Sequential Procurement Auctions and Their Effect on Investment Decisions*

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May 2015

Abstract

We characterize the optimal mechanism and investment level in an environment where (i) two projects are purchased sequentially, (ii) the buyer can commit to a two-period mechanism, and (iii) the winner of the first project can invest in a cost-reducing technology between auctions. We show that, in an attempt to induce more competition in the first period, the optimal mechanism gives an advantage to the first-period winner in the second auction. As a result of this advantage, the first-period winner invests more than the socially efficient level. Optimal advantages therefore create two channels for cost minimization in buyer-supplier relationships.

Keywords: *Procurement Auctions, Sequential Mechanisms, Mechanism Design, Cost-Reducing Investment. JEL C72, D44, D82, D92.*

*We are especially grateful to Soledad Arellano, Leandro Arozamena, Alessandro Bonatti, Vinicius Carrasco, Héctor Chade, Bob Gibbons, Heikki Rantakari, and Mike Whinston for their valuable feedback on this article. We would also like to thank the editor, David Martimort, and an anonymous referee for very thoughtful comments. Nicolás Figueroa was partially supported by the Complex Engineering Systems Institute, Millennium Nucleus Information and Coordination in Networks ICM/FIC RC130003, and by Fondecyt Grant N^o 1141124.

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1 Introduction

Managing relationships with suppliers is critical to the success of many firms. To achieve low procurement costs, a buyer may want to encourage competition among suppliers, but she may also hope that suppliers make investments that lower their costs of production. Two extreme views appear. The *arm's-length* model—prevalent in the U.S.—promotes competition in every period to strengthen a buyer's bargaining power (Porter, 1985). The *partnership model*—adopted in some Japanese industries—instead fosters long-term relationships by awarding advantages to suppliers with records of good performance. In return, these suppliers invest in specific assets that can reduce production costs.[1]

In this article we ask how the desires to promote competition and, at the same time, to encourage relationship-specific investments, may be best balanced to minimize a buyer's cost of procurement. Using the mechanism-design approach, we show that advantages to incumbents can optimally arise in fully competitive environments characterized by the repeated interaction between a buyer and multiple sellers. Even more so, we show that these advantages can generate strong incentives to invest in cost reduction. In dynamic settings therefore, contracts that lie in between the arm's length and partnership models can have a critical impact on the long-term cost structure of suppliers.

In our model, a buyer must purchase two projects sequentially from a pool of potential sellers. The sellers' costs for performing both tasks are distributed independently across time and across competitors, and they are private information to each firm. We assume that the first-period winner can improve his cost distribution for the second competition by undertaking a costly investment between auctions. This decision can be observable (and contractible) or not. In this context, we characterize (i) the cost-minimizing mechanism chosen by a seller with commitment power and (ii) the investment level carried out by the first-period winner. Because the return from investing in a better technology depends on how the second-period mechanism treats the first-period winner, the final degree of asymmetry between sellers is determined endogenously by the optimal mechanism.

[1]See Dyer (1994) and Dyer et al. (1998) for evidence from the Japanese automobile industry.

Our model fits many real-world examples of repeated buyer-supplier interactions in which winning bidders have the possibility to invest in order to reduce their costs, or to improve the value of the goods offered to the buyer. In the car industry for example, there is vast evidence that suppliers that win a contract in this industry get to better understand the manufacturers' needs, which allows for investments that better tailor products to the specific needs of the purchaser (Richardson, 1993). A similar phenomenon takes place in spectrum auctions, where firms that develop significant market positions invest in their brand name and in their knowledge about their customer base, thus developing a higher valuation for frequencies in future sales (Klemperer, 2004). In all these settings the buyer faces an important choice. Should she promote competition in every period to obtain lower prices? Or should she, instead, give an advantage to a previous winner in order to foster investments that will eventually result in lower prices?

Although in practice all sellers have the *potential* to improve their technologies, there are many instances in which significant investments take place only after becoming an established provider. In the defense industry for instance, technological capabilities developed through learning-by-doing play a key role in shaping a seller's cost structure. Winners of previous auctions thus have a natural advantage over losers, as developing a final product provides them with information that is unavailable to those sellers that stop at the design stage. A similar phenomenon occurs in purchases that involve sellers making relationship-specific investments. Dyer (1994, 1996) documents that suppliers of Toyota and Chrysler undertook important relationship-specific investments (manufacturer-specific software, human capital, proximity to plants, etc.) after becoming well-established providers to each firm.[2] It is often the case that these types of investments are prohibitively costly for those sellers who are not currently serving the buyer. Alternatively, sellers who have not been awarded previous contracts may lack the necessary information to identify which types of investments effectively add value to the relationship.

As a benchmark, we first consider the ex-post efficient mechanism: in each period

[2]Investments in plant proximity also take place in the case of contracts for the distribution of school meals in Chile, as winners build infrastructure in different regions of the country (Olivares et al., 2011).

the project is assigned to the lowest-cost supplier, and the efficient level of investment optimally balances the cost of such decision with the benefit of having a better competitor when the efficient allocation rule is in place. In this case, we show that the planner's and the winner's investment incentives are aligned. Consequently, the socially efficient level of investment can be implemented using two second-price procurement auctions.

We then use the mechanism-design approach to characterize the cost-minimizing mechanism under incentive constraints. When the buyer is able to commit to a two-period mechanism, we show that the optimal mechanism gives an *advantage* to the first-period winner in the second auction: it awards him the project even in some cases in which his cost is larger than the cost of other competitors. The reasoning is as follows. In the setting we study, the buyer cannot commit ex-ante to prevent the participation at $t = 2$ of a seller who did not participate in the first auction.[3] Hence, in order to induce participation in the first period, the buyer can threat a seller who skips the first auction with a low (yet, strictly positive) second-period rent. Such threat can be, in turn, implemented using a second-period auction that favors the first-period winner. Because the buyer can always extract the winner's additional rent through a lower first-period transfer, this form of relaxation of the first-period participation constraint entails no extra cost (in addition to the ex-post inefficiency introduced), and it reduces the total cost of procurement. Interestingly, this advantage (hence, the optimal mechanism) is independent of the investment carried out by the winner, and also bounded away from zero uniformly for any number of competitors. Hence, it is driven solely by the buyer's incentive to *encourage competition.*

A central result of this article is that this cost-minimizing mechanism generates *over-investment*—to the the best our knowledge, this is a new insight in the literature of investment in auctions. More specifically, we show that the buyer's and the winner's investment incentives are aligned (as in the efficiency benchmark case), and that the optimal level of investment is larger than the socially efficient one. Consequently, the only costs for incentive provision that the buyer faces come from the sellers' private

[3] Equivalently, the buyer does not extract all second-period rents at time zero using an entry fee. For example, the buyer may not know if a supplier in period 2 existed prior to the first-period competition.

information regarding their costs (adverse selection), and not from the non-observability of the investment decision (moral hazard).

From the point of view of the buyer, over-investing is optimal because it mitigates the ex-post inefficiency of allocating the second project to the first-period winner too frequently. The reason for why the first-period winner would like to over-invest is more subtle. In fact, an advantage gap could potentially induce the first-period winner to relax, knowing that he is likely to win anyway. However, a bigger advantage has the crucial property that it increases the sensitivity of the winner's second-period rent to the investment decision, as investing is valuable only in those events in which he wins. Because awarding an advantage yields a winning probability that is larger than the efficient one, the winner's marginal incentives become steeper, so he over-invests.

We conclude our analysis by studying the case in which the buyer cannot commit in advance to the form of the second-period auction: the investment problem thus becomes one of choosing a technology before a one-shot auction. In this case, we show that the investment falls below its socially efficient level (see also Dasgupta (1990), Piccione and Tan (1996), and Arozamena and Cantillón (2004) for similar results). This is because the optimal one-shot mechanism is biased against the first-period winner, which reduces the likelihood of winning below the socially efficient counterpart. In contrast to the commitment case, investment observability matters in this setting. In particular, due to a hold-up effect, investment is at its lowest level when investment is observable, as the buyer can *react* to larger investment levels with even more disadvantageous mechanisms.

Our results have important normative implications. First, cost-minimizing buyers with the ability to commit can use rewards in the form of future advantages as a tool to increase current competition, and thus to reduce the current costs of procurement. In this sense, the optimality of a mechanism lying between a sequence of second-price auctions (as in the American model) and the implicit guarantee of a long-term contract (as in the Japanese model) highlights the basic trade off between inducing competition and having the flexibility to change suppliers in future purchases. Second, strong investment incentives on behalf of incumbents are induced as a byproduct of these future advantages. This can be particularly important if the goal is to reduce long-run costs.

The features and predictions of our model resemble the experience of Chrysler during the eighties, when it transformed the structure of its relationship with providers. In order to promote investment and reduce costs in the long-run, Chrysler made it clear that long-term relationships—rather than squeezing suppliers' margins—was the new premise, and decided to start ranking suppliers based on past performance. This policy had the following consequences. First, Chrysler's supplier base was reduced from 2500 to 1140 companies (yet still large relative to its Japanese counterparts). Second, survivors gave up rents in order to ascend in the ranking, which is a clear signal of more intense competition. Third, the average length of a typical relationship increased from 2.1 to 4.4 years. Finally, suppliers undertook significant relationship-specific investments, due to an expectation of a business relationship beyond a particular contract (Dyer, 1996).

The article is organized as follows. Section 2 presents the model. Section 3 studies the efficiency benchmark. Section 4 characterizes the optimal mechanism and investment level under full commitment. Section 5 analyzes the case without commitment. The Appendix contains all the proofs.

Related Literature

Our model is related to several literatures. Relative to the the literature on dynamic auctions, our model is the first to consider investment incentives. Pesendorfer and Jofre-Bonet (2014) study a two-player version of our model in which cost distributions are exogenous in both periods. Lewis and Yildirim (2002) analyze an infinitely repeated game between a buyer and two sellers who become more efficient exogenously as they win auctions. Both articles show that the current leading firm is awarded an advantage in the optimal mechanism. Luton and McAfee (1986) instead analyze a two-period model in which sellers' costs exhibit a specific form of correlation, thus leading the buyer to choose a mechanism that disadvantages the first-period winner in the second auction. In contrast to these articles, in our setting the second-period asymmetry in cost distributions is determined endogenously by the optimal choice both of mechanism and investment level.

The interplay between auctions and investment incentives has previously been studied only in static contexts. Piccione and Tan (1996) show that, in an attempt to extract information rents, the optimal mechanism chosen by a buyer who lacks commitment is ex-post inefficient, which leads to under-investment. Dasgupta (1990) obtains the same result in a similar model, and also shows that commitment increases investment, but always below efficiency. Finally, Arozamena and Cantillón (2004) analyze the effects of allowing only one firm to invest before a first-price procurement auction, with this action being perfectly observable. They find that the firm under-invests as a result of its competitors responding aggressively to the investment decision.

To conclude, our article also relates to the literature on optimal regulation and advantages. In the seminal model of Baron and Besanko (1984) a regulator chooses an efficient pricing rule at $t = 2$ when a monopoly's costs are independent across time, which in turn induces efficient investment incentives. Finally, in the two-period model of Laffont and Tirole (1988) an incumbent can make a cost-reducing investment in the presence of a potential entrant in the last period of interaction. When this technology can be inherited by the entrant, the optimal replacement rule gives an advantage to the incumbent, as the latter would invest inefficiently low due to the possibility of replacement.

2 The Model

The Environment

Consider a risk-neutral buyer (she) who is interested in purchasing two projects, one at $t = 1$ and the other at $t = 2$. The set of competing sellers is $N = \{1, ..., n\}$, $n \geq 2$, all of which are risk neutral and live for the two periods. Providing these projects by herself is prohibitively costly to the buyer, and hence she is forced to buy the projects from the pool of potential suppliers (i.e., her reservation cos).

In each period, any seller's cost of undertaking the project is drawn from the interval $C = [\underline{c}, \bar{c}]$, and it is his private information. In the first period these costs are independent across sellers, and distributed according to a differentiable c.d.f. $F(\cdot)$ that satisfies $f(c) \equiv F'(c) > 0$, $c \in C$. Letting $\vec{c} := (c_1, ..., c_n)$ and $c_{-i} := (c_1, ..., c_{i-1}, c_{i+1}, ..., c_n)$, we use

$$f^n(\vec{c}) := \prod_{j=1}^{n} f(c_j) \text{ and } f^{n-1}(c_{-i}) := \prod_{j \neq i} f(c_j)$$

to denote the first-period (joint) densities.

In the second period, costs are independent across sellers, and also independent from those costs drawn in period 1. The sellers who were not awarded the first project draw their costs from the same distribution $F(\cdot)$. Instead, winning at $t = 1$ activates the option of investing in a cost-reducing technology. More specifically, if the first-period *winner* invests an amount $I \geq 0$ between auctions, his cost distribution now becomes $G(\cdot, I)$, with density $g(\cdot, I)$, and support C. Investing is costly according to a differentiable function $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ which is strictly increasing and convex, and that satisfies $\Psi(0) = \Psi'(0) = 0$.

The time-independence assumption admits the following interpretation (Lewis and Yildirim, 2002): the cost to perform each project consists of a common (and known) component across sellers (i.e., $\underline{c} \geq 0$) plus an idiosyncratic one that is transitory. The first-period winner can therefore invest in a technology that reduces this transitory component in a statistical sense.[4] As an example, consider a car manufacturer that is evaluating producing a new model. The transitory cost component then captures any new specifications that the new line of production requires. In this case, established suppliers are more likely to develop the know-how to make relationship-specific investments that can better accommodate to the new requirements by the car manufacturer.

We make the following assumptions regarding the cost distributions:

Assumption 1.

(i) *$c + F(c)/f(c)$ is strictly increasing in c.*

[4] In contrast, Lewis and Yildirim (2002) model learning by doing as a deterministic improvement of the common component.

(ii) $F(c) \leq G(c,0)$ *for all* $c \in C$.

(iii) *For each* $c \in C$, $I \mapsto G(c,I)$ *is twice continuously differentiable and strictly increasing and concave.*

Condition (i) corresponds to the standard monotonicity condition on virtual costs that makes the auction-design problem a "regular" one (Myerson, 1981). Condition (ii) captures the idea that the first-period winner can potentially improve his second-period distribution even in the absence of a costly investment decision (for instance, due to an acquired *know-how*). Finally, condition (iii) states that as investment increases, the winner's cost distribution puts more weight on lower costs (i.e., the family $\{G(\cdot,I) \mid I \geq 0\}$ satisfies first-order stochastic dominance as I decreases). The latter assumption is fairly general and weaker than the usual *monotone likelihood ratio* ordering (Milgrom, 1981).[5]

The previous assumptions are not hard to satisfy. The following family of distributions (used to model cost reduction in R&D; see Piccione and Tan, 1996), satisfies them:

Example 1. *Let* $F(\cdot)$ *be any twice differentiable and concave distribution. Define* $G(c,0) = F(c)^{\eta}$, $0 < \eta < 1$, *and* $G(c,I) = 1-(1-G(c,0))^{\gamma I+1}$, $\gamma > 0$. *Then,* $F(\cdot)$ *and* $\{G(\cdot,I) \mid I \geq 0\}$ *satisfy (i)-(iii) in Assumption 1.*

The Mechanisms

We focus on the case of a buyer that can commit to a two-period mechanism before any cost realization takes place. Hence, the revelation principle trivially applies. The case in which the buyer can commit to one-period contracts is analyzed in Section 5.

In what follows we consider mechanisms that are history-dependent only to the degree that second-period rules depend on the identity of the first-period winner: because costs are i.i.d. over time, this is without loss of generality (Lemma 2 in Section 4). Mechanisms of this sort are also typical in practice. For example, in defense procurement contractors must be awarded small projects before they can even compete in bigger competitions.

[5] In our context, MLRP corresponds to: For all $c' < c \in C$ and $0 \leq I' < I \in \mathbb{R}$, $\frac{f(c')}{f(c)} \leq \frac{g(c',I')}{g(c,I')} < \frac{g(c',I)}{g(c,I)}$.

Similarly in the Chilean purchasing system of school meals, where entrants (or previous losers) are awarded advantages in future auctions in order to avoid monopolization (Olivares et al., 2011).[6]

We will use subscript w to refer to the first-period winner at $t = 2$, and subscript (ℓ, i) at $t = 2$ if seller $i \in N$ was a first-period loser. Let $\Delta_n := \left\{\vec{x} \in \mathbb{R}^n_+ | \, x_1 + ... + x_n = 1\right\}$ denote the unit simplex.

Definition 1. *A direct mechanism* Γ *corresponds to a tuple*

$$\Gamma = (t^1, q^1, t^2_w, q^2_w, t^2_\ell, q^2_\ell)$$

where $t^1 : C^n \to \mathbb{R}^n$, $q^1 : C^n \to \Delta_n$, $t^2_w : C^n \to \mathbb{R}$, $q^2_w : C^n \to [0,1]$, $t^2_\ell : C^n \to \mathbb{R}^{n-1}$, $q^2_\ell : C^n \to [0,1]^{n-1}$, *such that* $q^2_w(\vec{c}) + \sum_{i \neq w} q^2_{\ell,i}(\vec{c}) = 1$ *for all* $\vec{c} \in C^n$. *If a mechanism implements a level of investment* $I \geq 0$, *we denote it by* $\Gamma(I)$.

In this definition, $t^1(\vec{c})$ is a vector of transfers in which the coordinate $t^1_i(\vec{c})$ corresponds to the transfer to seller $i \in N$ at time $t = 1$, conditional on the report $\vec{c} = (c_1, ..., c_n)$. The probability distribution $q^1(\vec{c})$ over $\{1, ..., N\}$ is such that $q^1_i(\vec{c})$ denotes the probability that competitor i wins the first procurement auction conditional on the report profile $\vec{c}$. The functions $t^2_w(\cdot)$, $q^2_w(\cdot)$, $t^2_\ell(\cdot)$ and $q^2_\ell(\cdot)$ are defined analogously.

For notational simplicity we omit the subscript I in the definition of a mechanism. This is because the buyer always implements only one mechanism and *only one* investment level. When investment is observable, the contract offered by the buyer also involves off-equilibrium transfers—contingent on all levels of investment—that prevent the first-period winner from deviating from the buyer's desired investment level. When investment is hidden instead, the buyer's optimization problem must include an additional constraint that makes the buyer's desired level of investment incentive compatible from the viewpoint of the first-period winner.

[6]Fully history-dependent mechanisms—which could be optimal when costs are highly persistent—are also difficult to implement due to the multidimensional nature of procurement contracting, in which a scoring rule to compare different offers is typically used (see, for example, Asker and Cantillon, 2010).

Any direct mechanism induces a sequential game of incomplete information among the potential sellers.[7] At each stage, the sellers' strategies are mappings from their current private histories to reports. In addition, the first-period winner must also decide how much to invest between both auctions. These strategies must maximize expected discounted payoffs at each stage of the game. Furthermore, the buyer's and sellers' conjecture about the investment level carried out by the first-period winner must be correct at time 2, before the second auction takes place. We assume that all agents discount future payoffs at the same rate, which can be chosen to be equal to 1.

3 Efficiency

It is clear that social surplus is maximized when the planner's actions (mechanism and investment level) are *sequentially efficient*: that is, when they maximize the "continuation surplus" at every information node. Therefore, efficiency requires that an efficient mechanism (e.g., a second-price auction) takes place at both auction stages: in the last stage, because the game ends; in the first-period, because the first-period cost-realization does not distort the investment decision between auctions. We show next that, when investment is unobservable, a second-price auction induces socially optimal investment incentives on behalf of the first-period winner; i.e., the social planner's and the first-period winner's investment incentives become aligned.[8] This is in fact a more general property of *Vickey-Clark-Groves* (VCG) mechanisms.[9]

Let Γ^e denote the ex-post efficient mechanism (i.e., in each period the project is

[7]For a general treatment of dynamic mechanism design, see Pavan, Segal and Toikka (2014).

[8]Arozamena and Cantillon (2004) derive a similar result in an investment stage before a one-shot auction takes place. See also Piccione and Tan (1996).

[9]We appreciate an anonymous referee for this observation. See Stegemann (1996) and Bergemann and Valimaki (2002) for analogous results in the context of information acquisition.

assigned to the the lowest-cost supplier). Total surplus is given by:

$$
\begin{aligned}
\mathcal{C}(\Gamma^e, I) &= n\int_C c[1-F(c)]^{n-1}f(c)dc + \int_C c[1-F(c)]^{n-1}g(c,I)dc \\
&\quad +(n-1)\int_C c[1-F(c)]^{n-2}[1-G(c,I)]f(c)dc + \Psi(I).
\end{aligned}
$$

The first term corresponds to the expected cost of the first procurement auction, and two the remaining ones are related to period 2. With this in hand, we have the following

Proposition 1. *The socially efficient level of investment, $I^e > 0$,*[10] *is the solution to*

$$
\max_{I\geq 0} \int_C [1-F(c)]^{n-1}G(c,I)dc - \Psi(I). \tag{1}
$$

Moreover, it can be implemented using a second price sealed-bid procurement auction at $t = 2$, regardless of the observability of the investment decision.

Proof: See the Appendix.

□

To understand why the planner's and the first-period winner's incentives are aligned, note that in a second-price auction the first-period winner chooses an investment that maximizes the expected value of the second-lowest cost (i.e., his payment to the buyer) minus his expected cost, conditional on winning. The planner instead chooses an investment that minimizes the difference between the first-period winner's and the next best cost, which are the effective savings generated by the investment decision. Both problems are trivially identical.

4 Cost Minimization Under Incentive Constraints

In this section we derive the optimal mechanism chosen by a buyer who can commit to the rules in both periods. The main results of this section are as follows. First, the

[10] We assume that $\frac{\partial G}{\partial I}(c, 0) \neq 0$ over a set of positive measure, so I^e is interior.

optimal two-period mechanism gives an *advantage* to the first-period winner in the second auction. Second, when facing this optimal mechanism, the first-period winner chooses the same investment level that the buyer chooses when this decision is observable. Hence, implementing the buyer's most-preferred technology comes at no extra cost. Third, the optimal investment level is larger than the efficient one, so *over-investment* occurs.

Preliminary Results

Consider a mechanism that implements a level of investment $I \geq 0$, $\Gamma(I)$. Let

$$Q_i^1(c_i') := \int_{C^{n-1}} q_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i},\ i \in N, \tag{2}$$

$$T_i^1(c_i') := \int_{C^{n-1}} t_i^1(c_i', c_{-i}) f^{n-1}(c_{-i}) dc_{-i},\ i \in N, \tag{3}$$

denote seller i's expected probability of winning the first procurement auction and his expected transfer, respectively, if he reports c_i' and the other players report truthfully.

The second-period expected probabilities are given by:

$$Q_w^2(c_w') = \int_{C^{n-1}} q_w^2(c_w', c_{-w}) f^{n-1}(c_{-w}) dc_{-w} \tag{4}$$

$$Q_{\ell,i}^2(c_i') = \int_{C^{n-1}} q_{\ell,i}^2(c_i', c_{-i}) f^{n-2}(c_{-w,i}) g(c_w, I) dc_{-i},\ i \neq w. \tag{5}$$

with expected transfers $T_w^2(\cdot)$, $T_{\ell,i}^2(\cdot)$ defined analogously.

We denote by $\Pi_w^2(c_w, c_w')$ the expected utility at $t = 2$ of a first-period winner with real cost c_w that declares c_w', and after the investment the decision has become sunk. The term $\Pi_{\ell,i}^2(c_i, c_i')$ is defined analogously:

$$\Pi_w^2(c_w, c_w') = T_w^2(c_w') - c_w Q_w^2(c_w') \tag{6}$$

$$\Pi_{\ell,i}^2(c_i, c_i') = T_{\ell,i}^2(c_i') - c_i Q_{\ell,i}^2(c_i'),\ i \neq w \tag{7}$$

Finally, we denote by $\Pi_i^1(c_i, c_i')$ seller i's expected discounted utility at $t = 1$ when his true cost is c_i and his report is c_i', and conditional on telling the truth at $t = 2$:

$$\begin{aligned}\Pi_i^1(c_i, c_i') &= T_i^1(c_i') - c_i Q_i^1(c_i') + Q_i^1(c_i') \int\limits_C [\Pi_w^2(c, c) - \Psi(I)] g(c, I) dc \\ &\quad + [1 - Q_i^1(c_i')] \int\limits_C \Pi_{\ell,i}^2(c, c) f(c) dc.\end{aligned} \tag{8}$$

The first two terms correspond to the expected payments and costs associated with the first auction. Second-period rents instead depend on being a winner or a loser in the first period, and on the cost of the investment carried out by the winner, $\Psi(I)$.[11]

Incentive-Compatible Mechanisms

When investment is observable the only incentive-compatibility constraints relate to truthful reporting:

$$IC_o : \begin{cases} \Pi_w^2(c_w, c_w) \geq \Pi_w^2(c_w, c_w'), \ \forall\ c_w, c_w' \in C, \\ \Pi_{\ell,i}^2(c_i, c_i) \geq \Pi_{\ell,i}^2(c_i, c_i'), \ \forall\ c_i, c_i' \in C, \ \forall\ i \neq w, \\ \Pi_i^1(c_i, c_i) \geq \Pi_i^1(c_i', c_i), \forall c_i, c_i' \in C, \forall i \in N, \end{cases}$$

where the subscript 'o' stands for "observable".

When investment is hidden (subscript 'h') in turn, the incentive-compatibility constraints must also make any investment level $K \neq I$ unprofitable the first-period winner. This is captured in the first constraint below:

$$IC_h : \begin{cases} I \in arg \max\limits_{K \geq 0} \int\limits_C \Pi_w^2(c, c) g(c, K) dc - \Psi(K) \\ \Pi_w^2(c_w, c_w) \geq \Pi_w^2(c_w, c_w'), \ \forall\ c_w, c_w' \in C \\ \Pi_{\ell,i}^2(c_i, c_i) \geq \Pi_{\ell,i}^2(c_i, c_i'), \ \forall\ c_i, c_i' \in C, \ \forall\ i \neq w \\ \Pi_i^1(c_i, c_i) \geq \Pi_i^1(c_i', c_i), \ \forall c_i, c_i' \in C, \ \forall i \in N. \end{cases}$$

[11] Notice that we have assumed that I is a scalar—this is because second-period rules are independent of first-period cost realizations. Specifying transfers for investment levels different than $I \geq 0$ is not necessary when this variable is hidden. See the next subsection.

Recall that, in the first constraint, $\Pi_w^2(c,c)$ assumes that the buyer computes probabilities and transfers as if the first-period winner has $G(\cdot,I)$ as his cost distribution, although we have not made that dependence explicit.

In the next lemma we state the usual characterization of incentive-compatible mechanisms with respect to cost revelation:

Lemma 1. (Incentive Compatibility). *A mechanism $\Gamma(I)$ is incentive-compatible with respect to cost revelation if and only if*

(i) *For all $i \in N$ and $I \geq 0$ $Q_i^1(\cdot)$ is non-increasing and, for all $c_i \in C$,*

$$\Pi_i^1(c_i, c_i) = \Pi_i^1(\bar{c}, \bar{c}) + \int_{c_i}^{\bar{c}} Q_i^1(s)ds$$

(ii) *For all $I \geq 0$, $Q_k^2(\ \cdot\ )$ is non-increasing, $k = w, (\ell, i)$, $i \neq w$, $i \in N$, and for all $c_k \in C$, $k = (w, I), (\ell, i)$, $\forall\ i \neq w$, $i \in N$*

$$\Pi_k^2(c_k, c_k) = \Pi_k^2(\bar{c}, \bar{c}) + \int_{c_k}^{\bar{c}} Q_k^2(s)ds.$$

Proof: See the Appendix. □

Voluntary Participation and Fully History-Dependent Mechanisms

Participation in the second period is standard, and ensured by assuming that

$$PC^2(I): \begin{cases} \Pi_w^2(c_w, c_w) - \Psi(I) \geq 0,\ \forall c_w \in C, \\ \Pi_{\ell,i}^2(c_i, c_i) \geq 0,\ \forall c_i \in C,\ i \neq w, \end{cases}$$

where the first constraint acknowledges that the winner must find it profitable to undertake the investment decision.

The first-period constraint instead depends on the type of contractual environment that the buyer can (or wishes to) promote. We follow Lewis and Yildirim (2002) and Pesendorfer and Jofre-Bonet (2014) by assuming that the buyer cannot prevent a seller from participating in the second auction just because this seller did not participate in the first one. In particular, this excludes the possibility of a buyer extracting all second period rents in the form of an entry fee before both auctions take place. This form of commitment can arise for two reasons. First, it could be that the buyer is just unable to implement the threat of using participation in the first auction as a necessary stage for participation in the second one (i.e., when a seller changes the name of his firm and asks for acceptance in the second period, after having strategically avoided the first auction). Second, it could be that the buyer wishes to credibly signal that her goal is not only rent extraction, but also her willingness to engage in long-term relationships intended to reduce long-term costs.[12]

In this context, the buyer must induce every seller to participate in both auctions, rather than allowing a seller to skip the first one and participate only in the second one. The first-period participation constraint then reads as

$$PC^1(I): \ \Pi_i^1(c_i, c_i) \geq \int\limits_C \Pi_{\ell,i}^2(c, c) f(c) dc, \ \forall \ c_i \in C, \ \forall \ i \in N,$$

where the right-hand side of the inequality is the ex-ante rent of seller i conditional on losing the first procurement. We have assumed in this constraint that the buyer commits to a second-period mechanism that is independent of the number of sellers who participated in $t = 1$. Consequently, from a time-zero perspective, the off-equilibrium rent for a seller who decides to skip the first auction is exactly the expected rent of a seller who participated in, but lost, the first competition.[13]

[12]In implementing their new buyer-supplier model, Chrysler made strong statements regarding how its new paradigm had moved away from rent extraction. See Dyer (1996).

[13]This is only a mild strengthening of our assumption that the buyer cannot threaten a non-participating seller with exclusion in $t = 2$, and can be justified similarly. In fact, if a seller is threatened with such punishing rules, he can always participate and submit an inconsequential bid.

The buyer chooses a mechanism $\Gamma(I)$ that minimizes the expected procurement cost

$$\mathcal{C} = \sum_{i=1}^{n} \int_{C} T_i^1(c) f(c) dc + \left[\int_{C} T_w^2(c) g(c, I) dc + \sum_{j \neq w} \int_{C} T_{\ell,j}^2(c) f(c) dc \right] \tag{9}$$

subject to $PC^1(I)$, $PC^2(I)$ and IC_o or IC_h, depending on the observability of investment.

So far we have discussed a restricted class of mechanisms, where second-period rules depend only on the identity of the winner, and not on the costs announced in $t = 1$. The next result states that this is without loss of generality:

Lemma 2. *It is optimal for the buyer to consider second-period mechanisms that are independent of the cost announcements made in* $t = 1$.

Proof: See the Appendix. □

The intuition should be straightforward: when costs are independent across time, the buyer cannot reduce expected costs using fully history-dependent mechanisms. This is because, after any possible history of cost realizations, the continuation game looks identical expect for the identity of the the first-period winner.[14]

The Optimal Mechanism

The following result presents the cost-minimizing mechanism when the buyer wishes to induce a level of investment $I \geq 0$, and the latter variable is observable. It corresponds to a mechanism that lies in between a sequence of second-price auctions (as in the American buyer-supplier model) and the implicit guarantee of a long-term contract (as in the Japanese *keiretsu* model), thus highlighting the trade off between inducing competition and having the flexibility to change suppliers in future purchases.

[14] This result does not rely on the assumption that buyer sets the same second-period mechanism on and off the equilibrium path, hence it is also valid in the unlikely event that the buyer can credibly commit to different threats on and off the equilibrium path.

Proposition 2. *Suppose that investment is observable and that the buyer wants to implement a level $I \geq 0$. The cost-minimizing mechanism, $\Gamma^*(I)$, is characterized by the allocation rules*

$$
\begin{aligned}
q_i^{1*}(\vec{c}) &= \mathbb{1}_{\{c_i < c_j,\, \forall j \neq i\}}, && (10)\\
q_w^{2*}(\vec{c}) &= \mathbb{1}_{\{c_w < k(c_i),\, \forall i \neq w\}}, && (11)\\
q_{\ell,i}^{2*}(\vec{c}) &= \mathbb{1}_{\{k(c_i) < \min\{c_w, k(c_j) |\, j \neq i\},\, i \neq w\}}, && (12)
\end{aligned}
$$

where $k(c) = c + \left(1 + \frac{1}{n-1}\right) \frac{F(c)}{f(c)}$, $c \in C$, *and by the transfers*

$$
\begin{aligned}
t_w^{2*}(\vec{c}) &= \mathbb{1}_{\{c_w < k(c_i),\, \forall i \neq w\}} \min\{k(c_i); i \neq w\}, && (13)\\
t_{\ell,i}^{2*}(\vec{c}) &= \mathbb{1}_{\{k(c_i) = \min\{c_w, k(c_j) |\, j \neq w\},\, i \neq w\}} \min\{c_w, k(c_j); j \neq w, i\}, && (14)\\
t_i^{1*}(\vec{c}) &= \mathbb{1}_{\{c_i < c_j,\, \forall j \neq i\}} \{\min\{c_j; j \neq i\} - [\Pi_w^{2*}(I) - \Psi(I) - \Pi_\ell^{2*}(I)]\} && (15)
\end{aligned}
$$

where

$$
\Pi_w^{2*}(I) := \int_C \Pi_w^{2*}(c,c) g(c,I) dc \quad \text{and} \quad \Pi_\ell^{2*}(I) := \int_C \Pi_\ell^{2*}(c,c) f(c) dc. \tag{16}
$$

In particular, the second period allocation rule is independent of the winner's cost distribution, and thus independent of the investment level that the buyer wishes to implement. Finally, $k(c) > c$ for $c > \underline{c}$, uniformly across all number of sellers.

Proof: See the Appendix. □

The allocation rule for $t = 1$ corresponds to the one derived by Myerson (1981) for n symmetric sellers in the "regular" case (monotone virtual costs). Thus, it is efficient. Instead, the second-period rule is inefficient, as it entitles the first-period winner with an *advantage* in the second competition: he can be awarded the second project even in situations in which he obtains a cost that is higher than a competitor.[15]

Even though this advantage decreases with the number of sellers (i.e., sacrificing efficiency becomes more costly as the number of sellers grows), it never disappears, as

$$k(c) := c + \left(1 + \frac{1}{n-1}\right)\frac{F(c)}{f(c)} \to c + \frac{F(c)}{f(c)} \text{ when } n \to \infty.$$

Furthermore, it can be shown that the optimal investment level decreases to zero as the number of competitors grows to infinity (Corollary 1), so the buyer entitles the winner with a strictly positive advantage in the limit, yet no investment is mandated. Consequently, the decision to favor the winner with high second-period rents is independent of any potential incentive to promote investment that the buyer may have.

In order to understand why awarding some degree of advantage is always optimal, we must look at the first-period participation constraint

$$PC^1(I): \ \Pi_i^1(c_i, c_i) \geq \int\limits_C \Pi_{\ell,i}^2(c,c)f(c)dc, \ \forall \ c_i \in C.$$

Observe that biasing the second auction *against* a first-period loser in fact relaxes this constraint; i.e., it reduces period-one participation costs by making skipping the first auction less attractive. While reducing procurement costs in this way does involve additional rents awarded to the first-period winner in the second period, this is not costly to the buyer. To see this, notice that from (15) the transfer to seller i at $t = 1$ takes the form

$$\begin{array}{rl} \min\{c_j; j \neq i\} - [\Pi_w^{2*}(I) - \Pi_\ell^{2*}(I)] + \Psi(I) & \text{if seller i wins,} \\ 0 & \text{otherwise,} \end{array} \tag{17}$$

from where we conclude that the buyer is always able to extract at $t = 1$ the additional rent that the first-period winner obtains due to the bias in the second auction.[16]

[15] Observe that, as it is standard in the auctions literature, the optimal mechanism can be implemented using modified second-price auctions (expressions (13)-(15)).

[16] This observation–i.e., that off-setting an increase in a second-period transfer with a decrease of the

Importantly, biasing the second-period auction in favor of the first-period winner has the effect that it lowers the cost of the first project relative to a standard static optimal auction where investment is not possible: (17) is lower than $\min\{c_j; j \neq i\}$ (when $\Psi \equiv 0$). Consequently, awarding future advantages reduces current procurement cost via *inducing more competition* today. The optimal advantage thus trades off the benefit of this first-period competition effect, with the inefficiency of assigning the second project to an incumbent with potentially high costs.[17]

This competition effect that future advantages can generate also appears in Lewis and Yildirim (2002), who study a fully dynamic setting with exogenous learning by doing, and more recently by Pesendorfer and Jofre-Bonet (2014), in the context that we analyze for the case restricted to the case of two players and exogenous distributions.[18] A key novelty of our analysis is to show that a buyer's incentives to promote competition can in fact induce strong investment incentives on incumbents as a byproduct.

Over-Investment

From Proposition (2), the optimal allocation rule (10)-(12) minimizes the total cost of procurement for all possible levels of investment that the buyer wishes to implement. Moreover, because it does not depend on the distribution of the winner, this allocation rule is also feasible when investment is not observable. The next result states that, under this cost-minimizing allocation rule, the buyer's and winner's investment incentives are perfectly aligned (as in the case of surplus maximization). Furthermore, the optimal level of investment that is implemented turns out to be larger than the efficient one:

Proposition 3. *When the buyer can commit to a two-period mechanism, regardless of*

same magnitude in the corresponding first-period transfer is costless to the buyer (i.e., it does not violate incentive compatibility)—was first made by Baron and Besanko (1984).

[17]Because (i) the buyer is able to extract the winner's incremental rent, and (ii) the outside option is determined by the rent of a loser, the distortion in the second-period mechanism depends only on the cost distribution of a loser.

[18]We extend Pesendorfer and Jofre-Bonet (2014) by allowing for an arbitrary number of players (thus showing that advantages survive in the limit), and by allowing for an investment decision. We also prove the optimality of mechanisms that condition on identity of the first-period winner only (Lemma 2).

the observability of investment, the optimal level, $I^* > 0$, solves

$$\max_{I \geq 0} \int_C [1 - F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I), \tag{18}$$

where $k(c) = c + (1 + 1/(n-1)) F(c)/f(c)$, $c \in C$. Hence, the optimal mechanism is given by $\Gamma^*(I^*)$ defined in Proposition 2, and I^* can be implemented using $\Gamma^*(I^*)$ at no extra cost. Furthermore, $I^* > I^e$, so over-investment occurs.

Proof: See the Appendix. □

The key to understand why the buyer's and seller's incentives to invest are aligned lies on the first-period transfer (15) (or, equivalently, (17)). More precisely, because the buyer (i) captures the winner's incremental rent due to the second-period bias and she (ii) compensates for the cost of investment, the buyer actually internalizes the winner's full benefit from improving his technology. In particular, under the cost-minimizing allocation rule (10)-(12), the winner's investment problem becomes

$$\begin{aligned} & \max_{I \geq 0} \int_C \Pi_w^{2*}(c, c) g(c, I) dc - \Psi(I) \\ \Leftrightarrow \quad & \max_{I \geq 0} \int_C [1 - F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I), \end{aligned}$$

which defines I^*. Consequently, $\Gamma^*(I^*)$ is optimal regardless of the observability of the investment decision.

The first-period winner over-invests because he is granted a non-trivial advantage gap in the second auction. Notice that because the socially optimal level of investment solves

$$\max_{I \geq 0} \int_C [1 - F(c)]^{n-1} G(c, I) dc - \Psi(I),$$

and $k^{-1}(c) < c$, we have that $[1 - F(k^{-1}(c))] > [1 - F(c)]$. As a result, a bigger advantage gap increases the *sensitivity* of the incumbent's rent to the investment decision, as the set of events over which he wins increases. As the incumbent's marginal incentives are steeper than under the efficient mechanism, he over-invests. This is useful for the buyer, as over-investing mitigates the extra cost of allocating the second project to the incumbent too frequently.

The evidence from Chrysler in the eighties shares many of the features that we have identified. By moving away from a static competitive bidding process towards a model of recognition of past performance, Chrysler was able to reduce substantially its production costs. Some noteworthy consequences of this new approach were that: some suppliers decided to gave up the right to pocket Chrysler's extra savings to boost their performance ratings for future contracts (more intense competition); the average length of a contract doubled to 4.4 years (the likelihood of continuation for incumbents increased); and, most importantly, suppliers made important investments in relationship-specific assets (e.g., software, facilities and proximity to Chrysler's plants). Moreover, analysts of this industry have emphasized that a key factor allowing for these changes was Chrysler's credible commitment to change its interaction with suppliers (for all this evidence, see Dyer 1996). In our model, this commitment is captured in a buyer who ties her hands when it comes to setting an entry fee before both auctions take place, and it is credibly signaled by the use of mechanisms that favor past winners in future competitions.

5 Cost Minimization in the Absence of Commitment

We now consider the case when the buyer cannot commit to a two-period mechanism; i.e., the second mechanism is chosen after the first auction takes place. We say that the buyer *lacks commitment* in this case, understanding that it corresponds to an intermediate degree of commitment (she can still commit to the rules of a one-shot auction).

When the buyer lacks commitment, the auction design problem becomes static. Hence, the optimal mechanism awards a disadvantage to the most efficient (in a distributional sense) supplier (Myerson, 1981).[19] Moreover, the size of the disadvantage grows as the

[19]We keep the standard commitment assumptions in the mechanism literature, so after the second

efficiency of this supplier improves. With theses insights in mind, we show in this section that (i) the buyer's lack of commitment induces investment levels *below* efficiency, and that (ii) the investment takes its lowest value when investment is observable. The intuition for (i) is straightforward, as investing in a better technology will reduce the probability of winning. For (ii), notice that when investment is observable the buyer adjusts the mechanism *after* the investment decision has taken place. This hold-up effect makes investing even less attractive than in the case in which investment is hidden.

Throughout the section we make the following mild assumptions:

Assumption 2.

(a) Hazard-rate ordering: for all $c \in C$ and $0 \leq I' < I$

$$\frac{F(c)}{f(c)} \leq \frac{G(c,I')}{g(c,I')} < \frac{G(c,I)}{g(c,I)}.$$

(b) Monotone virtual costs: for each $I \geq 0$, $J_I(c) := c + \frac{G(c,I)}{g(c,I)}$ is strictly increasing.

The hazard-rate ordering captures the idea that higher investment levels lead to higher virtual costs, and thus negatively impact the probability of winning. Intuitively, the buyer introduces an inefficiency in order to extract information rents from her best bidders.[20]

Finally, observe that because of the buyer's inability to commit to the second-period rules, he cannot decide the investment level, even when this decision is observable.

Investment Observability and Lack of Commitment

When investment is observable, the buyer chooses second-period rules after the investment has taken place, treating the investment decision as sunk.

period rules are announced, they cannot be changed.

[20]It is easy to see that (a) is implied by MLRP: For all $c' < c \in C$ and $0 \leq I' < I \in \mathbb{R}$, $\frac{f(c')}{f(c)} \leq \frac{g(c',I')}{g(c,I')} < \frac{g(c',I)}{g(c,I)}$.

Proposition 4. *If the first-period winner invests $I \geq 0$ and the buyer lacks commitment, the cost-minimizing mechanism at $t = 2$, $\hat{\Gamma}^2(I)$, has as allocation rule*

$$\begin{aligned}\hat{q}^2_w(c_w, c_{-w}) &= \mathbb{1}_{\{J_I(c_w)<J(c_i),\, \forall j \neq w\}} \\ \hat{q}^2_{\ell,i}(c_i, c_{-i}) &= \mathbb{1}_{\{J(c_i)<\min\{J_I(c_w),\min\{J(c_j),\, j\neq i,w\}\}\}}\end{aligned}$$

where $J_I(c) := c + \frac{G(c,I)}{g(c,I)}$ and $J(c) = c + \frac{F(c)}{f(c)}$. The optimal allocation rule for $t = 1$ is the efficient one.

Proof : Direct. □

As argued earlier, the second period mechanism hurts the first period winner ($J(c) < J_I(c)$). This results in an investment level below the efficient one:

Proposition 5. *Suppose that investment is observable and that the buyer lacks commitment. Then, the investment level chosen by the first-period winner, $\hat{I}^o$, solves*

$$\max_{I\geq 0} \int_C [1 - F(J^{-1}(J_I(c))]^{n-1} G(c, I)dc - \Psi(I). \tag{19}$$

Consequently, the optimal mechanism corresponds to $\hat{\Gamma}^2(\hat{I}^o)$. Furthermore, $\hat{I}^o < I^e$, so the winner under-invests.

Proof : See the Appendix. □

Compared to the case of full commitment, investment has now an additional effect on the winner's second-period rent: it negatively affects the allocation rule ($[1 - F(J^{-1}(J_I(c))]^{n-1}$ decreases with I). This is an example of the classic *hold-up* problem: the first-period winner incurs in a sunk investment to improve his cost distribution and, by changing the rules of the second auction, the buyer has the incentive to take advantage of it (reducing the first-period winner's information rents). The winner anticipates this behavior, and hence he underinvests.

Hidden Investment and Lack of Commitment

If investment is not observable, we have a situation equivalent to a simultaneous-move game between the first-period winner and the buyer. The action space for the first-period winner corresponds to $A_w = [0, +\infty)$. The action space for the buyer is, by a rationalizability argument, the set of one-shot mechanisms that are a best-response to some investment level by the first-period winner. Hence, we consider $A_b = \{\hat{\Gamma}^2(I) |\ I \geq 0\}$, where $\hat{\Gamma}^2(I)$ is the mechanism defined in Proposition 4.[21]

Definition 2. *A pure-strategy equilibrium under non-commitment and non-observable investment is a tuple* $(\Gamma, I) \in A_b \times A_w$ *such that*

(i) $\Gamma = \hat{\Gamma}^2(I)$

(ii) $I \in arg \max_{K \geq 0} \int_C \Pi_w^{2,\Gamma}(c,c) g(c,K) dc - \Psi(K)$

where $\Pi_w^{2,\Gamma}(c,c)$ *is the second-period utility of a first-period winner with cost* $c \in C$ *under the incentive-compatible mechanism* $\Gamma = \hat{\Gamma}^2(I)$.

We now state the main result of this subsection:

Proposition 6. *An equilibrium* $(\hat{I}^h, \hat{\Gamma}^2(\hat{I}^h))$ *exists and it is unique. In this equilibrium, the investment level* $\hat{I}^h$ *satisfies*

$$\hat{I}^h = \arg\max_{I \geq 0} \int_C [1 - F(J^{-1}(J_{\hat{I}^h}(c)))]^{n-1} G(c, I) dc - \Psi(I). \tag{20}$$

Also, $\hat{I}^o < \hat{I}^h < I^e < I^*$.

Proof : See the Appendix. □

Because any mechanism of the form $\hat{\Gamma}^2(I)$, $I \geq 0$, is taken as given by the winner when investment is hidden, the hold-up effect from the observable case disappears.

[21] Piccione and Tan (1996) use a similar construction in a slightly different environment.

Consequently, the negative effect of the buyer's lack of commitment on the winner's investment incentives is milder, leading to a level $\hat{I}^h$ that is larger than the one that arises when investment is observable, $\hat{I}^o$. Yet, investment is still below the social optimum, as the auction that arises in equilibrium at $t = 2$ always handicaps the first-period winner.

We conclude by showing how all the investment levels previously found vary with the number of sellers, and by discussing how the cost of procurement varies across different degrees of commitment.

Corollary 1. *Let $I^*(n)$ denote the investment level that arises under full commitment in the presence of n sellers. Suppose that $\frac{\partial G}{\partial I}(\cdot, 0)$ is integrable. Then $I^*(n) \searrow 0$ as $n \to \infty$. As a consequence, $I^e(n), \hat{I}^o(n)$ and $\hat{I}^h(n)$ (all defined correspondingly) go to zero as $n \to \infty$.*

Proof : See the Appendix. □

The intuition for why $I^*(n)$ decays to zero should be straightforward: because the probability of winning the second auction decays to zero as the number of sellers grows to infinity, investing in a better technology loses its full value in the limit.

Finally, it is easy to see that the total cost of procurement is lowest under full commitment. In fact, (i) by sequential rationality, the optimal first-period rule is always the efficient one, and (ii) the first-period transfers can always be designed such that participation is ensured for all sellers for both competitions. Hence, the cost-minimizing mechanisms under no commitment are feasible under full commitment. Also, the total cost of procurement trivially converges to $2\underline{c}$ as $n \to \infty$.

6 Conclusions

In this article we have shown that both advantages to incumbents and strong investment incentives on behalf of them can arise in fully competitive bidding settings. In dynamic procurement, the optimal size of any advantage must balance the benefit of inducing more competition today, with the future cost of entering in a long-term commitment with a supplier that can underperform in the future. Our work stresses that the latter cost need not be too high, as advantages in fact induce strong investment incentives as a byproduct.

An important takeaway from our analysis is that short-term commitment—through the impact that it has on suppliers' choices of technologies—can have an critical impact on long-term costs. The model adopted by Chrysler in the eighties in fact supports many of our findings. Crucially, although Chrysler did borrow ideas from the Japanese *keiretsu* model, it did it under the umbrella of the American model that promotes competition. Evidence of this is that Chrysler had a much larger supplier base than its Japanese counterparts, and had greater flexibility to drop underperforming suppliers (Dyer, 1996). Even though the mechanism design approach that we follow abstracts from any details of the relational contract developed between this manufacturer and its providers, it allows us to isolate the key competitive forces that arise in dynamic buyer-supplier interactions.

We conclude by discussing some of our modeling assumptions. First, in a general multi-period model there could be many past winners, and investment decisions must also incorporate the impact on all future mechanisms. Given our analysis, incorporating endogenous investment decisions in the framework of Lewis and Yildrim (2002) adds considerable technical complexities, at the expense of no obvious additional economic insights. Second, the assumption that there is no entry in the supply side is a reasonable one for the types of industries that we have in mind, and also because we are interested in short-term commitment. Nevertheless, advantages to incumbents can still survive in the presence of potential entrants in settings where technologies are partially transferable (Laffont and Tirole, 1988).

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7 Appendix: Proofs

Proof of Proposition 1: Let I^e be the minimizer of the social cost of procurement

$$\begin{aligned}\mathcal{C}(\Gamma^e, I) &= n\int_C c[1-F(c)]^{n-1}f(c)dc + \int_C c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc \\ &\quad +(n-1)\int_C c[1-F(c)]^{n-2}[1-G(c,I)]f(c)dc + \Psi(I) \qquad (21)\end{aligned}$$

Integrating by parts yields that

$$\begin{aligned}\int_C c[1-F(c)]^{n-1}\frac{\partial G}{\partial c}(c,I)dc &= (n-1)\int_C c[1-F(c)]^{n-2}f(c)G(c,I)dc \\ &\quad -\int_C [1-F(c)]^{n-1}G(c,I)dc \qquad (22)\end{aligned}$$

Hence, I^e is the solution to

$$\begin{aligned}&\min_{I\geq 0}\ (n-1)\int_C c[1-F(c)]^{n-2}f(c)dc - \int_C [1-F(c)]^{n-1}G(c,I)dc + \Psi(I) \\ \Leftrightarrow\ &\max_{I\geq 0}\int_C [1-F(c)]^{n-1}G(c,I)dc - \Psi(I). \qquad (23)\end{aligned}$$

For the implementation result, it remains to be checked is that the winner indeed chooses to invest I^e when the buyer sets a second-price auction in the second period (this mechanism is trivially efficient). By incentive compatibility, the first-period winner's expected rent in the second auction, conditional on his type, $\Pi_w^{2,SPA}(c,c)$, satisfies

$$\Pi_w^{2,SPA}(c,c) = \Pi_w^{2,SPA}(\bar{c},\bar{c}) + \int_c^{\bar{c}}[1-F(s)]^{n-1}ds,\ c\in C.$$

Integration by parts shows that maximizing $\int_C \Pi_w^{2,SPA}(c,c)g(c,I)dc - \Psi(I)$ over all positive levels of investment is equivalent to (1), so the first-period winner indeed invests I^e. This concludes the proof. $\square$

Proof of Lemma 1: We define $V_i^k(c_i) = \max\limits_{c_i'} \Pi_i^k(c_i, c_i')$. Because $V_i^k(c_i)$ is a maximum of linear functions, it follows that it is convex, and therefore differentiable a.e. It is easy to see that the following are equivalent: (a) The mechanism is incentive compatible (b) $V_i^k(c_i) = \Pi_i^k(c_i, c_i)$ for all i and $k = 1, 2$, (c) $-Q_i^k(c_i) \in \partial V_i^k(c_i)$ for all i and $k = 1, 2$.

We now prove the two implications required. For the sufficiency, notice that $-Q_i^k(c_i)$ is non-decreasing as it is a selection from the subdifferential of a convex function (see [22]). Because $-Q_i^k(c_i) \in \partial V_i^k(c_i)$ and $V_i^k(c_i)$ is convex we have that $V_i^k(c_i) = V_i^k(\bar{c}) + \int\limits_{c_i}^{\bar{c}} Q_i^k(s)ds$. For the necessity, notice that $V_i^k(c_i) = \int\limits_{c_i}^{c_i'} Q_i^k(s)ds \leq V_i^k(c_i') + Q_i^k(c_i')(c_i' - c_i)$. Therefore $Q_i^k(c_i) \in \partial V_i^k(c_i)$ and the mechanism is incentive compatible. $\square$

Proof of Lemma 2: Suppose that investment is observable. Let $t_{i,\rho}^{2,\vec{c}}(\vec{c'})$ and $q_{i,\rho}^{2,\vec{c}}(\vec{c'})$ denote the second-period transfer and allocation rule, respectively, for seller $\rho \in \{\ell, w\}$ given a first-period report $\vec{c}$ and a second period report $\vec{c'}$. Denote by $(t^{2,\vec{c}}, q^{2,\vec{c}})$ the resulting mechanism. Define the corresponding expected transfer, expected probability of winning, and expected utility, $T_{i,\rho}^{2,\vec{c}}(c_i')$, $Q_{i,\rho}^{2,\vec{c}}(c_i')$, and $\Pi_{i,\rho}^{2,\vec{c}}(c_i, c_i')$ respectively, as in Section 4.1, $\rho \in \{\ell, w\}$. It is clear that given any history $(\vec{c}, w)$, the continuation mechanism $(t^{2,\vec{c}}, q^{2,\vec{c}})$ is incentive compatible if and only if (ii) in Lemma 1 holds. The same characterization also holds for any IC first-period mechanism when truth-telling is induced at $t = 2$, as

$$
\begin{aligned}
\Pi_i^1(c_i, c_i') \quad := \quad & T_i^1(c_i') - c_i Q_i^1(c_i') \\
& + \underbrace{\int_{C^{n-1}} q_i^1(c_i', c_{-i}) \left(\int_C [\Pi_{i,w}^{2,(c_i',c_{-i})}(s,s) - \Psi(I)] g(s,I) ds \right) f^{n-1}(c_{-i}) dc_{-i}}_{\text{second-period rent if firm i wins first auction}} \\
& + \underbrace{\int_{C^{n-1}} [1 - q_i^1(c_i', c_{-i})] \left(\int_C \Pi_{i,\ell}^{2,(c_i',c_{-i})}(s,s) f(s) ds \right) f^{n-1}(c_{-i}) dc_{-i}}_{\text{second-period rent if firm i loses first auction}} \qquad (24)
\end{aligned}
$$

depends on c_i only through $c_i Q_i^2(c_i')$ in a linear way. In the previous expression, the investment level I mandated by the buyer can be a function of $\vec{c}$, but we omit such dependence.

Take any incentive-compatible mechanism. We can then write

$$
\begin{aligned}
T_i^1(c_i) &= \Pi_i^1(c_i, c_i) + c_i Q_i^2(c_i) \\
&\quad - \int_{C^{n-1}} q_i^1(\vec{c}) \left(\int_C [\Pi_{i,w}^{2,\vec{c}}(s,s) - \Psi(I)] g(s,I) ds \right) f^{n-1}(c_{-i}) dc_{-i} \\
&\quad - \int_{C^{n-1}} [1 - q_i^1(\vec{c})] \left(\int_C \Pi_{i,\ell}^{2,\vec{c}}(s,s) f(s) ds \right) f^{n-1}(c_{-i}) dc_{-i}, \qquad (25)
\end{aligned}
$$

and the total cost of procurement is given by $TC = \sum_{i=1}^{n} R_i$ where R_i is defined as

$$
\begin{aligned}
R_i &= \int_C T_i^1(c) f(c) dc \\
&+ \int_{C^n} q_i(\vec{c}) \left(\int_C T_{i,w}^{2,\vec{c}}(s) g(s,I) ds - \Psi(I) + \sum_{j \neq i} \int_C T_{i,\ell}^{2,\vec{c}}(s) f(s) ds \right) f(\vec{c}) d\vec{c}.
\end{aligned}
$$

Now, because $T_{i,\rho}^{2,\vec{c}}(s) = \Pi_{i,\rho}^{2,\vec{c}}(s,s) + s Q_{i,\rho}^{2,\vec{c}}(s)$ and

$$
\sum_{i=1}^{n} [1 - q_i^1(\vec{c})] \int_C \Pi_{i,\ell}^{2,\vec{c}}(s) f(s) ds = \sum_{i=1}^{n} q_i^1(\vec{c}) \sum_{j \neq i} \int_C \Pi_{j,\ell}^{2,\vec{c}}(s) f(s) ds,
$$

(where we used that $1 - q_i^1(\vec{c}) = \sum_{j \neq i} q_j^1(\vec{c})$) the total cost of procurement reduces to

$$
\begin{aligned}
TC &= \sum_{i=1}^{n} \left[\int_C [\Pi_i^1(\bar{c}, \bar{c}) + c_i Q_i^2(c_i)] f(c_i) dc_i + \int_C Q_i^2(c_i) F(c_i) dc_i \right] \\
&\quad + \int_{C^n} \sum_{i=1}^{n} q_i^1(\vec{c}) \left(\int_C s Q_{i,w}^{2,\vec{c}}(s) g(s,I) ds - \Psi(I) + \sum_{j \neq i} \int_C s Q_{j,\ell}^{2,\vec{c}}(s) f(s) ds \right) f^n(\vec{c}) d\vec{c}
\end{aligned}
$$

Suppose that if seller i skips the first auction and participates only in the second one, the second-period mechanism that is implemented gives him an expected second-period rent of $\Pi_i^{2,skip} \geq 0$ from a time-zero perspective (i.e., after seller i draws his first-period cost, but before the first auction takes place). This rent does not depend on his first-period cost, and thus it corresponds to a scalar. This is because (i) true costs are independent across time, and also because (ii) this seller does not participate at $t = 1$, so the second-period mechanism does not condition on his cost. Importantly, this mechanism need not coincide with the one that arises on the equilibrium path; it depends on the type of off-equilibrium threats that the buyer can make.

With this generic mechanism at hand, and under truth-telling in both periods, seller i's first-period participation constraint reads $\Pi_i^1(c, c) \geq \Pi^{2,skip}$. Thus,

$$\begin{aligned} TC &= \sum_{i=1}^{n}\left[\int\limits_C [\Pi_i^1(\bar{c},\bar{c}) - \Pi_i^{2,skip} + c_i Q_i^2(c_i)] f(c_i)dc_i + \int\limits_C Q_i^2(c_i)F(c_i)dc_i\right] \\ &\quad + \int_{C^n}\sum_{i=1}^{n} q_i^1(\vec{c})\left(\int_C sQ_{i,w}^{2,\vec{c}}(s)g(s,I)ds - \Psi(I) + \sum_{j\neq i}\int_C sQ_{j,\ell}^{2,\vec{c}}(s)f(s)ds \right. \\ &\qquad\qquad \left. + \sum_{j=1}^{n}\Pi_j^{2,skip}\right) f^n(\vec{c})d\vec{c} \end{aligned}$$

where we used that $\sum_{j=1}^n q_j^1(\vec{c}) = 1$. With this in hand, the second-period component of cost, $\mathcal{C}^2$, takes the form:

$$\begin{aligned} \mathcal{C}^2 &:= \int\limits_{C^n}\sum_{i=1}^{n} q_i^1(\vec{c})\mathcal{C}^2(\vec{c})f^n(\vec{c})d\vec{c}, \text{ where} \\ \mathcal{C}^2(\vec{c}) &:= \int\limits_C sQ_{i,w}^{2,\vec{c}}(s)g(s,I)ds - \Psi(I) + \sum_{j\neq i}\int\limits_C sQ_{j,\ell}^{2,\vec{c}}(s)f(s)ds + \sum_{j=1}^{n}\Pi_j^{2,skip} \end{aligned}$$

Observe that we can maximize $\mathcal{C}^2$ point-wise across events $q_i^1(\vec{c}) > 0$. Furthermore, since after any history $\vec{c}$ the optimization problem is the same one, the value of $\mathcal{C}^2(\vec{c})$ should be constant across all those histories. We conclude that we can restrict to second-period rules that are independent of first-period cost realizations. Moreover, as we can always set $\Pi_i^1(\bar{c},\bar{c}) = \Pi_i^{2,skip}$, choosing the minimizer of $\mathcal{C}^2(\vec{c})$ will not affect the cost of procurement at $t = 1$. Hence, it is optimal to restrict to second-period mechanisms that condition only on the identity of the first-period winner. This concludes the proof. □

Proof of Proposition 2: When the buyer commits to the same rules on and off the equilibrium path

$$\int_C \Pi^2_{\ell,i}(c,c)f(c)dc = \Pi^2_{\ell,i}(\bar{c},\bar{c}) + \int_C Q^2_{\ell,i}(c)F(c)dc$$

Hence, in the light of the previous Lemma 2, the total cost of second-period procurement under a direct mechanism $\Gamma(I)$ can be written as (choosing optimally $\Pi^2_{\ell,i}(\bar{c},\bar{c}) = 0$)

$$\mathcal{C}^2 = \int\limits_C sQ^2_w(s)g(s,I)ds + \Psi(I) + (n-1)\int\limits_C sQ^2_\ell(s)f(s)ds + n\int\limits_C Q^2_\ell(s)F(s)ds.$$

Using standard techniques, the total cost of procurement reduces to

$$\begin{aligned}
\mathcal{C} &= \int\limits_{C^n}\sum_{i=1}^n \left[c_i + \frac{F(c_i)}{f(c_i)}\right] q^1_i(c)f^n(\vec{c})d\vec{c} + \Psi(I) \\
&\quad + \int\limits_{C^n}\left[c_w q^2_w(\vec{c}) + \sum_{i\neq w}\left[c_i + \left(1+\frac{1}{n-1}\right)\frac{F(c_i)}{f(c_i)}\right] q^2_{\ell,i}(\vec{c})\right] f^{n-1}(c_{-w})g(c_w,I)d\vec{c}
\end{aligned} \tag{26}$$

Pointwise maximization yields the allocation rule (10)-(12). Furthermore, from the proof of Lemma 2, it is always optimal to set $\Pi^1_i(\bar{c},\bar{c}) = \Pi^2_{\ell,i}(I)$. Using (6)-(8) and the characterization of incentive compatibility

$$\begin{aligned}
T^1_i(c) &= \underbrace{\Pi^1_i(\bar{c},\bar{c}) + \int_c^{\bar{c}} Q^1_i(s)ds}_{\Pi^1_i(c,c)=} + cQ^1_i(c) - Q^1_i(c)[\Pi^2_w(I) - \Psi(I) - \Pi^2_{\ell,i}(I)] - \Pi^2_{\ell,i}(I) \\
&= \int_c^{\bar{c}} Q^1_i(s)ds + cQ^1_i(c) - Q^1_i(c)[\Pi^2_w(I) - \Psi(I) - \Pi^2_{\ell,i}(I)], \\
T^2_{\ell,i}(c) &= \underbrace{\Pi^2_\ell(\bar{c},\bar{c})}_{=0} + \int_c^{\bar{c}} Q^2_\ell(s)ds + cQ^2_\ell(c) \text{ and } T^2_w(c) = \underbrace{\Pi^2_w(\bar{c},\bar{c})}_{=0} + \int_c^{\bar{c}} Q^2_w(s)ds + cQ^2_w(c).
\end{aligned}$$

With this in hand it is easy to see that (13)-(15) induce participation and implement (10)-(12). This concludes the proof. □

Proof of Proposition 3: We first show that I^* solution to (18) is optimal for the buyer. Given $\Gamma(I)$ as in Proposition 2, the buyer's problem consists of finding the level of investment that minimizes the second-period cost of procurement

$$\mathcal{C}^2 := \int_C cQ_w^{2*}(c)g(c,I)dc + n\int_C Q_l^{2*}(c)F(c)dc + (n-1)\int_C cQ_l^{2*}(c)f(c)dc + \Psi(I)$$

(see (26) in the Proof of Proposition 2). It can be easily checked that

$$\begin{aligned} Q_w^{2*}(c) &= [1-F(k^{-1}(c))]^{n-1}, c \in C, \text{ and} \\ Q_l^{2*}(c) &= (1-G(k(c),I))(1-F(c))^{n-2}, c \in [\underline{c}, k^{-1}(\bar{c})] \text{ (and zero otherwise)} \end{aligned}$$

are the expected probabilities of winning the second project for a winner and any loser respectively. Integrating by parts,

$$\begin{aligned} \int_C cQ_w^{2*}(c)g(c,I)dc &= \bar{c}[1-F(k^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc \\ &\quad + (n-1)\int_C c[1-F(k^{-1}(c))]^{n-2}f(k^{-1}(c))\frac{G(c,I)}{k'(k^{-1}(c))}dc. \end{aligned}$$

Using $t = k^{-1}(c)$ in the last integral and replacing $k(c) = c + \left(1+\frac{1}{n-1}\right)\frac{F(c)}{f(c)}$ in the same expression yields

$$\begin{aligned} \int_C cQ_w^{2*}(c)g(c,I)dc &= \bar{c}[1-F(k^{-1}(\bar{c}))]^{n-1} - \int_C [1-F(k^{-1}(c))]^{n-1}G(c,I)dc \\ &\quad + (n-1)\int_{\underline{c}}^{k^{-1}(\bar{c})} c[1-F(c)]^{n-2}f(c)G(k(c),I)dc \\ &\quad + n\int_{\underline{c}}^{k^{-1}(\bar{c})} [1-F(c)]^{n-2}G(k(c),I)F(c)dc. \end{aligned}$$

With this in hand, and using that $Q_l^{2*}(c) = [1-F(c)]^{n-2}[1-G(k(c), I)]$, we conclude that

$$\begin{aligned}
\mathcal{C}^2 &= \bar{c}[1-F(k^{-1}(\bar{c}))]^{n-1} - \int\limits_C [1-F(k^{-1}(c))]^{n-1} G(c, I) dc \\
&\quad + n \int\limits_{\underline{c}}^{k^{-1}(\bar{c})} [1-F(c)]^{n-2} F(c) dc + (n-1) \int\limits_{\underline{c}}^{k^{-1}(\bar{c})} c[1-F(c)]^{n-2} f(c) dc + \Psi(I).
\end{aligned}$$

Thus I^* solves $\max\limits_{I \geq 0} \int\limits_C [1-F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I)$.

To show the implementation result, notice that the winner's investment problem under $\Gamma(I^*)$ corresponds to

$$\begin{aligned}
& \max_{I \geq 0} \int\limits_C \Pi_w^{2*}(c) g(c, I) dc - \Psi(I) \Leftrightarrow \max_{I \geq 0} \Pi_w^{2*}(\bar{c}, \bar{c}) + \int\limits_C Q_w^{2*}(c) G(c, I) dc - \Psi(I) \\
\Leftrightarrow \; & \max_{I \geq 0} \int\limits_C Q_w^{2*}(c) G(c, I) dc - \Psi(I) = \max_{I \geq 0} \int\limits_C [1-F(k^{-1}(c))]^{n-1} G(c, I) dc - \Psi(I)
\end{aligned}$$

where we used the characterization of IC from Lemma 1, integration by parts, the fact that $\Pi_w^{2*}(\bar{c}, \bar{c})$ does not depend on the winner's cost distribution, and the definition of $Q_w^{2*}(\cdot)$. Hence, the winner invests I^* as the buyer does.

To conclude, $I^* > I^e$ is a consequence of $1 - F(k^{-1}(\cdot)) > 1 - F(\cdot)$, of $\frac{\partial G}{\partial I}(\cdot, \cdot) > 0$ and that both

$$I \mapsto \int\limits_C [1-F(k^{-1}(c))]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I) \text{ and } I \mapsto \int\limits_C [1-F(c)]^{n-1} \frac{\partial G}{\partial I}(c, I) dc - \Psi'(I)$$

are decreasing functions. □

Proof of Proposition 5: Anticipating the selection of $\hat{\Gamma}^2(I)$ after having invested $I \geq 0$, a first-period winner will solve

$$\max_{I \geq 0} \int\limits_C \hat{\Pi}_w^2(c) g(c, I) dc - \Psi(I) \Leftrightarrow \max_{I \geq 0} \int\limits_C \hat{Q}_w^2(c) G(c, I) dc - \Psi(I).$$

Notice that under $\hat{\Gamma}^2(I)$, $\hat{Q}^2_w(c) = [1 - F(J^{-1}(J_I(c)))]^{n-1}$, which concludes the characterization result. To conclude, notice that

$$\begin{aligned}\frac{d}{dI}\int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1} G(c, I)dc &< \int\limits_C [1 - F(J^{-1}(J_I(c)))]^{n-1}\frac{\partial G}{\partial I}(c, I)dc \\ &< \int\limits_C [1 - F(c)]^{n-1}\frac{\partial G}{\partial I}(c, I)dc\end{aligned}$$

where in the first inequality we used that $I \mapsto J_I(c)$ and $J(\cdot)$ are strictly increasing, and in the second one that $J^{-1}(J_I(c)) > c$. Hence $\hat{I} < I^e$. □

Proof of Proposition 6: Observe that an equilibrium $(\Gamma(\hat{I}^h), \hat{I}^h)$ exists if and only if there exists a solution to

$$W^h(I) := \int_C [1 - F(J^{-1}(J_I(c)))]^{n-1}\frac{\partial G}{\partial I}(c, I)dc - \Psi'(I) = 0.$$

It is easy to see that $W^h(\cdot)$ is strictly decreasing. Now, using that $F(J^{-1}(J_I(c))) > F(c)$ and the definition of I^e, we have that $W^h(I^e) < 0$. Similarly, using the definition of $\hat{I}^o$ we conclude that $W^h(I^o) > 0$. The fact that $W^h(\cdot)$ is continuous ensure the existence of $\hat{I}^h \in (\hat{I}^o, I^e)$ such that $W^h(\hat{I}^h) = 0$. Finally, the strict monotonicity of $W^h(\cdot)$ guarantees that such $\hat{I}^h$ is unique in $\mathbb{R}_+$. □

Proof of Corollary 1: Observe that because the function $I \mapsto \frac{\partial G}{\partial I}(c, I)$ is decreasing and positive for each $c \in C$, we have that

$$[1 - F(k^{-1}(c))]^{n-1}\frac{\partial G}{\partial I}(c, I^*(n)) < \frac{\partial G}{\partial I}(c, 0),\ \forall c \in C,$$

and the latter function is integrable. Hence, we can apply the Dominated Convergence

Theorem which yields

$$\int_C [1 - F(k^{-1}(c))]^{n-1} \frac{\partial G}{\partial I}(c, I^*(n))dc \to 0, \quad \text{as} \;\; n \to \infty.$$

Using that $\Psi'(I) > 0$ for $I > 0$, we have that $I^*(n)$ must converge to zero. This concludes the proof. $\square$