JANUARY, 2007
DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS

WRITTEN QUALIFYING EXAMINATION
FOR
DOCTORAL CANDIDATES

Wed., January 24, 2007 Room 1-390 9:00 AM – 1:00 PM

CLOSED BOOK AND NOTES

Answer a total of five (5) questions (no more or less)

You must answer at least two (2) questions from Column A, (one (1) Math and one (1) Physics), and three (3) questions from Column B (Professional Area Subjects). Please answer each question in a separate blue book and indicate on the cover of the blue book which question is being answered.

Do not put your name on any of the blue books.

You will use a NUMBER (last four digits of your MIT ID) to identify yourself; make sure it appears on the cover of each of the blue books that you turn in to be graded.

Oral examinations will be held on Tuesday, January 30th. Please pick up your schedule on Monday, January 29th after 3:00 PM from the Aero Astro Student Services Office (33-208).

Results will be available from your advisor on Wednesday, January 31st after 3:00 PM.

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<tr>
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</tr>
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<tr>
<td>Mathematics (Discrete OR Continuous)</td>
<td>Autonomy</td>
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<tr>
<td>Physics (Dynamics OR Fields)</td>
<td>Communication and Networks</td>
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<td></td>
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Consider the following two-state Markov Chain:

1. What are the eigenvalues of the transition matrix for this Markov chain?
2. Please use the eigenvalues and eigenvectors to find the stationary distribution for this Markov chain.
Calculus

1) Determine the closest distance between the curve

\[ y = 1/\sqrt{x} \]

and the origin.

2) Given the first-order differential equation

\[ xy \, dx + f(x, y) \, dy = 0 \]

determine the necessary form that \( f(x, y) \) must take in order for this ODE to reduce to the
perfect differential

\[ d\phi = 0 \]

Determine the solution \( y(x) \) for your particular choice of \( f(x, y) \).

3) For the vector field

\[ \vec{v} = x^2 \hat{i} + \hat{j} \]

Evaluate the line integral

\[ I = \oint \vec{v} \cdot \hat{n} \, ds \]

around the ellipse defined by

\[ y = \pm \frac{1}{2} \sqrt{1-x^2} \]

4) The function \( y(x) \) satisfies

\[ x^2y^3 + \sin y = 0 \]

Obtain an expression for \( y'(x) \) in terms of \( x \) and \( y \).
This problem examines the dynamic behavior of two- and three-bladed propellers. We will make the following assumptions:

i) Aerodynamic forces are negligible.

ii) Each blade is a homogeneous slender bar of length $L$, and mass $m$.

iii) The mass of the hub is negligible.

While the propeller rotates steadily about the Z axis at rate $\dot{\phi} = p$, the XYZ coordinates are rotating about the Y axis at rate $\Omega$.

1) Determine the propeller’s tensor of inertia in the XYZ axes, as a function of the propeller’s angular position $\phi$, as shown.

2) Determine the XYZ moment, as a function of $\phi$, which must be applied to the propeller to rotate it at rate $\Omega$.

3) Repeat 1) and 2) for the case of a two-bladed propeller, with the blades at some angle $\phi$ and $\phi + \pi$.

4) A windmill must be yawed so that it always points into the wind. Based on your previous results, is it better for a windmill to have two blades or three blades? Explain.
Fields

During the second half of the 19th Century, James Clerk Maxwell derived a set of equations that fully describes previous experimental observations by Faraday, Ampere and Coulomb and reconciled them with conservation of electric charges and the lack of evidence of magnetic monopoles. In vacuum, these equations in differential form are as follows:

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]  
\text{(Gauss's Law for Electric Fields)}

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
\text{(Faraday's Law)}

\[ \nabla \cdot \vec{B} = 0 \]  
\text{(Gauss's Law for Magnetic Fields)}

\[ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]  
\text{(Ampere's Law and Charge Conservation)}

Suppose that an eminent experimental physicist discovers and announces the existence of magnetic monopoles. You are reconcile their presence with Maxwell's theory.

1) What effect does this discovery have on Gauss's Law for Magnetic Fields? Modify the equation if necessary.

2) Derive an equation for the conservation of magnetic monopoles, possibly in a time-dependent situation.

3) What modifications to the entire Maxwell set above are necessary to be fully consistent with the new observations? What is the physical interpretation of these modifications?

4) What is the effect of the discovery on the propagation of electromagnetic waves in free space?
Autonomy

1. Search

You are trying to find a path from the state S to state G in the following directed graph:

We will use A* search to find the lowest-cost path.

Assume the following:
- Each edge is labelled with its cost.
- Each node is labelled with its heuristic cost $h$.
- Given two or more equally good nodes, explore them in alphabetical order.
- The search stops as soon as the goal is expanded.

Using A* search, we construct a queue of partial paths, labelled by their total costs $f$. At each iteration of the search, we remove the lowest-cost path from the queue, until we find a complete path from the start to the goal. In the box below, please give the state of the queue at each step of the search. Please give each partial path on the queue in order from lowest-cost (left-most) to highest cost. Please include in each partial path the $f$ value. The queue initially contains a single partial path (S, $f=3$).
2. Linear Programming

Solve the following linear program using simplex, showing all simplex steps:

Maximize $z = 3x_1 + 2x_2 + 4x_3$
Subject to $x_1 + x_2 + x_3 \leq 6$
$x_1 + x_3 \leq 8$
$x_2 + x_3 \leq 5$
$x_1, x_2, x_3 \geq 0$

3. Inference

Consider the following logical theory:

$C \Rightarrow (A \land B)$
$(\neg D \lor E) \Leftrightarrow (A \land C)$
$C$

Find a valid interpretation for this theory. Please identify which inference algorithm (unit propagation, DPLL, resolution, etc.) you are using.
Communication

1. Consider the binary discrete-time communication system shown in Fig. 1. Assume that \( m = 0 \) and \( m = 1 \) are equally likely to occur. Under hypothesis \( H_m(m = 0, 1) \), the received signal \( r[n] \) is given by

\[
r[n] = y_m[n] + w[n] = s_m[n] * h[n] + w[n],
\]

where "*" denotes convolution, and \( w[n] \)'s are zero-mean statistically independent Gaussian random variables with variance \( \sigma^2 \). The two signals, \( s_0[n] \) and \( s_1[n] \) are shown in Figs. 2 and 3, respectively. The parameter \( \lambda \) in Fig. 3 satisfies \( 0 \leq \lambda \leq 1 \). The dispersive channel's impulse response is modeled by the linear time-invariant filter \( h[n] \) as shown in Fig. 4.

We wish to obtain a rule for making a decision about which signal was transmitted, based on the observed sequence \( r[n] \).

(a) Which samples of \( r[n] \) provide information in making the decision? Justify your reasoning.

(b) Find an optimum decision rule, which minimizes probability of error, based on observation of \( r[n] \). Simplify your processor as much as possible to minimize computation.

(c) Obtain an expression for probability of error, \( \Pr[\cdot] \), in terms of \( \lambda \) and \( Q(\cdot) \), where \( Q(\cdot) \) is defined as

\[
Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\alpha^2/2} d\alpha.
\]

![Figure 1](image-url)
Communication (cont'd)

Figure 2

Figure 3

Figure 4
Control
Consider an aircraft flying with the goal of tracking a desired path which is a slight perturbation of a straight line. The guidance logic is similar to the pure pursuit, wherein a reference point on the desired path is designated and a lateral acceleration command is generated according to the direction of the reference point, relative to vehicle velocity. The reference point is on the desired path at a constant distance \( L_1 \) forward of the vehicle. The lateral acceleration command is

\[
a_{s\text{cmd}} = \frac{V^2}{L_1} \sin \eta
\]

Answer the following - approximate values are marked:

1. (25%) Demonstrate that, under a small angle assumption, the control logic is a PD controller.

\[
a_{s\text{cmd}} \approx \frac{2V}{L_1} \left( \dot{d} + \frac{V}{L_1} \dot{d} \right)
\]

2. (30%) Show that if we ignore the inner loop vehicle dynamics so that the dynamics for the lateral error are approximately \( \ddot{d} \approx -a_{s\text{cmd}} \), then the closed loop transfer function can be written as

\[
\frac{d(s)}{d_{ref,pe}(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

(a) Give the appropriate values of \( \zeta \) and \( \omega_n \) in terms of \( V \) and \( L_1 \).

(b) What does Eq. 3 suggest about the performance of the closed-loop system?

3. (25%) Given these target pole locations, derive a lead and/or lag compensator of the form

\[
K(s) = \frac{k \omega_n^2}{s + b}
\]

that puts two of the closed poles for the system \( 1/s^2 \) at the same location as those in the transfer function in Eq. 3 (for simplicity assume \( L_1 = V \times (1 \text{ sec}) \)). Where is the third pole?

4. (10%) If it turns out that the actual vehicle has inner loop dynamics that cannot be ignored because of a time delay in the actuation (of approximately 0.5 sec), what do you predict will be the effect on the closed-loop system for small values of \( L_1 \)?

5. (10%) If the desired path happened to be a circle, and the aircraft was initially on the circle with the correct orientation, demonstrate that the control logic generates an acceleration command that is equal to the centripetal acceleration required to follow this path. When will this break down?
Fluids

A nozzle attached to a fire hydrant shoots a thin stream of water with density \( \rho \), speed \( V \), and area \( A \). The stream is aimed at a flexible curved deflector surface which has an adjustable angle \( \theta \). The deflector is mounted on a cart which can be moved at some speed \( V_c \). Load sensors measure the force components \( F_x \) and \( F_y \) that the surface imparts to the cart. Neglect friction and gravity in this problem.

![Diagram of a fire hydrant and a cart with a deflector surface, showing the forces \( F_x \) and \( F_y \), and the angle \( \theta \).]

1) With the cart motionless at \( V_c = 0 \) and the surface set at \( \theta = 90^\circ \), determine \( F_x \) and \( F_y \).

2) The cart is now moving at some \( V_c \neq 0 \), and the deflector is held at some arbitrary \( \theta \). Determine \( F_x \) and \( F_y \) as functions of \( V_c \) and \( \theta \).

3) The cart is stationary again \( (V_c = 0) \), but now the deflector angle is rotated at some steady rate \( \theta = \omega \). The radius to the tip of the deflector is \( R \), as shown. Determine \( F_x \) and \( F_y \) as functions of \( \theta \) and \( \omega \).
As a human factors consultant, you are asked to determine which of 2 competing Flight Management Systems produces superior pilot performance in terms of initially loading a flight plan. You have access to 10 subject matter experts that will test the systems.

a) Should you use a between or within subjects test and why?

b) Each of the two companies tells you they already have experimental data that demonstrates average pilot performance. Company A used 8 subjects and found a mean time of flight plan loading to be 191 seconds, with a standard deviation of 38s. Company B used 10 subjects with a mean of 199s, standard deviation of 12s. Assuming all populations are normally distributed. Given α=.05, is one system better or worse than another? Show your calculations.

c) You decide you do not trust the companies’ numbers and decide to run your own experiment. In addition to testing for loading times, what other metrics could you measure and why?

d) You run an experiment and compute a p value of .065. Explain, in general, what a p value means and then specifically, discuss whether or not your p value is significant (α=.05).

### Important Formulas

**Variance Test**

\[
F = \frac{S_1^2}{S_2^2}
\]

DOF num: \(n_1-1\), denom: \(n_2-1\)

\(s_1 > s_2\)

**Difference in Means Tests**

Independent z test, large population

\[
z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}
\]
Between subjects, independent t-test:

With unequal variance:
\[
t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
\text{DOF = smaller of (n}_1-1\text{) or (n}_2-1\text{)}
\]

With equal variance:
\[
t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
\text{DOF = } n_1 + n_2 - 2
\]

Within-subjects, dependent t-test:
\[
t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{n}} = \frac{\sum D}{n} s_D = \sqrt{\frac{\sum D^2 - (\sum D)^2}{n-1}} \\
\text{DOF = } n-1
\]
\[D = X_1 - X_2\]
### Table A.4

CRITICAL VALUES OF $t$

For particular number of degrees of freedom, entry represents the critical value of $t$ corresponding to a specified upper tail area ($a$).

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>Upper Tail Areas</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.25</td>
</tr>
<tr>
<td>1</td>
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<tr>
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(Continued)
1. By accident, a scuba diver drops his compressed air bottle to the ground. The valve breaks off such that compressed air rushes out through a 1 cm diameter hole at the end of the bottle. The 20 cm diameter, 20kg heavy bottle is pressurized at 150 bar and at an ambient temperature of 15 deg C. The ambient pressure at sea level is 1 bar. The internal volume is $0.015 \text{ m}^3$.

a) What determines the rate of change of air mass inside the bottle? Write down an equation that captures the underlying concept.

b) Describe the attributes of the air flow out of the bottle. What are the parameters that govern the mass flow through the small hole at the end of the bottle?

c) Combine your findings from above and determine the pressure of the air inside the bottle as a function of time.

d) At what time will the flow out of the bottle unchoke?

e) Assuming that the air velocity and the mass flow out of the bottle stay constant at their initial values (at $t = 0$), determine the steady-state velocity at which the bottle is propelled. You can assume a drag coefficient of 1.5 and a friction coefficient of 1.0 between aluminum and concrete.
Real-time Software Systems

1. Scheduling

Suppose we have three tasks, with parameters T1=(0, 50, 25, 25), T2=(0, 60, 10, 20), T3=(125, 125, 25, 50). The 4 parameters are (phase, period, execution time, relative deadline). Please schedule this job using Rate-Monotonic scheduling. Identify any missed deadlines.

2. Deadlocks and Livelocks

Describe one way (if any) that a concurrent embedded system can guarantee the absence of deadlocks.

Describe one way (if any) that a concurrent embedded system can guarantee the absence of livelocks.

3. Concurrency and Mutual Exclusion

Consider a java-like language that uses the synchronized keyword to provide mutual exclusion; only one thread at a time may enter a block of code labelled synchronized.

Now imagine that there are two kinds of concurrent processes, Producer and Consumer. The Producer and Consumer classes are set up such that only a single Producer at a time can add new data to the queue using push, but multiple Consumers can retrieve data from the queue using pop.

```java
public class Consumer extends Thread {
    public void run() {
        while (true) {
            synchronized(producer) {
                if (queue.isEmpty() && !producing)
                    producer.wait();
            }
            Data data = queue.pop();
            synchronized(producer) {
                producer.notify();
            }
            processData(data);
        }
    }
}
```

```java
public class Producer extends Thread {
    public void run() {
        while (true) {
            Data data = generateData();
            synchronized(consumer) {
                if (queue.isFull())
                    consumer.wait();
                producing = true;
                queue.push(data);
                producing = false;
                consumer.notifyAll();
            }
        }
    }
}
```

However, this implementation of the single-producer/multiple-consumer problem contains a race condition. For example, every now and then, pop() fails in a thread, complaining there is no data, even though the same thread checked to make sure there was data. What is the failure of this implementation (there may be more than one failure)? Suggest how the implementation can be fixed, while allowing multiple consumers to process data concurrently. You can either write new pseudo-code, new Java, or describe the necessary changes in English.
A product is made of several alternating layers of 200 mm thick aluminum and 20 mm thick styrofoam. The aluminum has a modulus of 70 GPa and a Poisson's ratio of 0.30. The (isotropic) styrofoam has a modulus of 4.0 GPa, and a major Poisson's ratio of 0.35. The structural configuration, as shown, is supported at one end with pins, and at the other end with roller supports. The overall length is 4 meters and the critical loading is a mid-span load of up to 2000 Newtons (this includes a factor to account for dynamic effects).

Find:

(a) the maximum stress, $\sigma_{xx'}$ in the beam and its location.

(b) the maximum shear stress, $\sigma_{xy'}$ in the beam and its location.

(c) the maximum deflection of the beam and its location.
Vehicle Design and Performance

1) Describe the phases of vehicle design. For each phase, state the objectives for that phase and describe the deliverables.

2) Describe a commonly occurring organizational problem in vehicle design and suggest ways in which it can be resolved using management tools.

3) Not all budgets are financial. Describe the various budgets that are tracked during vehicle design as well as the process by which margin is allocated.

4) Describe the role of prototyping in vehicle design.

Note: Prototyping is defined here as the process of developing a model (software, hardware, or operational) of a portion of the vehicle.