together, with the DS providing descent propulsion and stabilization on the surface (landing gear). After surface exploration is complete, the CM-AS lift off together with the AS providing propulsion and the DS being left behind on the surface. The AS fires again in LLO to provide trans-Earth injection for the CM. The CM enters Earth's atmosphere directly for a ballistic reentry.

Lunar Orbit Rendezvous: The CM-SM and LM-AS-DS leave low Earth orbit together with the SM providing both the ΔV for trans-lunar injection and circularization in LLO. The LM-AS-DS descend to the lunar surface with the DS providing descent propulsion and stabilization on the surface (landing gear). During surface exploration the CM-SM waits in orbit. After surface exploration is complete, the LM-AS lift off together with the AS providing propulsion and the DS being left behind on the surface. The LM-AS and CM-SM rendezvous in orbit and the astronauts transfer from the LM back to the CM. The empty LM-AS are abandoned in lunar orbit. The SM fires to provide trans-Earth injection for the CM and is subsequently jettisoned. The CM enters Earth's atmosphere directly for a ballistic reentry.

a) **Compute the total departure mass in LEO [kg] for both architectures and compare.**
b) **Explain why one architecture is superior to another in terms of departure mass.**
c) **At what ratio of masses LM/CM would both architectures be equivalent in terms of departure mass?**
d) **What criteria, other than mass, must be considered for architecture selection?**

Data to be used: I_{sp}=400 [s], CM=6000 [kg], LM=2000 [kg], assume all tanks and engines are massless.

ΔV:	LEO→ LLO	3400 m/s	(trans-lunar injection)
	circularize at LLO	1100 m/s	
	LLO→ Lunar Surface	2200 m/s	
	Lunar Surface → LLO	2200 m/s	
	LLO→ LEO	1100 m/s	(trans-Earth injection)